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General Certificate of Education (A-level)
June 2012

Mathematics MM03

(Specification 6360)

Mechanics 3

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MM03

Q	Solution	Marks	Total	Comments
1(a)	$I = \int_0^{0.5} 4\times10^4 t^2(1-2t)\ \mathrm{d}t$	M1		Attempt to integrate
		A1		Use of correct limits, PI
	$= 4\times10^4\left[\frac{1}{3}t^3 - \frac{1}{2}t^4\right]_0^{0.5}$	A1F		Correct integration
	$= 417$ (or $\frac{1250}{3}$) Ns	A1F	4	Accept $416.\dot{6}$ or 416.7
(b)	$416.\dot{6} = 60v + 60\times5$	M1A1F		A1F correct sign
	$v = 1.94$	A1F	3	AWRT 1.94, accept 1.95 ISW
	Total		**7**	
2	Dimension of g is LT^{-2} Dimension of s is L Dimension of h is L Dimension of m_1 and m_2 is M	B 1		B1 for dimensions of the five quantities
	Dimension of $\frac{g}{s}[s(m_1+m_2)+\frac{hm_1^2}{m_1+m_2}]$ is			
	$\frac{LT^{-2}}{L}[LM+\frac{LM^2}{M}] \cong MLT^{-2}+MLT^{-2}$	M1		Correct substitution of dimensions
	$\cong MLT^{-2}$	A1		
	which is a force	B1	4	
	Total		**4**	

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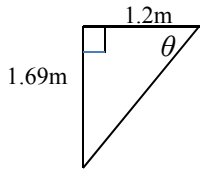
Q	Solution	Marks	Total	Comments
3(a)	$x = ut\cos\alpha$	M1		
	$t = \dfrac{x}{u\cos\alpha}$	A1		
	$y = -\dfrac{1}{2}gt^2 + ut\sin\alpha$	M1		Must have correct signs
	$y = -\dfrac{1}{2}g\left(\dfrac{x}{u\cos\alpha}\right)^2 + u\left(\dfrac{x}{u\cos\alpha}\right)\sin\alpha$	M1		
	$y = -\dfrac{gx^2}{2u^2\cos^2\alpha} + \dfrac{x\sin\alpha}{\cos\alpha}$			
	$y = -\dfrac{gx^2}{2u^2}(1+\tan^2\alpha) + x\tan\alpha$	A1		
	$k = -\dfrac{10(2k)^2}{2u^2}(1+\tan^2\alpha) + 2k\tan\alpha$	M1		
	$u^2 = -20k(1+\tan^2\alpha) + 2u^2\tan\alpha$			
	$20k\tan^2\alpha - 2u^2\tan\alpha + u^2 + 20k = 0$	A1	7	AG
(b)	Pass through $P \Longrightarrow$ Discriminant ≥ 0			
	$(-2u^2)^2 - 4(20k)(u^2+20k) \geq 0$	M1A1		OE must be seen
	$4u^4 - 80ku^2 - 1600k^2 \geq 0$			
	$u^4 - 20ku^2 - 400k^2 \geq 0$	A1	3	AG
	Total		**10**	

MM03

Q	Solution	Marks	Total	Comments
4(a)				
	$\theta = \tan^{-1}\dfrac{1.69}{1.2} = 54.623^\circ$	B1		AWRT 55°
	$u\cos 60^\circ = v\cos 54.623^\circ$	M1		$v = 0.864u$
	$eu\sin 60^\circ = v\sin 54.623^\circ$	M1		
	$e = \dfrac{v\sin 54.623^\circ}{\dfrac{v\cos 54.623^\circ}{\cos 60^\circ}\times\sin 60^\circ}$	m1		OE, dependent on both M1s
	$e = 0.813$ or 0.812	A1	5	ISW
(b)	$I = 0.15u\sin 60^\circ + 0.15v\sin 54.623^\circ$	M1A1		Single angle values needed for A1
	$= 0.15u\sin 60^\circ + 0.15\times\dfrac{u\cos 60^\circ}{\cos 54.623^\circ}\times\sin 54.623^\circ$	m1		
	$= 0.236u$	A1	4	AG (condone 0.2355 or negative result)
(c)	Attempt at considering motion parallel or perpendicular to AC	M1		
	$t = \dfrac{1.2}{u\cos 60^\circ}$	M1		
	$t = \dfrac{12}{5u}$ or $\dfrac{2.4}{u}$	A1	3	OE, No ISW
	Alternative:			
	$CP = \dfrac{1.2}{\cos 54.623^\circ}$ $(= 2.072703844$ m)	(M1)		
	$t = \dfrac{\dfrac{1.2}{\cos 54.623^\circ}}{\dfrac{u\cos 60^\circ}{\cos 54.623^\circ}}$	(M1)		
	$= \dfrac{12}{5u}$ or $\dfrac{2.4}{u}$	(A1)	(3)	(OE), No ISW
(d)	Velocity (momentum) parallel to the cushion is unchanged, or, Restitution only affects motion perpendicular to the cushion	E1	1	Accept 'horizontal component of velocity is unchanged'
	Total		**13**	

Q	Solution	Marks	Total	Comments
5(a)	$0 = 15t\sin 30 - \frac{1}{2}g\cos 25t^2$	M1A1		Accept wrong angle(s) for M1 but **not** sin and cos in wrong places
	$t = \frac{15\sin 30}{\frac{1}{2}g\cos 25}$	M1		
	$t = 1.69$ sec.	A1F	4	AWRT 1.69
(b)	$\perp$ to plane $\dot{y} = 15\sin 30 - g\cos 25 \times \frac{15\sin 30}{\frac{1}{2}g\cos 25}$	M1		
	$\dot{y} = -7.5$ ms^{-1}	A1F		Or –7.51, ft from their answer in (a)
	$\parallel$ to plane $\dot{x} = 15\cos 30 - g\sin 25 \times \frac{15\sin 30}{\frac{1}{2}g\cos 25}$	M1		
	$\dot{x} = 5.995766$ or 6.00 ms^{-1}	A1F		Accept 5.99
	Restitution: Rebound $\dot{y} = \frac{2}{3} \times 7.5 = 5$ ms^{-1}	M1		Or 5.01
	$\dot{x}$ unchanged	B1		PI, dependent on the last M1
	Speed of rebound $= \sqrt{5.995766^2 + 5^2}$	m1		Dependent on the last three M1s
	$= 7.81$ ms^{-1}	A1F	8	
	Total		**12**	

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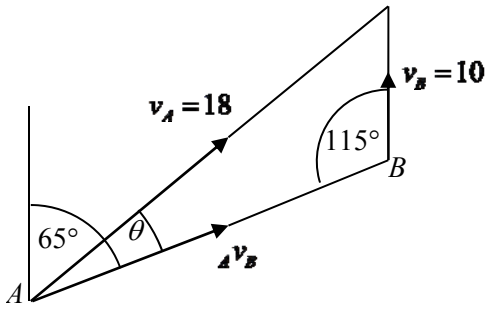
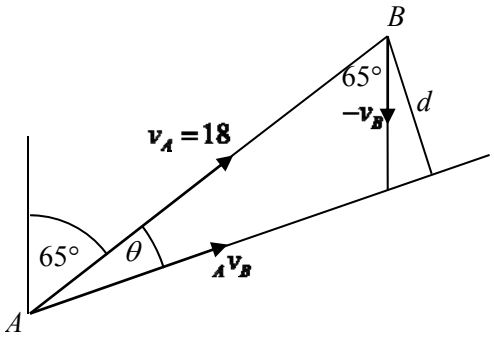
Q	Solution	Marks	Total	Comments
6(a)		B1		For any appropriate diagram PI by correct method
	$\frac{\sin\theta}{10} = \frac{\sin 115^\circ}{18}$	M1		
	$\theta = 30.2^\circ$	A1		
	Bearing = 035°	A1	4	Accept 034.8°
(b)(i)		B1		For any appropriate diagram PI by correct method
	$_Av_B{}^2 = 18^2 + 10^2 - 2(18)(10)\cos 65^\circ$	M1		
	$_Av_B = 16.4881$ ms^{-1}	A1		OE
	$\frac{\sin 65^\circ}{16.4881} = \frac{\sin\theta}{10}$	M1		
	$\theta = 33.3446^\circ$	A1F		
	$d = 12 \times \sin 33.3446^\circ$	m1		OE
	$d = 6.60$ km	A1F	7	Dependent on the previous two M1s (AWRT 6.6 km)
(ii)	$t = \frac{12 \times \cos 33.3446^\circ}{16.4881} = 0.607987$ hours	M1		Or 0.608 hours
		A1F		LHS values
	(= 36.5 min)	A1F	3	Correct time
	Total		**14**	

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Q6 (b)(i) Alternative:

$r_A = [(18\cos 25)\mathbf{i} + (18\sin 25)\mathbf{j}]t$

$r_B = [(12\cos 25)\mathbf{i} + (12\sin 25)\mathbf{j}] + 10\mathbf{j}t$ M1 for both

${}_A r_B = (-12\cos 25 + 18t\cos 25)\mathbf{i} + (-12\sin 25 + 18t\sin 25 - 10t)\mathbf{j}$ m1 A1

$\left|{}_A r_B\right|^2 = (-12\cos 25 + 18t\cos 25)^2 + (-12\sin 25 + 18t\sin 25 - 10t)^2$ A1

$\frac{\mathrm{d}\left|{}_A r_B\right|^2}{\mathrm{d}t} = (36\cos 25)(-12\cos 25 + 18t\cos 25) + (36\sin 25 - 20)(-12\sin 25 + 18t\sin 25 - 10t) = 0$ m1

$t = 0.608$ A1

$d = 6.60$ km or 6.6 km A1

The corresponding marks awarded for finding the closest approach time:

$\frac{\mathrm{d}\left|{}_A r_B\right|^2}{\mathrm{d}t} = (36\cos 25)(-12\cos 25 + 18t\cos 25) + (36\sin 25 - 20)(-12\sin 25 + 18t\sin 25 - 10t) = 0$ M1 A1

$t = 0.608$ (or better) A1

(b)(i) Alternative (**Not in the specification**):

${}_A r_B = (-12\cos 25 + 18t\cos 25)\mathbf{i} + (-12\sin 25 + 18t\sin 25 - 10t)\mathbf{j}$ M1 A1

$[(-12\cos 25 + 18t\cos 25)\mathbf{i} + (-12\sin 25 + 18t\sin 25 - 10t)\mathbf{j}] \cdot [(18\sin 65)\mathbf{i} + (18\cos 65 - 10)\mathbf{j}] = 0$ m1

$(-12\cos 25 + 18t\cos 25)(18\sin 65) + (-12\sin 25 + 18t\sin 25 - 10t)(18\cos 65 - 10) = 0$ A1

$271.85\,t = 165.27$ m1

$t = 0.608$ (or better) A1

$d = 6.60$ km or 6.6 km A1

The corresponding marks awarded for finding the closest approach time:

$(-12\cos 25 + 18t\cos 25)(18\sin 65) + (-12\sin 25 + 18t\sin 25 - 10t)(18\cos 65 - 10) = 0$ M1

$271.85\,t = 165.27$ A1

$t = 0.608$ (or better) A1

(b)(ii) FT from their answers in part (b)(i)

MM03

Q	Solution	Marks	Total	Comments
7(a)	$2m(3i+j)+m(2i-5j)=2mv_A+m(2i+j)$	M1A1		
	$8i-3j=2v_A+(2i+j)$			
	$v_A=3i-2j$	A1	3	
(b)	$I=m(2i+j)-m(2i-5j)$	M1A1		
	$I=6mj$	A1	3	AG
(c)	$I=6mj \Rightarrow$ Line of centres along j	B1		PI
	Restitution along j : $1+2=e(5+1)$	M1A1		
	$e=0.5$	A1	4	
	Accept energy methods			
(d)	${}_Av_B=i-3j$			
	${}_Ar_B=-0.1j+(i-3j)t$	M1A1		
	$1.1^2=t^2+(-0.1-3t)^2$	M1		OE
	$10t^2+0.6t-1.2=0$			
	$t=\dfrac{-0.6\pm\sqrt{0.6^2-4(10)(-1.2)}}{2(10)}$ $(=0.31770677)$	m1		Dependent on both M1s
	$t=0.318$ or 0.317 sec.	A1	5	
	Total		**15**	
	TOTAL		**75**	