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General Certificate of Education (A-level)
June 2012

Mathematics MM05

(Specification 6360)

Mechanics 5

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

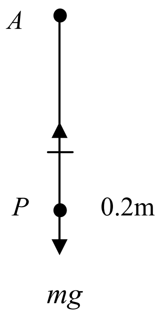
Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

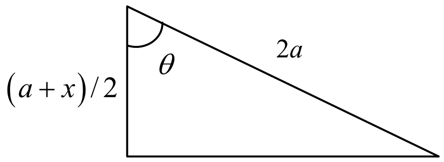
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MM05

Q	Solution	Marks	Total	Comments
1(a)	$T = 2\pi\sqrt{\frac{l}{g}}$			
	$T = 2\pi\sqrt{\frac{0.5}{9.8}}$	M1		
	T = 1.42 s	A1	2	
(b)	$f = \frac{1}{T}$			
	$f = 0.705\ \text{s}^{-1}$	B1F	1	
	Total		**3**	
2(a)				
	$T = 0.4g$			
	$T = 0.2k$	M1		Both, accept use of λ and l
	$0.2k = 0.4g$	A1		
	$k = 2g$			
	$k = 19.6\text{Nm}^{-1}$	A1F	3	
(b)	extension = $0.2 + x$	B1		
(i)	$mg - T = m\ddot{x}$	M1		
	$0.4g - 19.6(x + 0.2) = 0.4\ddot{x}$	A1F		ft stiffness
	$0.4\ddot{x} = -19.6x$			
	$\ddot{x} = -49x$ $(h = -49)$	A1F	4	ft stiffness
(ii)	$\ddot{x} = -\omega^2 x$ SHM (constant < 0)	B1F	1	ft stiffness provided $h < 0$
(iii)	$T = \frac{2\pi}{\omega}$			
	$T = \frac{2\pi}{7}$	M1		
	$T = 0.898$ sec	A1	2	AG
(iv)	max $v = a\omega$			
	$= 0.1 \times 7$	M1		
	$= 0.7\,\text{ms}^{-1}$	A1F	2	ft stiffness provided $h < 0$
	Total		**12**	

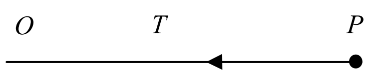
MM05 (cont)

Q	Solution	Marks	Total	Comments
3(a)				
	$2a\cos\theta = (a+x)/2$	M1		
	$4a\cos\theta = a+x$			
	$x = 4a\cos\theta - a$	A1		
	$V = -2\times Wa\cos\theta - 2\times W3a\cos\theta$	M1A1		
	$+\dfrac{2W(4a\cos\theta - a)^2}{2a}$	M1		
	$V = -8Wa\cos\theta + Wa(4\cos\theta - 1)^2$	A1	6	AG
(b)(i)	$\dfrac{dv}{d\theta} = 8Wa\sin\theta + Wa\times 2(4\cos\theta - 1)\times(-4\sin\theta)$	M1A1		
	$= 8Wa\sin\theta(2 - 4\cos\theta)$	M1		$\dfrac{dv}{d\theta} = 0$
	$\dfrac{dv}{d\theta} = 0 \quad \Rightarrow \quad 2 - 4\cos\theta = 0$	M1		solvable form
	$\theta = \dfrac{\pi}{3}$	A1F	5	ft θ in range
(ii)	$\dfrac{d^2v}{d\theta^2} = 8Wa\sin\theta(4\sin\theta) + 8Wa\cos\theta(2 - 4\cos\theta)$	M1 A1F		
	$\theta = \dfrac{\pi}{3} \Rightarrow$			
	$\dfrac{d^2v}{d\theta^2} = 8Wa\dfrac{\sqrt{3}}{2}\times 4\dfrac{\sqrt{3}}{2} + 8Wa\dfrac{1}{2}\times 0$ $= 8Wa\times 3 = 24Wa > 0$			
	minimum $\therefore$ stable	A1F	3	ft one slip
	Total		**14**	

MM05 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$m\ddot{x} = -4mn^2x - 2mk\dot{x}$	M1A1		
	$\ddot{x} + 2k\dot{x} + 4n^2x = 0$	A1	3	AG
(b)(i)	$k = n$			
	$p^2 + 2np + 4n^2 = 0$			
	$(p+n)^2 + 3n^2 = 0$	M1		
	$p = -n \pm n\sqrt{3}\mathrm{i}$	A1		
	$x = \mathrm{e}^{-nt}\left(A\cos\sqrt{3}nt + B\sin\sqrt{3}nt\right)$	A1F		ft provided in correct form
	$t = 0,\ x = a \Rightarrow a = A$	A1		
	$\dot{x} = \mathrm{e}^{-nt}(-\sqrt{3}nA\sin\sqrt{3}nt + \sqrt{3}nB\cos\sqrt{3}nt)$			
	$-n\mathrm{e}^{-nt}(A\cos\sqrt{3}nt + B\sin\sqrt{3}nt)$	m1		
	$t = 0, \dot{x} = 0 \Rightarrow 0 = \sqrt{3}nB - nA$			
	$B = \dfrac{a}{\sqrt{3}} \quad \left(= \dfrac{\sqrt{3}a}{3}\right)$	A1F		
	$x = \mathrm{e}^{-nt}\left(a\cos\sqrt{3}nt + \dfrac{\sqrt{3}a}{3}\sin\sqrt{3}nt\right)$	A1F	7	AG
(ii)	$x = 0,\ \ a\cos\sqrt{3}nt + \dfrac{\sqrt{3}}{3}a\sin\sqrt{3}nt = 0$	M1		Condone verification with $t > 0$
	$(\mathrm{e}^{-nt} \neq 0)$			
	$\tan\sqrt{3}nt = -\sqrt{3}$	M1A1	3	AG
(c)(i)	$k = 2n$			
	$p^2 + 4np + 4n^2 = 0$	M1		
	$(p+2n)^2 = 0$	M1		
	$x = \mathrm{e}^{-2nt}(A + Bt)$	A1	3	
(ii)	critical damping	A1F	1	
	Total		**17**	

MM05 (cont)

Q	Solution	Marks	Total	Comments
5(a)(i)	The only force acting on P is the tension, which is radial	B1	1	
(ii)	$r^2\dot{\theta} = r\left(r\dot{\theta}\right)$	M1		
	$= 3aU$	A1	2	
(b)(i)	$\dot{r} = 0$ (so motion only transverse)	B1		
	$r^2\dot{\theta}$ is constant	M1		used
	$3aU = 4aV$			
	$V = \dfrac{3U}{4}$	A1	3	AG
(ii)	Energy: $\left(\dfrac{\lambda x^2}{2l}\right) = \dfrac{2mg\left(2a\right)^2}{2a}$	M1		
	$+\dfrac{1}{2}mU^2$	M1		
	$= 4mga + \dfrac{1}{2}mU^2$	A1	3	
(iii)	$4mga + \dfrac{1}{2}mU^2 = \dfrac{1}{2}m\left(\dfrac{3U}{4}\right)^2 + \dfrac{2mg(3a)^2}{2a}$	M1A1		ft wrong but dimensionally correct EPE
	$4ga + \dfrac{1}{2}U^2 = \dfrac{9U^2}{32} + 9ag$			
	$\dfrac{7U^2}{32} = 5ag$	m1		
	$U = \sqrt{\dfrac{160ag}{7}}$	A1F	4	
(iv)				
	$\text{T} = m \times \text{acc}^n$			
	$\dfrac{2mg \times 3a}{a} = m \times \text{acc}^n$	M1		
	$\text{acc}^n = 6g$	A1		
	along PO	A1	3	
	Total		**16**	

MM05 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$\frac{\mathrm{d}m}{\mathrm{d}t} = km$			
	$\int_{m_0}^{2m_0} \frac{\mathrm{d}m}{m} = \int_0^T k\,\mathrm{d}t$	M1		set up and separate
	$\left[\ln m\right]_{m_0}^{2m_0} = \left[kt\right]_0^T$	A1		integration
	$\ln\frac{2m_0}{m_0} = kT$	M1		limits or constant
	$T = \frac{1}{k}\ln 2$	A1	4	AG
(b)	$mg\delta t = (m+\delta m)(v+\delta v) - mv$	M1A1		
	$mg\delta t = m\delta v + v\delta m + \delta v\delta m$			
	$mg = m\frac{\mathrm{d}v}{\mathrm{d}t} + v\frac{\mathrm{d}m}{\mathrm{d}t}$	m1		
	$mg = m\frac{\mathrm{d}v}{\mathrm{d}t} + vkm$			
	$\frac{\mathrm{d}v}{\mathrm{d}t} = g - kv$	A1	4	AG
				Alternative for (c):
(c)	$v = \frac{g}{k} - \left(\frac{g}{k} - U\right)\mathrm{e}^{-kt}$			$\frac{\mathrm{d}v}{\mathrm{d}t} = g - kv$
	$\frac{\mathrm{d}v}{\mathrm{d}t} = -\left(\frac{g}{k} - U\right) \times -k\mathrm{e}^{-kt} = \left(\frac{g}{k} - U\right)k\mathrm{e}^{-kt}$	M1		$\int \frac{\mathrm{d}v}{g - kv} = \int \mathrm{d}t$
	Subs:			$-\frac{1}{k}\ln(g - kv) = t + c$ M1
	$\left(\frac{g}{k} - U\right)\mathrm{e}^{-kt} = g - k\left\{\frac{g}{k} - \left(\frac{g}{k} - U\right)\mathrm{e}^{-kt}\right\}$	M1		$t = 0$, $v = U$: $c = -\frac{1}{k}\ln(g - kU)$
	Completion	A1	3	$t = -\frac{1}{k}\ln\left(\frac{g - kv}{g - kU}\right)$ M1
				$\mathrm{e}^{-kt} = \frac{g - kv}{g - kU}$
				$g - kv = (g - kU)\mathrm{e}^{-kt}$
				$v = \frac{g}{k} - \left(\frac{g}{k} - U\right)\mathrm{e}^{-kt}$ A1
(d)	at $2m_0$, $kt = \ln 2$			
	$v = \frac{9.8}{0.7} - \left(\frac{9.8}{0.7} - 2\right) \times \frac{1}{2}$	M1		
	$v = 8$	A1	2	
	Total		**13**	
	TOTAL		**75**	