Version 1.0

General Certificate of Education (A-level)
June 2011

Mathematics MM04

(Specification 6360)

Mechanics 4

Final

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from: aqa.org.uk

Set and published by the Assessment and Qualifications Alliance.

The Assessment and Qualifications Alliance (AQA) is a company limited by guarantee registered in England and Wales (company number 3644723) and a registered charity (registered charity number 1073334).
Registered address: AQA, Devas Street, Manchester M15 6EX.

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

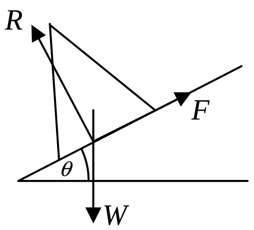
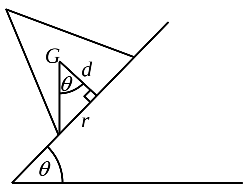
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

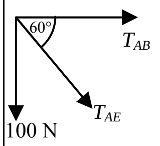
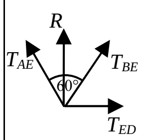
MM04

Q	Solution	Marks	Total	Comments
1(a)	$\begin{pmatrix}-1\\1\\0\end{pmatrix}+\begin{pmatrix}4\\0\\-2\end{pmatrix}+\begin{pmatrix}-3\\-1\\2\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$	B1	1	Clear use of $\sum \mathbf{F}_i = \mathbf{0}$
(b)	$\mathbf{r}_1\times\mathbf{F}_1=\begin{vmatrix}\mathbf{i}&0&-1\\\mathbf{j}&2&1\\\mathbf{k}&1&0\end{vmatrix}=\begin{pmatrix}-1\\-1\\2\end{pmatrix}$	M1		Any attempt at $\mathbf{r}\times\mathbf{F}$ or $\mathbf{F}\times\mathbf{r}$
	$\mathbf{r}_2\times\mathbf{F}_2=\begin{vmatrix}\mathbf{i}&3&4\\\mathbf{j}&-1&0\\\mathbf{k}&0&-2\end{vmatrix}=\begin{pmatrix}2\\6\\4\end{pmatrix}$			
	$\mathbf{r}_3\times\mathbf{F}_3=\begin{vmatrix}\mathbf{i}&4&-3\\\mathbf{j}&0&-1\\\mathbf{k}&-5&2\end{vmatrix}=\begin{pmatrix}-5\\7\\-4\end{pmatrix}$	A3,2,1		-1 each 'type' of error
	$\begin{pmatrix}-1\\-1\\2\end{pmatrix}+\begin{pmatrix}2\\6\\4\end{pmatrix}+\begin{pmatrix}-5\\7\\-4\end{pmatrix}=\begin{pmatrix}-4\\12\\2\end{pmatrix}$	m1 A1F	6	Summing 3 vector product expressions Follow through their three answers for $\mathbf{r}\times\mathbf{F}$ $\mathbf{F}\times\mathbf{r}$ loses final A1
(c)	Resultant force $=\begin{pmatrix}0\\0\\0\end{pmatrix}$ and moment about O is non zero $\Rightarrow$ couple	E2,1	2	E1 $\sum \mathbf{F} = 0$ E1 Moment $\neq 0$
	Total		**9**	

MM04 (cont)

Q	Solution	Marks	Total	Comments
2(a)	Volume of cone $=\frac{1}{3}\pi r^2 h=\frac{8h^3\pi}{3}$	B1		Correct volume
	Alternative: $\left[\pi\int_0^{8h}\left(\frac{1}{8}x\right)^2 dx=\frac{8h^3}{3}\pi\right]$	(B1)		
	$\pi\int xy^2 dx=\pi\int_0^{8h} x\left(\frac{x}{8}\right)^2 dx=\pi\int_0^{8h}\frac{x^3}{64}dx$	M1		Use of $\int xy^2\,dx$
	$=\left[\frac{x^4}{256}\right]_0^{8h}$	A1		Correct integration of kx^3
	$=16h^4$			
	$\bar{x}=\frac{\pi\int xy^2 dx}{\pi\int y^2 dx}=\frac{16h^4\pi}{8h^3\frac{\pi}{3}}$	M1		Attempt at formula for $\bar{x}$
	$=6h$	A1	5	AG
(b)(i)		B1		3 forces
		B1	2	Through a common point at base of cone
(ii)	 G above point of rotation			
	$\tan\theta=\frac{r}{d}$	M1		$\tan\theta$ seen anywhere
	$=\frac{h}{8h-6h}=\frac{1}{2}$	A1		Ratio correct
	$\theta=\tan^{-1}\frac{1}{2}=26.6°$	A1F	3	Use of $\tan^{-1}$ to get θ; ft incorrect ratio
(c)	Resolve perp to plane $N=W\cos\alpha$; parallel to plane $F=W\sin\alpha$	B1		Both correct
	Limiting friction $F=\mu N\Rightarrow\tan\alpha=\frac{6}{13}$	M1		Eliminate W, obtain $\tan\alpha$
	$\alpha=24.8°<26.6°$ or $\frac{6}{13}<\frac{1}{2}$	A1		Comparison
	$\Rightarrow$ slides first	A1F	4	Conclusion; ft (b)(ii)
	Total		**14**	

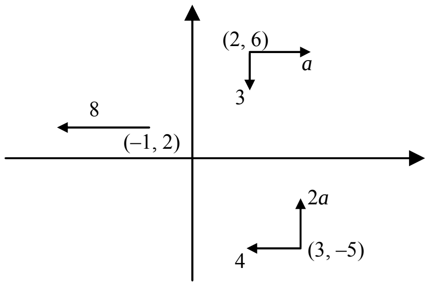
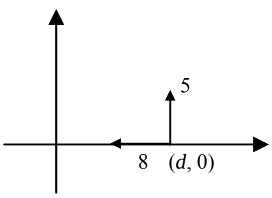
MM04 (cont)

Q	Solution	Marks	Total	Comments
3(a)	System is symmetrical about vertical line through *B*, hence reactions are equal	E1	1	
(b)	Resolve vertically $2R = 250$	M1		Attempt to resolve
	$R = 125$ N	A1	2	
(c)	 At *A* resolve vertically $T_{AE}\sin 60° + 100 = 0$	M1		Attempt at equation involving *AE*
	$\lvert T_{AE}\rvert = \dfrac{100}{\sin 60°} = 115.4(...) = 115\text{N}$	A1		AG $\left[\text{or } \dfrac{200\sqrt{3}}{3} \text{ seen}\right]$
	At *A* resolve horizontally $T_{AB} + T_{AE}\cos 60° = 0$	M1		Attempt at equation involving *AB*
	$\lvert T_{AB}\rvert = \dfrac{100}{\sin 60°}\times\cos 60° = 57.73(...) = 57.7\text{N}$	A1	4	AG $\left[\text{or } \dfrac{100\sqrt{3}}{3} \text{ seen}\right]$
(d)	By symmetry, $T_{BE} = T_{BD} = T$	M1		Use of symmetry
	Resolve vertically at *B*: $2T\cos 30° + 50 = 0$	A1		
	$\lvert T_{BE}\rvert = \dfrac{25}{\cos 30°} = 28.86 = 28.9\text{N}$	A1	3	Accept $\dfrac{50\sqrt{3}}{3}$
	Alternative: At *E* resolve vertically			
	 $R + T_{AE}\cos 30° + T_{BE}\cos 30° = 0$	(M1)		3 term equation involving R, T_{AE}, T_{BE}
	$125 - \dfrac{100}{\sin 60°}\cos 30° + T_{BE}\cos 30° = 0$	(A1)		Correct equation, +/– signs and values correct
	$\lvert T_{BE}\rvert = 28.86 = 28.9\text{N}$	(A1)	(3)	Accept $\dfrac{50\sqrt{3}}{3}$
(e)	At *E* resolve horizontally $T_{ED} + T_{BE}\cos 60° = T_{AE}\cos 60°$	M1		3 term equation including T_{ED}, T_{BE}, T_{AE}
	$\lvert T_{ED}\rvert = \dfrac{25}{\cos 30°}\cos 60° - \dfrac{100}{\sin 60°}\cos 60° =$ 43.3(...) = 43.3N	A1		Correct equation, +/– signs and values correct
	= 43.3N	A1F	3	Follow through T_{AE}; accept $\dfrac{75\sqrt{3}}{3}$
	Total		**13**	

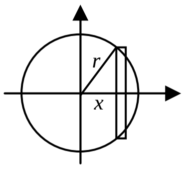
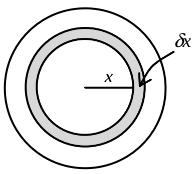
MM04 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$\mathrm{MI}_{\mathrm{DISC}} = \frac{1}{2}mr^2 = \frac{1}{2}(1.5)(0.2)^2$	M1		Use of $\frac{1}{2}mr^2$
	$= 0.03\,\mathrm{kg\,m^2}$	A1	2	AG
(b)(i)	$V = r\omega$ hence $2 = 0.2\omega$	M1		
	$\omega = 10\ \mathrm{rad\,s^{-1}}$	A1	2	Attempt to use $r\omega$
(ii)	KE gained by pulley $= \frac{1}{2}I\omega^2$			
	$= \frac{1}{2}(0.03)(10)^2$	M1		Attempt at KE for pulley
	$= 1.5\,\mathrm{J}$	A1		Correct KE for pulley
	Conservation of energy: Gain $\mathrm{KE_{PULLEY}} + \mathrm{KE_{BUCKET}}$ = loss in PE for bucket			
	$1.5 + \frac{1}{2}(1)(2)^2 = 1gd$	M1		3 term equation
	$d = \frac{5}{14}$	A1	4	AG
(iii)	$g - T = 0.2\ddot{\theta}$	M1A1		Equation of motion for bucket $mg - T = mr\ddot{\theta}$
	$0.2T = 0.03\ddot{\theta}$	M1A1		Equation of motion for pulley $rT = I\ddot{\theta}$ A1 for correct substitutions in each case
	$0.2T = \frac{0.03(9.8 - T)}{0.2}$			
	$T = 4.2\,\mathrm{N}$	A1	5	CAO
	Alternative:			
	$a = r\ddot{\theta} \Rightarrow \ddot{\theta} = \frac{a}{0.2}$	(M1)		Connecting a, $\ddot{\theta}$
	For bucket $u = 0$, $v = 2$, $s = \frac{5}{14}$			
	using $v^2 = u^2 + 2as$ gives $a = \frac{28}{5} = 5.6$			
	$\Rightarrow \ddot{\theta} = \frac{5.6}{0.2} = 28\ \mathrm{rad\,s^{-1}}$	(A1)		
	Either $g - T = 0.2\ddot{\theta}$ or $0.2T = 0.03\ddot{\theta}$	(M1A1)		One other equation
	$T = 4.2\,\mathrm{N}$	(A1)	(5)	CAO
	Total		**13**	

MM04 (cont)

Q	Solution	Marks	Total	Comments
5(a)				
	Moments about O:			
	$-3(2)-a(6)+2a(3)-4(5)+8(2)$	M1 A2,1		One correct $F\times d$ pairing seen –1 each type of error
	$=-6-6a+6a-20+16$			
	$=-10 \Rightarrow$ no 'a' $\Rightarrow$ independent	A1		'a' cancels / comment about no 'a'
	Magnitude = 10	A1F	5	Must be positive value stated; ft single slip
	Alternative:			
	$\mathbf{r}_1\times\mathbf{F}_1=\begin{vmatrix}\mathbf{i} & -1 & -8\\ \mathbf{j} & 2 & 0\\ \mathbf{k} & 0 & 0\end{vmatrix}=\begin{pmatrix}0\\0\\16\end{pmatrix}$	(M1)		One correct $\mathbf{r}\times\mathbf{F}$ seen
	$\mathbf{r}_2\times\mathbf{F}_2=\begin{vmatrix}\mathbf{i} & 2 & a\\ \mathbf{j} & 6 & -3\\ \mathbf{k} & 0 & 0\end{vmatrix}=\begin{pmatrix}0\\0\\-6-6a\end{pmatrix}$	(A1)		Second $\mathbf{r}\times\mathbf{F}$ correct
	$\mathbf{r}_3\times\mathbf{F}_3=\begin{vmatrix}\mathbf{i} & 3 & -4\\ \mathbf{j} & -5 & 2a\\ \mathbf{k} & 0 & 0\end{vmatrix}=\begin{pmatrix}0\\0\\6a-20\end{pmatrix}$			
	Total moment $=16-6-6a+6a-20$	(A1)		$\sum$ (k components)
	$=-10$ 'a' cancels / no 'a'	(A1)		Comment about 'a'
	Magnitude = 10	(A1F)	(5)	ft single slip
(b)(i)	$\mathbf{F}=\begin{pmatrix}4\\-3\end{pmatrix}+\begin{pmatrix}-4\\8\end{pmatrix}+\begin{pmatrix}-8\\0\end{pmatrix}=\begin{pmatrix}-8\\5\end{pmatrix}$	M1 A1	2	Substitute a = 4 and add forces
(ii)	 Moments about O: $-10 = 5d$	M1		y-component × d = answer (a)
	$d=-2$	A1F	2	ft (a)
	Total		**9**	

MM04 (cont)

Q	Solution	Marks	Total	Comments
6(a)	 Let density = ρ $m=\frac{4}{3}\pi r^3\rho \Rightarrow \rho=\frac{3m}{4\pi r^3}$	B1		ρ, m linked anywhere
	Mass of elemental disc / cylinder $=\pi\left(r^2-x^2\right)\delta x\rho$	M1		Attempt to use $\pi r^2 h\rho$
	MI of elemental disc = $\frac{1}{2}\left[\pi\left(r^2-x^2\right)\delta x\rho\right]\left[r^2-x^2\right]$	A1		Use of $\frac{1}{2}mr^2$
	MI of sphere = $\int_{-r}^{r}\frac{1}{2}\pi\rho\left(r^2-x^2\right)^2\,\mathrm{d}x$			
	$=\frac{3m}{8r^3}\int_{-r}^{r}\left(r^4+x^4-2r^2x^2\right)\mathrm{d}x$	m1		Attempt at integral, dep on first M1
	$=\frac{3m}{8r^3}\left[r^4x+\frac{x^5}{5}-2r^2\frac{x^3}{3}\right]_{-r}^{r}$	A1F		Their expression integrated correctly – correct number of terms only
	$=\frac{3m}{8r^3}\left[2\left(r^5+\frac{r^5}{5}-\frac{2r^5}{3}\right)\right]$			Correct limit used, ρ replaced to obtain answer given
	$=\frac{3m}{4r^3}\times\frac{8r^5}{15}=\frac{2}{5}mr^2$	A1	6	AG
	Alternative:			
	 Let density $=\rho$ $m=4\pi r^3\rho$ $\Rightarrow\rho=\frac{3m}{4\pi r^3}$	(B1)		ρ, m linked anywhere
	Mass of elemental shell $=4\pi x^2\delta x\rho$	(M1)		Attempt to use surface area of sphere $\times\,\delta x\times\rho$
	MI of elemental shell about diameter $=\frac{2}{3}\left(4\pi x^2\delta x\rho\right)x^2$	(A1)		Use of $\frac{2}{3}mr^2$
	MI of sphere $=\int_0^r\frac{8}{3}\pi x^4\rho\,\mathrm{d}x$			
	$=\frac{2m}{r^3}\int_0^r x^4\,\mathrm{d}x$	(m1)		Attempt at integral; dep on first M1
	$=\frac{2m}{r^3}\left[\frac{x^5}{5}\right]_0^r$	(A1F)		Their expression integrated correctly
	$=\frac{2mr^5}{5r^3}$			Correct limits used, ρ replaced
	$=\frac{2mr^2}{5}$	(A1)	(6)	AG

MM04 (cont)

Q	Solution	Marks	Total	Comments
6(b)(i)	$\text{MI}_{\text{ROD}} = \frac{4}{3}(3m)(2l)^2 = 16ml^2$	B1	1	Use of $\frac{4}{3}ml^2$ with 'm' = $3m$ and 'l' = $2l$
(ii)	$\text{MI}_{\text{SPHERE (about } G)} = \frac{2}{5}(5m)(l)^2 = 2ml^2$	B1		Use of $\frac{2}{5}ml^2$ with 'm' = $5m$ and 'l' = l
	$\text{MI}_{\text{SPHERE (about } P)} = 2ml^2 + 5m(5l)^2$	M1		Use of parallel axis theorem
	$= 127ml^2$	A1		
	$\text{I}_{\text{PENDULUM}} = 127ml^2 + 16ml^2 = 143ml^2$	A1	4	AG
(iii)	Angular momentum of clay before collision = $mv(3l) = 3mvl$	B1		
	Angular momentum for pendulum after collision $= \text{I}_1\omega = 143ml^2\omega$	B1		Correct for pendulum
	Angular momentum for clay after collision = $\text{I}_2\omega = m(3l)^2\omega$	B1		Correct for clay
	Conservation of angular momentum: $3mvl = 143ml^2\omega + 9ml^2\omega$	M1A1		Attempt at conservation of momentum
	$3mvl = 152ml^2\omega$			
	$\omega = \frac{3v}{152l}$	A1F	6	ft one slip
	Total		**17**	
	TOTAL		**75**	