



General Certificate of Education

Mathematics 6360

MM04 Mechanics 4

Mark Scheme

2007 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available to download from the AQA Website: www.aqa.org.uk

Copyright © 2007 AQA and its licensors. All rights reserved.

COPYRIGHT

AQA retains the copyright on all its publications. However, registered centres for AQA are permitted to copy material from this booklet for their own internal use, with the following important exception: AQA cannot give permission to centres to photocopy any material that is acknowledged to a third party even for internal use within the centre.

Set and published by the Assessment and Qualifications Alliance.

Key to mark scheme and abbreviations used in marking

| | | | |
|--------------|--|-----|----------------------------|
| M | mark is for method | | |
| m or dM | mark is dependent on one or more M marks and is for method | | |
| A | mark is dependent on M or m marks and is for accuracy | | |
| B | mark is independent of M or m marks and is for method and accuracy | | |
| E | mark is for explanation | | |
| ✓ or ft or F | follow through from previous incorrect result | MC | mis-copy |
| CAO | correct answer only | MR | mis-read |
| CSO | correct solution only | RA | required accuracy |
| AWFW | anything which falls within | FW | further work |
| AWRT | anything which rounds to | ISW | ignore subsequent work |
| ACF | any correct form | FIW | from incorrect work |
| AG | answer given | BOD | given benefit of doubt |
| SC | special case | WR | work replaced by candidate |
| OE | or equivalent | FB | formulae book |
| A2,1 | 2 or 1 (or 0) accuracy marks | NOS | not on scheme |
| -x EE | deduct x marks for each error | G | graph |
| NMS | no method shown | c | candidate |
| PI | possibly implied | sf | significant figure(s) |
| SCA | substantially correct approach | dp | decimal place(s) |

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

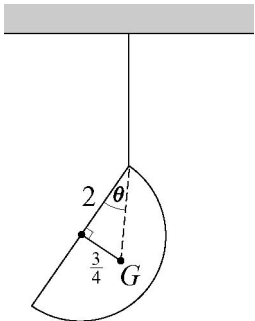
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

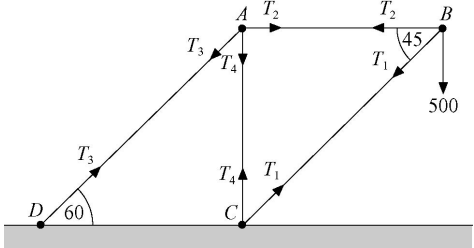
MM04

| Q | Solution | Mark | Total | Comments |
|---------|--|----------------------|-----------|---|
| 1(a)(i) | $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ -3 \\ 5 \end{pmatrix} + \mathbf{F} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ | M1 | | sum of forces = 0 must be seen for M1 |
| | $\Rightarrow \mathbf{F} = \begin{pmatrix} -5 \\ 1 \\ -8 \end{pmatrix}$ | B1 A1 | 3 | $\pm (5\mathbf{i} - \mathbf{j} + 8\mathbf{k})$ seen correct sign |
| | (ii) $ \mathbf{F} = \sqrt{5^2 + 1^2 + 8^2} = \sqrt{90} = 3\sqrt{10}$ | M1 A1 | 2 | $\sqrt{\text{their } \mathbf{F} \text{ components}}$ AG |
| | (b) Moment = $\mathbf{r} \times \mathbf{F}$ | | | |
| | $= \begin{vmatrix} \mathbf{i} & 1 & 1 \\ \mathbf{j} & -1 & 2 \\ \mathbf{k} & 6 & 3 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & 0 & 4 \\ \mathbf{j} & 3 & -3 \\ \mathbf{k} & -2 & 5 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & 0 & -5 \\ \mathbf{j} & 3 & 1 \\ \mathbf{k} & -2 & -8 \end{vmatrix}$ | M1 M1 | | attempt at one $\mathbf{r} \times \mathbf{F}$ (all attempted) |
| | $= \begin{pmatrix} -15 \\ 3 \\ 3 \end{pmatrix} + \begin{pmatrix} 9 \\ -8 \\ -12 \end{pmatrix} + \begin{pmatrix} -22 \\ 10 \\ 15 \end{pmatrix}$ | A1✓ A1✓ | | any three components correct all components correct |
| | $= \begin{pmatrix} -28 \\ 5 \\ 6 \end{pmatrix}$ | A1✓ | 5 | sum of vectors; ✓ their \mathbf{F} from part (a) |
| | 1 st Alternative for (b): | | | |
| | $\overrightarrow{QP} = \begin{pmatrix} 1 \\ -4 \\ 8 \end{pmatrix}$ | (M1) (A1) | | intention to use $\mathbf{r} \times \mathbf{F}$ about Q \overrightarrow{QP} obtained correctly |
| | Moments about Q | | | |
| | $QP \times \mathbf{F}_1 = \begin{vmatrix} \mathbf{i} & 1 & 1 \\ \mathbf{j} & -4 & 2 \\ \mathbf{k} & 8 & 3 \end{vmatrix} = \begin{pmatrix} -28 \\ 5 \\ 6 \end{pmatrix}$ | (M1) (A1) (A1) | (5) | determinant attempted one component correct all correct |
| | 2 nd Alternative for (b): | | | |
| | $\overrightarrow{PQ} = \begin{pmatrix} -1 \\ 4 \\ -8 \end{pmatrix}$ | (M1) (A1) | | intention to use $\mathbf{r} \times \mathbf{F}$ about P \overrightarrow{PQ} obtained correctly |
| | $\begin{vmatrix} \mathbf{i} & -1 & -5 \\ \mathbf{j} & 4 & 1 \\ \mathbf{k} & -8 & -8 \end{vmatrix} = \begin{pmatrix} -24 \\ 32 \\ 19 \end{pmatrix}$ | (M1) | | one determinant correct |
| | $\begin{vmatrix} \mathbf{i} & -1 & 4 \\ \mathbf{j} & 4 & -3 \\ \mathbf{k} & -8 & 5 \end{vmatrix} = \begin{pmatrix} -4 \\ -27 \\ -13 \end{pmatrix}$ | (A1) | | both correct |
| | $\begin{pmatrix} -24 \\ 32 \\ 19 \end{pmatrix} + \begin{pmatrix} -4 \\ -27 \\ -13 \end{pmatrix} = \begin{pmatrix} -28 \\ 5 \\ 6 \end{pmatrix}$ | (A1) | (5) | all correct |
| | Total | | 10 | |

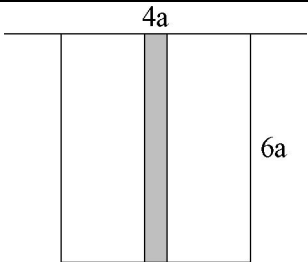
MM04 (cont)

| Q | Solution | Mark | Total | Comments |
|-------|--|---|-------|---|
| 2(a) | $\text{volume} = \pi \int y^2 dx$ $= \pi \int_0^2 (4 - x^2) dx$ $= \pi \left[4x - \frac{x^3}{3} \right]_0^2$ $= \pi \left[8 - \frac{8}{3} - 0 \right]$ $= \frac{16\pi}{3}$ | <p>M1</p> <p>A1</p> <p>A1</p> | 3 | <p>evidence of attempt at $\int y^2 dx$</p> <p>integrating</p> <p>AG</p> |
| (b) | $\frac{16\pi}{3} \bar{x} = \pi \int_0^2 x(4 - x^2) dx$ $= \pi \int_0^2 (4x - x^3) dx$ $= \pi \left[2x^2 - \frac{x^4}{4} \right]_0^2$ $= \pi [8 - 4 - 0]$ $= 4\pi$ $\Rightarrow \bar{x} = \frac{3}{4}$ | <p>M1</p> <p>A1</p> <p>m1</p> <p>A1</p> | 4 | <p>attempt at $\int xy^2 dx$</p> <p>integrating correctly</p> <p>equation to find \bar{x} (dependent on first M1)</p> |
| (c) |  $\tan \theta = \frac{\frac{3}{4}}{2}$ $= \frac{3}{8}$ $\Rightarrow \theta = 20.6^\circ$ | <p>M1</p> <p>A1✓</p> <p>A1✓</p> | 3 | <p>$\tan \theta$ seen</p> <p>structure correct $\frac{\bar{x}}{2}$</p> <p>accept AWWF 20° – 21°; ✓ their \bar{x}</p> |
| Total | | | 10 | |

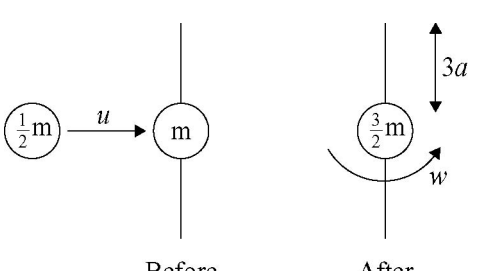
MM04 (cont)

| Q | Solution | Mark | Total | Comments |
|---------|---|---|-----------|--|
| 3(a)(i) |  <p>Resolve vertically at B:</p> $T_1 \sin 45^\circ + 500 = 0$ $\Rightarrow T_1 = \frac{-500}{\sin 45^\circ} = -500\sqrt{2} \text{ or } -707 \text{ N}$ <p>[magnitude = 707 N]</p> <p>Resolve horizontally at B:</p> $T_2 + T_1 \cos 45^\circ = 0$ $\Rightarrow T_2 = -T_1 \cos 45^\circ = 500 \text{ N}$ <p>Resolve horizontally at A:</p> $T_2 = T_3 \sin 30^\circ$ $\Rightarrow T_3 = \frac{T_2}{\sin 30^\circ} = 1000 \text{ N}$ | <p>M1A1</p> <p>M1A1 A1✓</p> <p>M1A1 A1✓</p> | <p>9</p> | <p>forces can be marked as tensions and/or compressions; signs must be consistent NB if moments are used, reaction forces at C, D must be identified for first M1</p> <p>✓ their T_1</p> <p>✓ their T_2</p> |
| (ii) | AD and AB are in tension and could be replaced by ropes. BC is in thrust and cannot be replaced by ropes. | B1 B1 E1 | 3 | identification of AD/AB identification of BC (can be implied) reference to tension/thrust |
| (b) | magnitude = $T_3 = 1000 \text{ N}$ | B1✓ | 1 | |
| | Total | | 13 | |

MM04 (cont)

| Q | Solution | Mark | Total | Comments |
|--------------|---|--|-----------|---|
| 4(a) | On point of toppling, take moments about bottom right corner $W(2a) = P \cos \theta (8a)$ $P = \frac{W}{4 \cos \theta}$ | M1 A1,A1 A1 | 4 | attempt at moments A1 each side |
| (b) | On point of sliding vertically, $N + P \sin \theta = W$ horizontally, $F = P \cos \theta$ friction $F = \mu N$ $\Rightarrow P \cos \theta = \mu(W - P \sin \theta)$ $P \cos \theta = \mu W - \mu P \sin \theta$ $P(\cos \theta + \mu \sin \theta) = \mu W$ $P = \frac{\mu W}{\cos \theta + \mu \sin \theta}$ | M1A1 M1A1 M1A1 A1 | 7 | substitute; use of $F = \mu N$ AG |
| (c) | Slides before topples \Rightarrow $\frac{\mu W}{\cos \theta + \mu \sin \theta} < \frac{W}{4 \cos \theta}$ $4\mu \cos \theta < \cos \theta + \mu \sin \theta$ $4\mu < 1 + \mu \tan \theta$ $\tan \theta = 1 \Rightarrow 3\mu < 1$ $\mu < \frac{1}{3}$ | M1 A1 A1 M1 A1 | 5 | inequality formed elimination of fractions / cancel W \div by $\cos \theta$ and use of $\tan \theta = 1$ collect μ terms |
| Total | | | 16 | |
| 5(a) |  <p>mass = $m = 24a^2 \rho$ $\therefore \rho = \frac{m}{24a^2}$ Mass of strip = $6a \delta x \rho$ MI of rectangle $= \sum \frac{4}{3} (6a \delta x \rho) (3a)^2 = \sum 72a^3 \rho \delta x$ $= \int_0^{4a} 72a^3 \frac{m}{24a^2} dx$ $= [3max]_0^{4a} = 12ma^2$</p> | B1 M1 A1 m1 A1 | 5 | use of area \times density use of $\frac{4}{3}ml^2$ m, l correct integrating - dependent on first M1 AG |

MM04 (cont)

| Q | Solution | Mark | Total | Comments |
|-----|--|---|-----------|--|
| 5 | <p>Alternative for (a):</p> $\rho = \frac{m}{24a^2}$ <p>Mass of strip = $4a\delta x\rho$</p> <p>MI of rectangle = $\sum (4a\delta x\rho)x^2$</p> $= \int_0^{6a} 4a \frac{m}{24a^2} x^2 dx$ $= \left[\frac{mx^3}{18a} \right]_0^{6a} = 12ma^2$ | <p>(B1)</p> <p>(M1)</p> <p>(m1)</p> <p>(A1, A1)</p> | (5) | <p>use of mx^2</p> <p>integration attempt</p> <p>AG</p> |
| (b) |  <p>Before</p> <p>After</p> <p>angular momentum before</p> $= \frac{1}{2}mu(3a) = \frac{3mua}{2}$ <p>angular momentum after</p> $= Iw + \frac{1}{2}m(3a)^2 w$ $= 12ma^2 w + \frac{9ma^2}{2} w$ $= \frac{33ma^2 w}{2}$ <p>use C of momentum to set</p> $\frac{3mua}{2} = \frac{33ma^2 w}{2}$ $\Rightarrow w = \frac{u}{11a}$ | <p>M1A1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A1</p> | 7 | <p>'ka' required for M1</p> <p>either term correct both correct</p> <p>use of $I = 12ma^2$ anywhere</p> <p>equation – C of m ('their' expression)</p> |
| | Total | | 12 | |

MM04 (cont)

| Q | Solution | Mark | Total | Comments |
|------|---|--|-----------|--|
| 6(a) | | | | |
| (i) | KE = $\frac{1}{2}(4m)(a\dot{\theta})^2 + \frac{1}{2}(2m)(a\dot{\theta})^2 + \frac{1}{2}(10ma^2)\dot{\theta}^2$ $= 2ma^2\dot{\theta}^2 + ma^2\dot{\theta}^2 + 5ma^2\dot{\theta}^2$ $= 8ma^2\dot{\theta}^2$ | B1 B1 M1 A1 | 4 | $a\dot{\theta}$ used disc KE particles KE AG |
| (ii) | PE lost = $4mga\theta - 2mga\theta$ $= 2mga\theta$ C of E $\Rightarrow 8ma^2\dot{\theta}^2 = 2mga\theta$ $a\dot{\theta}^2 = \frac{g\theta}{4}$ | B1 M1 A1 | 3 | PE seen - any term C of E AG |
| (b) | differentiating $2a\dot{\theta}\ddot{\theta} = \frac{g\dot{\theta}}{4}$ $\Rightarrow a\ddot{\theta} = \frac{g}{8}$ For P, $T - 2mg = 2ma\ddot{\theta} \Rightarrow T = 2mg + \frac{mg}{4} = \frac{9mg}{4}$ For Q, $4mg - S = 4ma\ddot{\theta} \Rightarrow S = 4mg - \frac{mg}{2} = \frac{7mg}{2}$ | M1 A1 M1 A1 M1A1 A1 | 7 | equation for P for $\frac{9mg}{4}$ equation for Q for $\frac{7mg}{2}$ |
| | Alternative for (b): Use $C = I\ddot{\theta}$ for disc $Sa - Ta = 10ma^2\ddot{\theta}$ $\Rightarrow S - T = 10ma\ddot{\theta}$ For P, $T - 2mg = 2ma\ddot{\theta}$ For Q, $4mg - S = 4ma\ddot{\theta}$ Solving For T For S | (M1) (A1) (M1) (M1) (M1) (A1) (A1) | (7) | M1 for LHS attempt RHS correct |
| | Total | | 14 | |
| | TOTAL | | 75 | |