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General Certificate of Education Advanced Level Examination June 2011

# **Mathematics**

**MM03** 

**Unit Mechanics 3** 

Wednesday 22 June 2011 9.00 am to 10.30 am

# For this paper you must have:

• the blue AQA booklet of formulae and statistical tables. You may use a graphics calculator.

#### Time allowed

• 1 hour 30 minutes

## Instructions

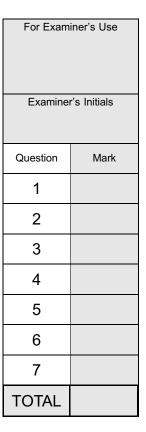
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer all questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take  $g = 9.8 \text{ m s}^{-2}$ , unless stated otherwise.

#### Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

#### **Advice**

 Unless stated otherwise, you may quote formulae, without proof, from the booklet.





		Answer all questions in the spaces provided.	
1		A ball of mass $0.2\mathrm{kg}$ is hit directly by a bat. Just before the impact, the is travelling horizontally with speed $18\mathrm{ms^{-1}}$ . Just after the impact, the travelling horizontally with speed $32\mathrm{ms^{-1}}$ in the opposite direction.	
(a	1)	Find the magnitude of the impulse exerted on the ball.	(2 marks)
(b)		At time $t$ seconds after the ball first comes into contact with the bat, the exerted by the bat on the ball is $k(0.9t-10t^2)$ newtons, where $k$ is a contact $0 \le t \le 0.09$ . The bat stays in contact with the ball for 0.09 seconds.	
		Find the value of $k$ .	(4 marks)
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2	The time, $t$ , for a single vibration of a piece of taut string is believed to depend on
	the length of the taut string, $l$ , the tension in the string, $F$ , the mass per unit length of the string, $q$ , and a dimensionless constant, $k$ ,
	such that
	$t = k l^{\alpha} F^{\beta} q^{\gamma}$
	where $\alpha$ , $\beta$ and $\gamma$ are constants.
	By using dimensional analysis, find the values of $\alpha$ , $\beta$ and $\gamma$ . (5 marks)
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3 (In this question, use  $g = 10 \,\mathrm{m \, s^{-2}}$ .)

A golf ball is hit from a point O on a horizontal golf course with a velocity of  $40\,\mathrm{m\,s^{-1}}$  at an angle of elevation  $\theta$ . The golf ball travels in a vertical plane through O. During its flight, the horizontal and upward vertical distances of the golf ball from O are x and y metres respectively.

(a) Show that the equation of the trajectory of the golf ball during its flight is given by

$$x^{2} \tan^{2} \theta - 320x \tan \theta + (x^{2} + 320y) = 0$$
 (6 marks)

**(b) (i)** The golf ball hits the top of a tree, which has a vertical height of 8 m and is at a horizontal distance of 150 m from O.

Find the two possible values of  $\theta$ .

(5 marks)

(ii) Which value of  $\theta$  gives the shortest possible time for the golf ball to travel from O to the top of the tree? Give a reason for your choice of  $\theta$ . (2 marks)

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4		The unit vectors ${\bf i}$ , ${\bf j}$ and ${\bf k}$ are directed due east, due north and vertically respectively.	upwards
		A helicopter, $A$ , is travelling in the direction of the vector $-2\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$ constant speed $140 \mathrm{km} \mathrm{h}^{-1}$ . Another helicopter, $B$ , is travelling in the difference $2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ with constant speed $60 \mathrm{km} \mathrm{h}^{-1}$ .	
(a	)	Find the velocity of $A$ relative to $B$ .	(5 marks)
(b)		Initially, the position vectors of $A$ and $B$ are $(4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$ km and $(-3\mathbf{i} + 6\mathbf{j} + 3\mathbf{k})$ km respectively, relative to a fixed origin.	
		Write down the position vector of $A$ relative to $B$ , $t$ hours after they leave positions.	e their initial (2 marks)
(с	)	Find the distance between $A$ and $B$ when they are closest together.	(8 marks)
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5		A ball is dropped from a height of 2.5 m above a horizontal floor. The ball repeatedly on the floor.	bounces
(a	)	Find the speed of the ball when it first hits the floor.	(2 marks)
(b	)	The coefficient of restitution between the ball and the floor is $e$ .	
	(i)	Show that the time taken between the first contact of the ball with the floor second contact of the ball with the floor is $\frac{10e}{7}$ seconds.	and the (3 marks)
	(ii)	Find, in terms of $e$ , the time taken between the second contact and the third of the ball with the floor.	contact (1 mark)
(c)		Find, in terms of $e$ , the total vertical distance travelled by the ball from when dropped until its third contact with the floor.	n it is (5 marks)
(d	)	State a modelling assumption for answering this question, other than the ball particle.	being a (1 mark)
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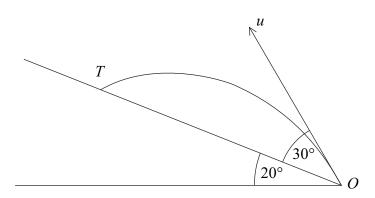


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A projectile is fired from a point O on a plane which is inclined at an angle of  $20^{\circ}$  to the horizontal. The projectile is fired up the plane with velocity  $u \, \text{m s}^{-1}$  at an angle of  $30^{\circ}$  to the inclined plane. The projectile travels in a vertical plane containing a line of greatest slope of the inclined plane.

The projectile hits a target T on the inclined plane.



- (a) Given that  $OT = 200 \,\mathrm{m}$ , determine the value of u. (7 marks)
- (b) Find the greatest perpendicular distance of the projectile from the inclined plane.

  (4 marks)

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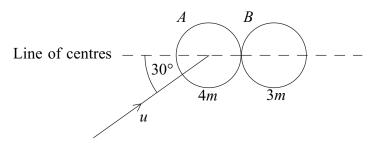
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7 Two smooth spheres, A and B, have equal radii and masses 4m and 3m respectively. The sphere A is moving on a smooth horizontal surface and collides with the sphere B, which is stationary on the same surface.

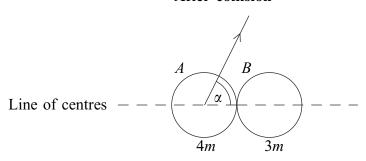
Just before the collision, A is moving with speed u at an angle of 30° to the line of centres, as shown in the diagram below.

## **Before collision**



Immediately after the collision, the direction of motion of A makes an angle  $\alpha$  with the line of centres, as shown in the diagram below.

## After collision



The coefficient of restitution between the spheres is  $\frac{5}{9}$ .

- (a) Find the value of  $\alpha$ . (10 marks)
- (b) Find, in terms of m and u, the magnitude of the impulse exerted on B during the collision. (3 marks)



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