Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

General Certificate of Education
Advanced Level Examination
January 2011

Mathematics MFP4

Unit Further Pure 4

Friday 28 January 2011 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Do not write outside the box

Answer **all** questions in the spaces provided.

1 Let $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ x & y & z \\ y+z & z+x & x+y \end{vmatrix}$.

(a) Use a row operation to show that $(x+y+z)$ is a factor of Δ. *(2 marks)*

(b) Hence, or otherwise, express Δ as a product of linear factors. *(2 marks)*

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

2 The non-zero vectors **a** and **b** have magnitudes a and b respectively.

Let $c = |\mathbf{a} \times \mathbf{b}|$ and $d = |\mathbf{a} \cdot \mathbf{b}|$.

By considering the definitions of the vector and scalar products, or otherwise, show that

$$c^2 + d^2 = a^2 b^2$$

(3 marks)

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

Do not write outside the box

3 (a) Find the values of t for which the system of equations

$$\begin{aligned} tx + 2y + \quad 3z &= a \\ 2x + 3y - \quad tz &= b \\ 3x + 5y + (t+1)z &= c \end{aligned}$$

does not have a unique solution. *(3 marks)*

(b) For the integer value of t found in part **(a)**, find the relationship between a, b and c such that this system of equations is consistent. *(3 marks)*

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

Turn over ▶

4 The non-singular matrix $\mathbf{X} = \begin{bmatrix} 3 & 1 & -3 \\ 2 & 4 & 3 \\ -4 & 2 & -1 \end{bmatrix}$.

(a) (i) Show that $\mathbf{X}^2 - \mathbf{X} = k\mathbf{I}$ for some integer k. *(3 marks)*

(ii) Hence show that $\mathbf{X}^{-1} = \frac{1}{20}(\mathbf{X} - \mathbf{I})$. *(2 marks)*

(b) The 3×3 matrix $\mathbf{Y}$ has inverse $\mathbf{Y}^{-1} = \begin{bmatrix} 60 & 0 & 0 \\ 0 & 0 & -10 \\ 0 & 20 & 0 \end{bmatrix}$.

Without finding $\mathbf{Y}$, determine the matrix $(\mathbf{XY})^{-1}$. *(3 marks)*

QUESTION PART REFERENCE

QUESTION PART REFERENCE

Do not write outside the box

5 The planes Π_1 and Π_2 have vector equations $\mathbf{r} \cdot \begin{bmatrix} 6 \\ 2 \\ 9 \end{bmatrix} = 5$ and $\mathbf{r} \cdot \begin{bmatrix} 10 \\ -1 \\ -11 \end{bmatrix} = 4$ respectively.

(a) Write down cartesian equations for Π_1 and Π_2. *(1 mark)*

(b) Find a vector equation for the line of intersection of Π_1 and Π_2. *(5 marks)*

(c) The plane Π_3 has cartesian equation $5x + 3y + 11z = 28$.

Use your answer to part **(b)** to find the coordinates of the point of intersection of Π_1, Π_2 and Π_3. *(4 marks)*

(d) Determine a vector equation for the plane which passes through the point (4, 1, 9) and which is perpendicular to both Π_1 and Π_2. *(3 marks)*

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

Do not write outside the box

6 The plane Π has equation $\mathbf{r} \,.\begin{bmatrix}12\\15\\16\end{bmatrix} = 11$ and the point Q has coordinates $(1,\ 1,\ -1)$.

(a) Show that Q is in Π. *(1 mark)*

(b) (i) Write down cartesian equations for the line l which passes through Q and is perpendicular to Π. *(2 marks)*

(ii) Deduce the direction cosines of l. *(2 marks)*

(c) The points M and N are on l, and each is 50 units from Π.

Find the coordinates of M and N. *(3 marks)*

(d) Given that the point $P(5,\ 1,\ -4)$ is in Π, determine the area of triangle PMN. *(3 marks)*

QUESTION PART REFERENCE

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

7 Let $\mathbf{Y} = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}$.

(a) Show that 4 is a repeated eigenvalue of $\mathbf{Y}$, and find the other eigenvalue of $\mathbf{Y}$. *(7 marks)*

(b) For each eigenvalue of $\mathbf{Y}$, find a full set of eigenvectors. *(5 marks)*

(c) The matrix $\mathbf{Y}$ represents the transformation T.

Describe the geometrical significance of the eigenvectors of $\mathbf{Y}$ in relation to T. *(3 marks)*

QUESTION PART REFERENCE

QUESTION PART REFERENCE

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

Turn over ▶

8 The plane transformation T is represented by the matrix $\mathbf{M} = \begin{bmatrix} -3 & 8 \\ -1 & 3 \end{bmatrix}$.

(a) The quadrilateral $ABCD$ has image $A'B'C'D'$ under T.

Evaluate $\det \mathbf{M}$ and describe the geometrical significance of both its sign and its magnitude in relation to $ABCD$ and $A'B'C'D'$. *(3 marks)*

(b) The line $y = px$ is a line of invariant points of T, and the line $y = qx$ is an invariant line of T.

Show that $p = \frac{1}{2}$ and determine the value of q. *(5 marks)*

(c) (i) Find the 2×2 matrix $\mathbf{R}$ which represents a reflection in the line $y = \frac{1}{2}x$. *(2 marks)*

(ii) Given that T is the composition of a shear, with matrix $\mathbf{S}$, followed by a reflection in the line $y = \frac{1}{2}x$, determine the matrix $\mathbf{S}$ and describe the shear as fully as possible.

(5 marks)

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

Turn over ▶

Do not write outside the box

QUESTION PART REFERENCE

END OF QUESTIONS