



General Certificate of Education (A-level)
June 2012

Mathematics

MM04

(Specification 6360)

Mechanics 4

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
✓ or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

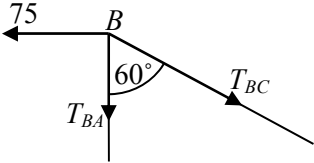
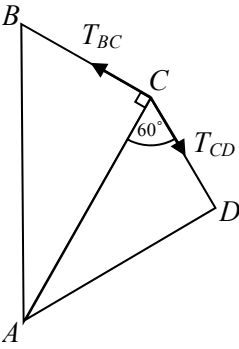
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

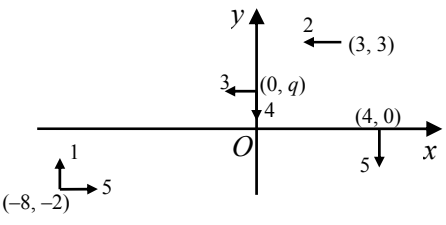
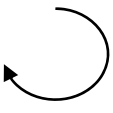
MM04

Q	Solution	Marks	Total	Comments
1(a)	$M = \left(\frac{4-2}{2}, \frac{-1+1}{2}, \frac{4+6}{2} \right) = (1, 0, 5)$	B1		mid-point found
	$\overrightarrow{PM} = -\begin{pmatrix} -2 \\ -1 \\ 4 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$	B1	2	AG
	alternative			
	$\overrightarrow{PQ} = \begin{pmatrix} 4 \\ 1 \\ 6 \end{pmatrix} - \begin{pmatrix} -2 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \\ 2 \end{pmatrix}$	(B1)		
	$\overrightarrow{PM} = \frac{1}{2} \begin{pmatrix} 6 \\ 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$	(B1)	(2)	AG
(b)	Moment = $\mathbf{r} \times \mathbf{F}$			
	$= \begin{vmatrix} \mathbf{i} & 3 & a \\ \mathbf{j} & 1 & 1 \\ \mathbf{k} & 1 & -2 \end{vmatrix}$	M1		attempt at $\mathbf{r} \times \mathbf{F}$ or $\mathbf{F} \times \mathbf{r}$
	$= \begin{pmatrix} -3 \\ a+6 \\ 3-a \end{pmatrix}$	A2,1	3	one component correct $\Rightarrow$ A1 $\mathbf{F} \times \mathbf{r}$ attempt $\Rightarrow$ M1A1A0
(c)	Magnitude = $\sqrt{(-3)^2 + (a+6)^2 + (3-a)^2}$	M1		attempt at magnitude of their moment
	Hence $9 + (a+6)^2 + (3-a)^2 = 50$	A1F		forms equation magnitude ² = 50
	$a^2 + 3a + 2 = 0$	m1		attempts to solve a quadratic – real roots
	$(a+2)(a+1) = 0$			
	$a = -2$ or -1	A1	4	both values obtained; CAO No further penalty for $\mathbf{F} \times \mathbf{r}$ attempt which is correct ie (3, -a-6, a-3) as components
	Total		9	

MM04 (cont)

Q	Solution	Marks	Total	Comments
2(a)	Take moments at A $2lP = \frac{200\sqrt{3}}{3} \left(\frac{3}{2}l \cos 30^\circ \right)$ $P = 75\text{N}$ alternative At B, perpendicular to AB $P = T_{BC} \cos 30^\circ$ At C, parallel to BC $T_{BC} = T_{CD} \cos 30^\circ$ At D, parallel to CD $T_{CD} = \frac{200\sqrt{3}}{3} \cos 30^\circ$ $\Rightarrow P = \frac{200\sqrt{3}}{3} \times (\cos 30^\circ)^3 = 75\text{N}$	M1 A1 A1 (M1) (A1) (A1)	3 (3)	evidence of force $\times$ perpendicular distance correct equation AG Sufficient equations to find P All correct AG
(b)	 At B, resolve horizontally $T_{BC} \cos 30^\circ = 75$ $\Rightarrow T_{BC} = 86.6\text{N}$ BC in tension Resolve vertically $T_{BA} + T_{BC} \cos 60^\circ = 0$ $\Rightarrow T_{BA} = -T_{BC} \cos 60^\circ$ $\Rightarrow T_{BA} = 43.3\text{N}$ BA in compression	M1 A1 E1 M1 A1F E1	6	Equation involving T_{BC} or $50\sqrt{3}$ Equation involving T_{BA} or $25\sqrt{3}$ ft their T_{BC}
(c)	 Resolve perpendicular to AC $T_{BC} = T_{CD} \cos 30^\circ$ $\Rightarrow T_{CD} = \frac{86.6...}{\cos 30^\circ} = 100\text{N}$	M1 A1F	2	Equation involving T_{CD} ft their T_{BC}
(d)	CD in tension AC in compression AD in compression	B2,1	2	B1 two correct B2 all correct
Total			13	

MM04 (cont)

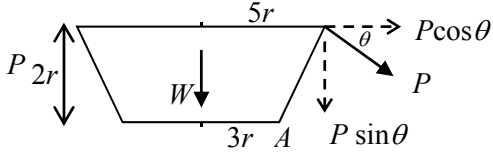
Q	Solution	Marks	Total	Comments
3(a)	$\begin{pmatrix} -2 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -5 \end{pmatrix} + \begin{pmatrix} p \\ 1 \end{pmatrix} + \begin{pmatrix} -3 \\ -4 \end{pmatrix} = \begin{pmatrix} p-5 \\ -8 \end{pmatrix}$	B1	1	
(b)(i)	Parallel to y-axis $\Rightarrow p - 5 = 0$ $p = 5$	M1 A1	2	set i component = 0 (seen or implied)
(ii)	 Moments about O for given system $-5(4) + 2(3) + 3q + 5(2) - 1(8)$ $= 3q - 12$ Moments about O for equivalent system $= -8(3)$ $= -24$ Hence $3q - 12 = -24$ $3q = -12$ $q = -4$	M1 A2,1F B1 M1 A1F	6	$F \times d$ for at least four components -1 each type of error, ft (a), (b)(i) $(12 - 3q)$ scores M1A2) ± 24 seen ft (a) allow $\pm 3 \times$ their j component attempt at moment equation – must see clear use of Force $\times$ distance on RHS ft error with p from (b)(i)
(c)	$ C = 24$ 	B1F B1	2	Should match part (b) – must be positive accept 'clockwise'
Total			11	

MM04 (cont)

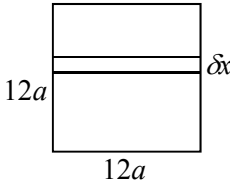
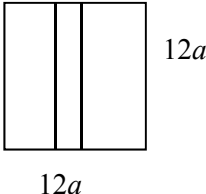
Q	Solution	Marks	Total	Comments
4(a)	$G = \text{mid-point of } BC$ $PG = \sqrt{2}l \text{ or } PG^2 = 2l^2$ $MI_p = MI_G + m PG^2$ $= \frac{ml^2}{3} + m(2l^2)$ $= \frac{7ml^2}{3}$	B1 M1 A1 A1	4	correct distance, seen/used use of parallel axis theorem $\frac{ml^2}{3}$ used AG
(b)	$MI_{\text{particles}} = 3ml^2 + 3ml^2 + 4m(5l^2)$ $= 26ml^2$ $MI_{\text{rods}} = \frac{ml^2}{3} + \frac{7ml^2}{3}$ $= \frac{8ml^2}{3}$ $MI_{\text{system}} = 26ml + \frac{8ml^2}{3}$ $= \frac{86ml^2}{3}$	M1 A1 A1 M1 A1F	5	MI of three particles $3ml^2$ seen use of $5l^2$ with $4m$ MI of two rods (a) + (b) ft error in (a)
(c)	Gain in KE $= \frac{1}{2} I \dot{\theta}^2$ $= \frac{1}{2} \left(\frac{86}{3} ml^2 \right) \dot{\theta}^2$ $= \frac{43}{3} ml^2 \dot{\theta}^2$ Loss in PE for rod BC only $= mgh$ $= 2mgl$ Loss in PE for $4m$ particle $= 4mg(3l)$ $= 12mgl$ Gain for $3m$ particle at A $= \text{loss for } 3m \text{ particle at } B = 3mgl$ (System) total loss of PE $= 14mgl$ $\therefore \frac{43}{3} ml^2 \dot{\theta}^2 = 14mgl$ $\dot{\theta} = \sqrt{\frac{42g}{43l}}$	B1F M1 A1 A1 A1 m1 A1F	7	use of KE formula with MI from (b) use of mgh seen loss for one rod only loss for $4m$ particle total loss for system conservation of energy equation – dependent on use of KE, PE for rod <u>and</u> particles ft error in (a) or (b) Condone $\dot{\theta}^2 = \frac{42g}{43l}$
	Total		16	

Q	Solution	Marks	Total	Comments
4(c)	Alternative 1 PE before motion = $mgl + 4mg(2l)$ $= 9mgl$ PE after motion $= -3mgl - mgl - 4mgl + 3mgl$ $= -5mgl$ KE before = 0 KE after = $\frac{43}{3}ml^2\dot{\theta}^2$ C of E $\Rightarrow 9mgl = \frac{43}{3}ml^2\dot{\theta}^2 - 5mgl$ $\Rightarrow \frac{43}{3}ml^2\dot{\theta}^2 = 14mgl$ $\Rightarrow \dot{\theta} = \sqrt{\frac{42g}{43l}}$ Alternative 2 Centre of mass of system at $(\frac{17}{12}l, \frac{3}{4}l)$ Change in height of centre of mass = $\frac{3}{4}l + (\frac{17}{12}l - l) = \frac{7}{6}l$ Total PE loss = $12mg(\frac{7}{6}l) = 14mgl$ KE gain = $\frac{43}{3}ml^2\dot{\theta}^2$ C of E $\Rightarrow \frac{43}{3}ml^2\dot{\theta}^2 = 14mgl$ $\dot{\theta} = \sqrt{\frac{42g}{43l}}$	(M1) (A1) (A1) (B1F) (M1) (A1) (A1F) (M1) (A1) (B1F) (M1) (A1F)	(7) (7)	<i>mgh</i> used total PE correct total PE correct use of KE formula with MI from (b) attempt at C of E equation correct equation Centre of mass attempted Change in height seen/used <i>mgh</i> used Total loss found use of KE formula with MI from (b) C of E equation formed

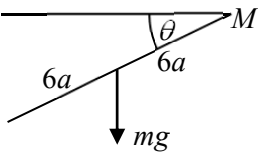
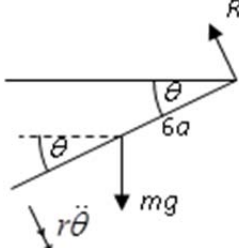
MM04 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$\pi \int xy^2 dx = \pi \int_0^{2r} x(x+3r)^2 dx$ $= \pi \int_0^{2r} (x^3 + 6rx^2 + 9r^2x) dx$ $= \pi \left[\frac{x^4}{4} + 2rx^3 + \frac{9r^2x^2}{2} \right]_0^{2r}$ $= \pi [4r^4 + 16r^4 + 18r^4]$ $= 38r^4\pi$ $\bar{x} = \frac{38r^4\pi}{98\pi r^3/3} = \frac{57r}{49}$	M1 A1 m1 A1 M1 A1F	6	(use of π must be consistent) attempt to integrate $\int xy^2 dx$ - must involve three terms correct integration Limits used correctly correctly evaluated in terms of r^4 use of $\frac{\pi \int xy^2 dx}{\text{volume}}$ ft 'their' $\int xy^2 dx$
(b)(i)	 Moments at A: $W(3r) = P \sin \theta (2r) + P \cos \theta (2r)$ $3W = 2P(\cos \theta + \sin \theta)$ $\frac{3W}{2(\cos \theta + \sin \theta)} = P$	M1A1 A1 A1	4	M1 attempt at moments — evidence of force $\times$ perpendicular distance A1 two terms correct A1 all terms correct AG Must see evidence of factorising
(ii)	Min value of P is when $\cos \theta + \sin \theta$ is at a maximum. Max value of $\cos \theta + \sin \theta$ is $\sqrt{2}$ Max P value = $\frac{3W}{2\sqrt{2}}$	M1 A1 A1	3	Attempt to maximise denominator $\sqrt{2}$ seen Or equiv eg $\frac{6\sqrt{2}W}{8}, \frac{3}{4}\sqrt{2}W$ etc
(iii)	$\theta = 45^\circ$	B1	1	
Total			14	

MM04 (cont)

Q	Solution	Marks	Total	Comments
6(a)	 $m = 144a^2 \rho$ $\Rightarrow \rho = \frac{m}{144a^2}$ Mass of strip $= 12a \delta x \rho$ $MI_{square} = \sum 12a \delta x \rho x^2$ $= \int_0^{12a} 12a x^2 \frac{m}{144a^2} dx$ $= \int_0^{12a} \frac{mx^2}{12a} dx$ $= \left[\frac{mx^3}{36a} \right]_0^{12a}$ $= 48ma^2$ Alternative  $m = 144a^2 \rho \Rightarrow \rho = \frac{m}{144a^2}$ Mass of strip $= 12a \rho \delta x$ MI of strip about end $= \sum \frac{4}{3} (12a \rho \delta x) (6a)^2$ $= \sum 576 \rho a^3 \delta x$ $= \int_0^{12a} \frac{576a^3 m}{144a^2} dx = \int_0^{12a} 4am dx$ $= \left[4amx \right]_0^{12a}$ $= 48ma^2$	B1 M1 A1 M1 A1 (B1) (M1) (A1) (M1) (A1)	5 (5)	seen anywhere – connection between ρ and m Use of $\sum mx^2$ Correct integral formed attempt at integration – must be of the form $\int kx^2 dx$ AG seen anywhere – connection between ρ and m Use of $\frac{4}{3} ml^2$ Correct integral Attempt at integration AG

MM04 (cont)

Q	Solution	Marks	Total	Comments
6(b)(i)	 Using $C = I\dot{\theta}$ $mg \, 6a \cos \theta = 48ma^2 \ddot{\theta}$ $\ddot{\theta} = \frac{g \cos \theta}{8a}$ Alternative PE lost = $6magsin\theta$ KE gained = $\frac{1}{2}(48ma^2)\dot{\theta}^2$ Conservation of energy $\Rightarrow$ $\frac{1}{2}(48ma^2)\dot{\theta}^2 = 6magsin\theta$ $\dot{\theta}^2 = \frac{g \sin \theta}{4a}$ Differentiate $2\dot{\theta}\ddot{\theta} = \frac{g \cos \theta}{4a} \dot{\theta}$ $\Rightarrow \ddot{\theta} = \frac{g \cos \theta}{8a}$	M1 A1 A1	3	Attempt at equation - one side correct both sides correct AG
(b)(ii)	 Using NSL $mg \cos \theta - R = m(6a)\ddot{\theta}$ $R = mg \cos \theta - \frac{6mg}{8} \cos \theta$ $= \frac{mg \cos \theta}{4}$	M1 A1 A1	3	attempt at $F = ma$ fully correct substituting $\ddot{\theta}$ to obtain answer
6(b)(iii)	Consider frictional forces/resistances	E1	1	Any sensible modelling comment
	Total		12	
	TOTAL		75	