Centre Number			Candidate Number		
Surname					
Other Names					
Candidate Signature					



General Certificate of Education Advanced Level Examination January 2011

Mathematics

MFP4

Unit Further Pure 4

Friday 28 January 2011 9.00 am to 10.30 am

For this paper you must have:

• the blue AQA booklet of formulae and statistical tables. You may use a graphics calculator.

Time allowed

1 hour 30 minutes

Instructions

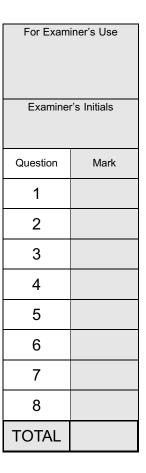
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer all questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

 Unless stated otherwise, you may quote formulae, without proof, from the booklet.





Answer all questions in the spaces provided.

1 Let
$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ x & y & z \\ y+z & z+x & x+y \end{vmatrix}$$
.

- (a) Use a row operation to show that (x + y + z) is a factor of Δ . (2 marks)
- (b) Hence, or otherwise, express Δ as a product of linear factors. (2 marks)

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2	he non-zero vectors a and b have magnitudes a and b respectively.	
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Let
$$c = |\mathbf{a} \times \mathbf{b}|$$
 and $d = |\mathbf{a} \cdot \mathbf{b}|$.

By considering the definitions of the vector and scalar products, or otherwise, show that

$$c^2 + d^2 = a^2b^2 \tag{3 marks}$$

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3 (a) Find the values of t for which the system of equations

$$tx + 2y + 3z = a$$

$$2x + 3y - tz = b$$

$$3x + 5y + (t+1)z = c$$

does not have a unique solution.

(3 marks)

(b) For the integer value of t found in part (a), find the relationship between a, b and c such that this system of equations is consistent. (3 marks)

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4 The non-singular matrix $\mathbf{X} = \begin{bmatrix} 3 & 1 & -3 \\ 2 & 4 & 3 \\ -4 & 2 & -1 \end{bmatrix}$.

(a) (i) Show that $X^2 - X = kI$ for some integer k.

(3 marks)

(ii) Hence show that $\mathbf{X}^{-1} = \frac{1}{20}(\mathbf{X} - \mathbf{I})$.

- (2 marks)
- **(b)** The 3×3 matrix **Y** has inverse $\mathbf{Y}^{-1} = \begin{bmatrix} 60 & 0 & 0 \\ 0 & 0 & -10 \\ 0 & 20 & 0 \end{bmatrix}$.

Without finding Y, determine the matrix $(XY)^{-1}$.

(3 marks)

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		<pre>6</pre>]	[10]	
5	The planes Π_1 and Π_2 have vector equations ${\bf r}$.	2	$=$ 5 and \mathbf{r} .	-1	=4
		<u>_</u> 9_		$\lfloor -11 \rfloor$	
	respectively				

- (a) Write down cartesian equations for Π_1 and Π_2 . (1 mark)
- (b) Find a vector equation for the line of intersection of Π_1 and Π_2 . (5 marks)
- (c) The plane Π_3 has cartesian equation 5x + 3y + 11z = 28.

 Use your answer to part (b) to find the coordinates of the point of intersection of Π_1 , Π_2 and Π_3 .

 (4 marks)
- (d) Determine a vector equation for the plane which passes through the point (4, 1, 9) and which is perpendicular to both Π_1 and Π_2 . (3 marks)

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6	The plane Π has equation $\mathbf{r} \cdot \begin{bmatrix} 12 \\ 15 \\ 16 \end{bmatrix} = 11$ and the point Q has coordinates $(1, 1, -1)$.	
(a)	Show that Q is in Π . (1 mark))
(b) (i)	Write down cartesian equations for the line l which passes through Q and is perpendicular to Π . (2 marks))
(ii)	Deduce the direction cosines of <i>l</i> . (2 marks))
(c)	The points M and N are on l , and each is 50 units from Π .	
	Find the coordinates of M and N . (3 marks))
(d)	Given that the point $P(5, 1, -4)$ is in Π , determine the area of triangle PMN .)
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7 Let
$$\mathbf{Y} = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}$$
.

- (a) Show that 4 is a repeated eigenvalue of Y, and find the other eigenvalue of Y.

 (7 marks)
- (b) For each eigenvalue of Y, find a full set of eigenvectors. (5 marks)
- (c) The matrix Y represents the transformation T.

 Describe the geometrical significance of the eigenvectors of Y in relation to T.

 (3 marks)

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8 The plane transformation T is represented by the matrix $\mathbf{M} = \begin{bmatrix} -3 & 8 \\ -1 & 3 \end{bmatrix}$.

(a) The quadrilateral ABCD has image A'B'C'D' under T.

Evaluate det M and describe the geometrical significance of both its sign and its magnitude in relation to ABCD and A'B'C'D'. (3 marks)

(b) The line y = px is a line of invariant points of T, and the line y = qx is an invariant line of T.

Show that $p = \frac{1}{2}$ and determine the value of q. (5 marks)

- (c) (i) Find the 2 × 2 matrix **R** which represents a reflection in the line $y = \frac{1}{2}x$. (2 marks)
 - (ii) Given that T is the composition of a shear, with matrix S, followed by a reflection in the line $y = \frac{1}{2}x$, determine the matrix S and describe the shear as fully as possible.

(5 marks)

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