Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
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7	
8	
TOTAL	

General Certificate of Education
Advanced Level Examination
June 2011

Mathematics MFP4

Unit Further Pure 4

Wednesday 22 June 2011 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Do not write outside the box

Answer **all** questions in the spaces provided.

1 The matrices $\mathbf{A}$ and $\mathbf{B}$ are given in terms of p by

$$\mathbf{A} = \begin{bmatrix} 1 & p & 4 \\ -3 & 2 & 1 \\ 2 & -1 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} p & 1 & 5 \\ 9 & p & -1 \\ 2 & 0 & 1 \end{bmatrix}$$

(a) Find each of $\det \mathbf{A}$ and $\det \mathbf{B}$ in terms of p. *(3 marks)*

(b) Without finding $\mathbf{AB}$, determine all values of p for which $\mathbf{AB}$ is singular. *(3 marks)*

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2 The plane transformation T is the composition of a reflection in the line $y = x\tan\alpha$ followed by an anticlockwise rotation about O through an angle β.

Determine the matrix which represents T, and hence describe T as a single transformation. *(6 marks)*

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3 Given the vectors $\mathbf{p} = \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix}$, $\mathbf{q} = \begin{bmatrix} 7 \\ -2 \\ 4 \end{bmatrix}$ and $\mathbf{r} = \begin{bmatrix} 2 \\ 3 \\ t \end{bmatrix}$, where t is a scalar parameter, determine the value of t in each of the following cases:

(a) $\mathbf{p} \times \mathbf{q}$ is parallel to $\mathbf{r}$; *(3 marks)*

(b) $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ are linearly dependent. *(3 marks)*

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4 The system of equations S is given in terms of the real parameters a and b by

$$\begin{aligned} 2x + y + 3z &= a + 1 \\ 5x - 2y + (a+1)z &= 3 \\ ax + 2y + 4z &= b \end{aligned}$$

(a) Find the two values of a for which S does not have a unique solution. *(4 marks)*

(b) In the case when $a = 2$, determine the value of b for which S has infinitely many solutions. *(4 marks)*

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5 (a) (i) Find the eigenvalues and corresponding eigenvectors of $\mathbf{A} = \begin{bmatrix} 1 & 3 \\ -2 & 8 \end{bmatrix}$. *(6 marks)*

(ii) Hence write down each of the matrices $\mathbf{U}$, $\mathbf{D}$ and $\mathbf{U}^{-1}$ such that $\mathbf{A} = \mathbf{UDU}^{-1}$, where $\mathbf{D}$ is a diagonal matrix. *(4 marks)*

(b) A 2×2 matrix $\mathbf{M}$ has distinct real eigenvalues λ and μ, with corresponding eigenvectors $\mathbf{v}_1$ and $\mathbf{v}_2$.

(i) By considering the diagonalised form of $\mathbf{M}$, determine the eigenvalues of $\mathbf{M}^3$. *(2 marks)*

(ii) Write down the eigenvectors of $\mathbf{M}^3$. *(1 mark)*

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6 (a) The transformation U of three-dimensional space is represented by the matrix

$$\begin{bmatrix} 1 & 4 & -3 \\ 2 & -1 & 0 \\ 1 & 1 & -1 \end{bmatrix}$$

(i) Write down a vector equation for the line L with cartesian equation

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{6}$$ *(2 marks)*

(ii) Find a vector equation for the image of L under U, and deduce that it is a line through the origin. *(4 marks)*

(b) The plane transformation V is represented by the matrix $\begin{bmatrix} 1 & 4 \\ 2 & -1 \end{bmatrix}$.

L_1 is the line with equation $y = \frac{1}{2}x + k$, and L_2 is the image of L_1 under V.

(i) Find, in the form $y = mx + c$, the cartesian equation for L_2. *(4 marks)*

(ii) Deduce that L_2 is parallel to L_1 and find, in terms of k, the distance between these two lines. *(3 marks)*

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7 Let $\Delta = \begin{vmatrix} n(n+1) & n+1 & -1 \\ 0 & 1 & n \\ 1 & -(n+1) & 1 \end{vmatrix}$.

(a) (i) Show that $(n^2 + n + 1)$ is a factor of Δ. *(2 marks)*

(ii) Hence, or otherwise, express Δ in factorised form. *(2 marks)*

(b) By expanding Δ directly, show that

$$\Delta = [n(n+1)]^2 + \mathrm{f}(n)$$

where $\mathrm{f}(n)$ can be expressed as the sum of two squares. *(2 marks)*

(c) Hence express the number 12 321 as the sum of three squares. *(2 marks)*

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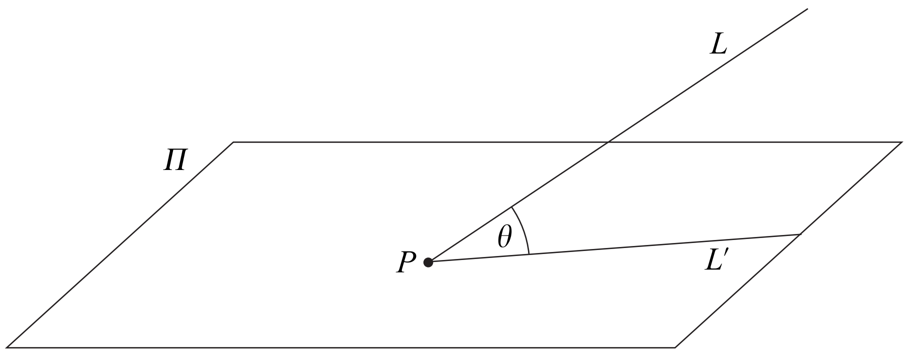
8 The diagram shows the plane Π and the lines L and L'. The plane Π and the line L have equations

$$\mathbf{r}\,.\begin{bmatrix}3\\-2\\6\end{bmatrix} = 37 \quad \text{and} \quad \mathbf{r} = \begin{bmatrix}1\\2\\-7\end{bmatrix} + \lambda\begin{bmatrix}2\\1\\2\end{bmatrix}$$

The line L does not lie in Π, and intersects it at the point P.



(a) Determine the value of θ, the angle between L and Π, giving your answer to the nearest $0.1°$. *(4 marks)*

(b) Find the coordinates of P. *(4 marks)*

(c) The line L' lies in Π and is such that the angle between L and L' is θ, the angle between L and Π.

(i) Find a vector which is parallel to Π and perpendicular to L. *(3 marks)*

(ii) Hence, or otherwise, find a vector equation for L' in the form $\mathbf{r} = \mathbf{a} + \mu\mathbf{b}$. *(4 marks)*

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END OF QUESTIONS

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