Centre Number						Candidate Number				
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Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
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5	
6	
7	
TOTAL	

General Certificate of Education
Advanced Level Examination
June 2010

Mathematics MM03

Unit Mechanics 3

Tuesday 22 June 2010 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\mathrm{m\,s^{-2}}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions in the spaces provided.

1 A tank containing a liquid has a small hole in the bottom through which the liquid escapes. The speed, $u\,\mathrm{m\,s^{-1}}$, at which the liquid escapes is given by

$$u = CV\rho g$$

where $V\ \mathrm{m}^3$ is the volume of the liquid in the tank, $\rho\,\mathrm{kg\,m^{-3}}$ is the density of the liquid, g is the acceleration due to gravity and C is a constant.

By using dimensional analysis, find the dimensions of C. *(5 marks)*

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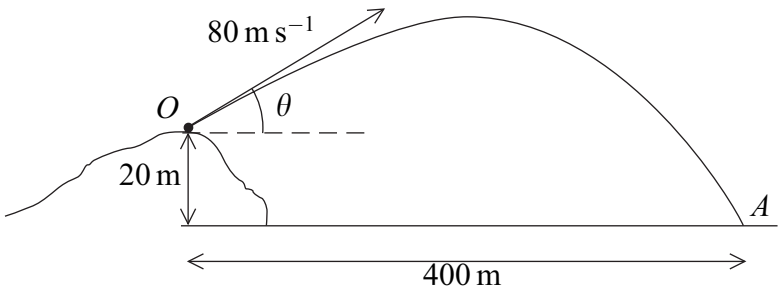
2 A projectile is fired from a point O on top of a hill with initial velocity $80\,\text{m s}^{-1}$ at an angle θ above the horizontal and moves in a vertical plane. The horizontal and upward vertical distances of the projectile from O are x metres and y metres respectively.

(a) (i) Show that, during the flight, the equation of the trajectory of the projectile is given by

$$y = x\tan\theta - \frac{gx^2}{12\,800}(1 + \tan^2\theta) \qquad \textit{(5 marks)}$$

(ii) The projectile hits a target A, which is 20 m vertically below O and 400 m horizontally from O.



Show that

$$49\tan^2\theta - 160\tan\theta + 41 = 0 \qquad \textit{(2 marks)}$$

(b) (i) Find the two possible values of θ. Give your answers to the nearest 0.1°. *(3 marks)*

(ii) Hence find the shortest possible time of the flight of the projectile from O to A. *(2 marks)*

(c) State a necessary modelling assumption for answering part **(a)(i)**. *(1 mark)*

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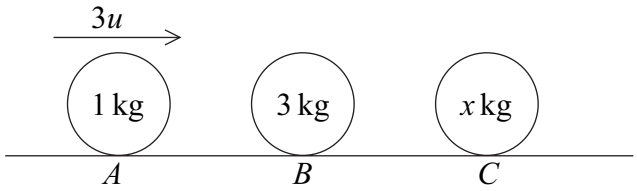
3 Three smooth spheres, A, B and C, of equal radii have masses 1 kg, 3 kg and x kg respectively. The spheres lie at rest in a straight line on a smooth horizontal surface with B between A and C. The sphere A is projected with speed $3u$ directly towards B and collides with it.



The coefficient of restitution between each pair of spheres is $\frac{1}{3}$.

(a) Show that A is brought to rest by the impact and find the speed of B immediately after the collision in terms of u. *(6 marks)*

(b) Subsequently, B collides with C.

Show that the speed of C immediately after the collision is $\frac{4u}{3+x}$.

Find the speed of B immediately after the collision in terms of u and x. *(6 marks)*

(c) Show that B will collide with A again if $x > 9$. *(2 marks)*

(d) Given that $x = 5$, find the magnitude of the impulse exerted on C by B in terms of u. *(2 marks)*

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4 The unit vectors **i**, **j** and **k** are directed east, north and vertically upwards respectively.

At time $t = 0$, the position vectors of two small aeroplanes, A and B, relative to a fixed origin O are $(-60\mathbf{i} + 30\mathbf{k})$ km and $(-40\mathbf{i} + 10\mathbf{j} - 10\mathbf{k})$ km respectively.

The aeroplane A is flying with constant velocity $(250\mathbf{i} + 50\mathbf{j} - 100\mathbf{k})\,\text{km h}^{-1}$ and the aeroplane B is flying with constant velocity $(200\mathbf{i} + 25\mathbf{j} + 50\mathbf{k})\,\text{km h}^{-1}$.

(a) Write down the position vectors of A and B at time t hours. *(3 marks)*

(b) Show that the position vector of A relative to B at time t hours is $((-20 + 50t)\mathbf{i} + (-10 + 25t)\mathbf{j} + (40 - 150t)\mathbf{k})$ km. *(2 marks)*

(c) Show that A and B do not collide. *(4 marks)*

(d) Find the value of t when A and B are closest together. *(6 marks)*

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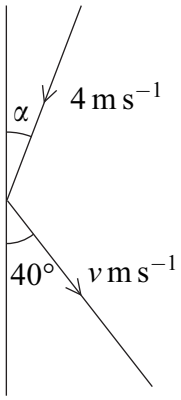
5 A smooth sphere is moving on a smooth horizontal surface when it strikes a smooth vertical wall and rebounds.

Immediately before the impact, the sphere is moving with speed $4\,\text{m s}^{-1}$ and the angle between the sphere's direction of motion and the wall is α.

Immediately after the impact, the sphere is moving with speed $v\,\text{m s}^{-1}$ and the angle between the sphere's direction of motion and the wall is 40°.

The coefficient of restitution between the sphere and the wall is $\frac{2}{3}$.



(a) Show that $\tan\alpha = \frac{3}{2}\tan 40°$. *(3 marks)*

(b) Find the value of v. *(3 marks)*

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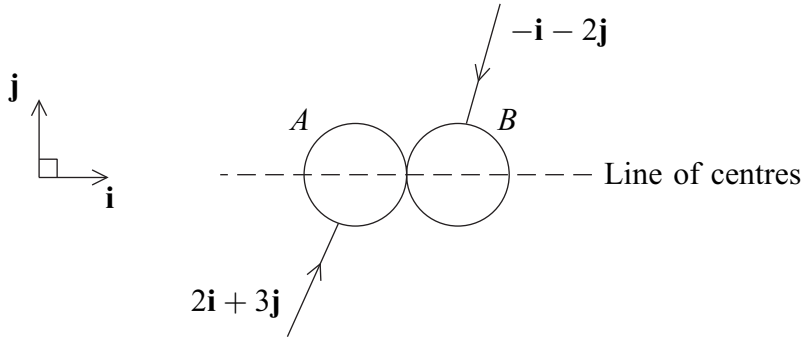
6 Two smooth spheres, A and B, have equal radii and masses 1 kg and 2 kg respectively.

The sphere A is moving with velocity $(2\mathbf{i} + 3\mathbf{j})\,\mathrm{m\,s^{-1}}$ and the sphere B is moving with velocity $(-\mathbf{i} - 2\mathbf{j})\,\mathrm{m\,s^{-1}}$ on the same smooth horizontal surface.

The spheres collide when their line of centres is parallel to the unit vector **i**, as shown in the diagram.



(a) Briefly state why the components of the velocities of A and B parallel to the unit vector **j** are not changed by the collision. *(1 mark)*

(b) The coefficient of restitution between the spheres is 0.5.

Find the velocities of A and B immediately after the collision. *(6 marks)*

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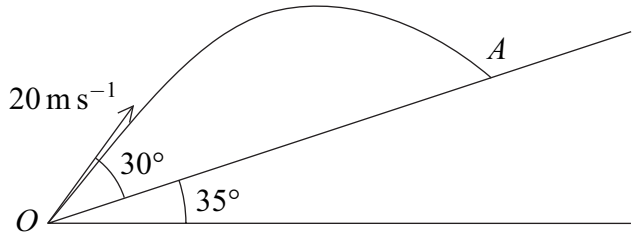
7 A ball is projected from a point O on a smooth plane which is inclined at an angle of 35° above the horizontal. The ball is projected with velocity $20\,\text{m s}^{-1}$ at an angle of 30° above the plane, as shown in the diagram. The motion of the ball is in a vertical plane containing a line of greatest slope of the inclined plane. The ball strikes the inclined plane at the point A.



(a) Find the components of the velocity of the ball, parallel and perpendicular to the plane, as it strikes the inclined plane at A. *(7 marks)*

(b) On striking the plane at A, the ball rebounds. The coefficient of restitution between the plane and the ball is $\frac{4}{5}$.

Show that the ball next strikes the plane at a point lower down than A. *(6 marks)*

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END OF QUESTIONS