

General Certificate of Education  
June 2007  
Advanced Level Examination



**MATHEMATICS**  
**Unit Further Pure 4**

**MFP4**

Friday 22 June 2007 9.00 am to 10.30 am

**For this paper you must have:**

- a 12-page answer book
  - the **blue** AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP4.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

**Information**

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

**Advice**

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

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Answer **all** questions.

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**1** Given that  $\mathbf{a} \times \mathbf{b} = 3\mathbf{i} + \mathbf{j} + \mathbf{k}$  and that  $\mathbf{a} \times \mathbf{c} = -\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ , determine:

(a)  $\mathbf{c} \times \mathbf{a}$  ; (1 mark)

(b)  $\mathbf{a} \times (\mathbf{b} + \mathbf{c})$  ; (2 marks)

(c)  $(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{a} \times \mathbf{c})$  ; (2 marks)

(d)  $\mathbf{a} \cdot (\mathbf{a} \times \mathbf{c})$  . (1 mark)

**2** Factorise completely the determinant  $\begin{vmatrix} y & x & x+y-1 \\ x & y & 1 \\ y+1 & x+1 & 2 \end{vmatrix}$ . (6 marks)

**3** Three points,  $A$ ,  $B$  and  $C$ , have position vectors

$$\mathbf{a} = \begin{bmatrix} 1 \\ 7 \\ -1 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 5 \\ 1 \\ 1 \end{bmatrix} \text{ and } \mathbf{c} = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$$

respectively.

(a) Using the scalar triple product, or otherwise, show that  $\mathbf{a}$ ,  $\mathbf{b}$  and  $\mathbf{c}$  are coplanar. (2 marks)

(b) (i) Calculate  $(\mathbf{b} - \mathbf{a}) \times (\mathbf{c} - \mathbf{a})$  . (3 marks)

(ii) Hence find, to three significant figures, the area of the triangle  $ABC$  . (3 marks)

4 Consider the following system of equations, where  $k$  is a real constant:

$$\begin{array}{rrcrcl} kx & + & 2y & + & z & = & 5 \\ x & + & (k+1)y & - & 2z & = & 3 \\ 2x & - & ky & + & 3z & = & -11 \end{array}$$

- (a) Show that the system does not have a unique solution when  $k^2 = 16$ . (3 marks)
- (b) In the case when  $k = 4$ , show that the system is inconsistent. (4 marks)
- (c) In the case when  $k = -4$ :
- (i) solve the system of equations; (5 marks)
- (ii) interpret this result geometrically. (1 mark)

5 The line  $l$  has equation  $\mathbf{r} = \begin{bmatrix} 3 \\ 26 \\ -15 \end{bmatrix} + \lambda \begin{bmatrix} 8 \\ -4 \\ 1 \end{bmatrix}$ .

- (a) Show that the point  $P(-29, 42, -19)$  lies on  $l$ . (1 mark)
- (b) Find:
- (i) the direction cosines of  $l$ ; (2 marks)
- (ii) the acute angle between  $l$  and the  $z$ -axis. (1 mark)
- (c) The plane  $\Pi$  has cartesian equation  $3x - 4y + 5z = 100$ .
- (i) Write down a normal vector to  $\Pi$ . (1 mark)
- (ii) Find the acute angle between  $l$  and this normal vector. (4 marks)
- (d) Find the position vector of the point  $Q$  where  $l$  meets  $\Pi$ . (4 marks)
- (e) Determine the shortest distance from  $P$  to  $\Pi$ . (3 marks)

**Turn over for the next question**

**Turn over ►**

6 The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ -1 & 1 \\ 1 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & t \end{bmatrix}$$

(a) Find, in terms of  $t$ , the matrices:

(i) **AB**; (3 marks)

(ii) **BA**. (2 marks)

(b) Explain why **AB** is singular for all values of  $t$ . (1 mark)

(c) In the case when  $t = -2$ , show that the transformation with matrix **BA** is the combination of an enlargement, E, and a second transformation, F. Find the scale factor of E and give a full geometrical description of F. (6 marks)

7 (a) The matrix  $\mathbf{M} = \begin{bmatrix} -1 & 2 \\ -2 & 3 \end{bmatrix}$  represents a shear.

(i) Find  $\det \mathbf{M}$  and give a geometrical interpretation of this result. (2 marks)

(ii) Show that the characteristic equation of **M** is  $\lambda^2 - 2\lambda + 1 = 0$ , where  $\lambda$  is an eigenvalue of **M**. (2 marks)

(iii) Hence find an eigenvector of **M**. (3 marks)

(iv) Write down the equation of the line of invariant points of the shear. (1 mark)

(b) The matrix  $\mathbf{S} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  represents a shear.

(i) Write down the characteristic equation of **S**, giving the coefficients in terms of  $a, b, c$  and  $d$ . (2 marks)

(ii) State the numerical value of  $\det \mathbf{S}$  and hence write down an equation relating  $a, b, c$  and  $d$ . (2 marks)

(iii) Given that the only eigenvalue of **S** is 1, find the value of  $a + d$ . (2 marks)

**END OF QUESTIONS**