General Certificate of Education
Advanced Level Examination
January 2010

Mathematics MFP4

Unit Further Pure 4

Monday 25 January 2010 9.00 am to 10.30 am

For this paper you must have:
- a 12-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The **Examining Body** for this paper is AQA. The **Paper Reference** is MFP4.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The 2×2 matrix $\mathbf{M}$ represents the plane transformation T. Write down the value of $\det \mathbf{M}$ in each of the following cases:

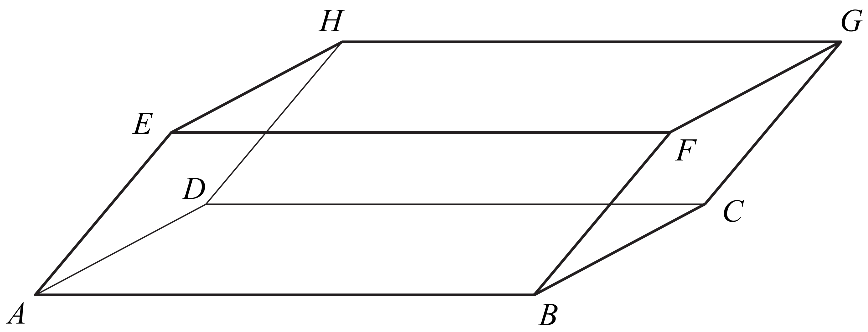
(a) T is a rotation;

(b) T is a reflection;

(c) T is a shear;

(d) T is an enlargement with scale factor 3. *(4 marks)*

2 The diagram shows the parallelepiped $ABCDEFGH$.



The position vectors of A, B, C, D and E are, respectively,

$$\mathbf{a} = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 5 \\ 3 \\ 1 \end{bmatrix}, \quad \mathbf{c} = \begin{bmatrix} -3 \\ 10 \\ 4 \end{bmatrix}, \quad \mathbf{d} = \begin{bmatrix} -7 \\ 10 \\ 7 \end{bmatrix} \quad \text{and} \quad \mathbf{e} = \begin{bmatrix} 3 \\ 4 \\ 10 \end{bmatrix}$$

(a) Show that the area of $ABCD$ is 37. *(4 marks)*

(b) Find the volume of $ABCDEFGH$. *(2 marks)*

(c) Deduce the distance between the planes $ABCD$ and $EFGH$. *(2 marks)*

3 The matrices $\mathbf{A}$ and $\mathbf{B}$ are defined in terms of a real parameter t by

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 1 \\ 2 & t & 4 \\ 3 & 2 & -1 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 15 & -4 & -1 \\ -2t & 4 & 2 \\ 17 & -4 & -3 \end{bmatrix}$$

(a) Find, in terms of t, the matrix $\mathbf{AB}$ and deduce that there exists a value of t such that $\mathbf{AB}$ is a scalar multiple of the 3×3 identity matrix $\mathbf{I}$. *(5 marks)*

(b) For this value of t, deduce $\mathbf{A}^{-1}$. *(2 marks)*

4 (a) Determine the two values of k for which the system of equations

$$\begin{aligned} x \quad - 2y + \quad kz &= 5 \\ (k+1)x + 3y \qquad\quad &= k \\ 2x \quad + y + (k-1)z &= 3 \end{aligned}$$

does not have a unique solution. *(4 marks)*

(b) Show that this system of equations is consistent for one of these values of k, but is inconsistent for the other.

(You are not required to find any solutions to this system of equations.) *(8 marks)*

5 The plane transformations $\mathrm{T_A}$ and $\mathrm{T_B}$ are represented by the matrices $\mathbf{A}$ and $\mathbf{B}$ respectively, where $\mathbf{A} = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 2 & 5 \\ 5 & 13 \end{bmatrix}$.

(a) Find the equation of the line which is the image of $y = 2x + 1$ under $\mathrm{T_A}$. *(3 marks)*

(b) The rectangle $PQRS$, with area 4.5 cm^2, is mapped onto the parallelogram $P'Q'R'S'$ under $\mathrm{T_B}$. Determine the area of $P'Q'R'S'$. *(2 marks)*

(c) The transformation $\mathrm{T_C}$ is the composition

'$\mathrm{T_B}$ followed by $\mathrm{T_A}$'

By finding the matrix which represents $\mathrm{T_C}$, give a full geometrical description of $\mathrm{T_C}$. *(5 marks)*

Turn over for the next question

6 (a) Find the value of p for which the planes with equations

$$\mathbf{r} \cdot \begin{bmatrix} 6 \\ -3 \\ 2 \end{bmatrix} = 42 \quad \text{and} \quad \mathbf{r} \cdot \begin{bmatrix} 4p+1 \\ p-2 \\ 1 \end{bmatrix} = -7$$

(i) are perpendicular; *(3 marks)*

(ii) are parallel. *(3 marks)*

(b) In the case when $p = 4$:

(i) write down a cartesian equation for each plane; *(2 marks)*

(ii) find, in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{d}$, an equation for l, the line of intersection of the planes. *(6 marks)*

(c) Determine a vector equation, in the form $\mathbf{r} = \mathbf{u} + \beta\mathbf{v} + \gamma\mathbf{w}$, for the plane which contains l and which passes through the point $(30, 7, 30)$. *(2 marks)*

7 (a) It is given that $\Delta = \begin{vmatrix} 16-q & 5 & 7 \\ -12 & -1-q & -7 \\ 6 & 6 & 10-q \end{vmatrix}$.

(i) By using row operations on the first two rows of Δ, show that $(4-q)$ is a factor of Δ. *(2 marks)*

(ii) Express Δ as the product of three linear factors. *(4 marks)*

(b) It is given that $\mathbf{M} = \begin{bmatrix} 16 & 5 & 7 \\ -12 & -1 & -7 \\ 6 & 6 & 10 \end{bmatrix}$.

(i) Verify that $\begin{bmatrix} 2 \\ 5 \\ -7 \end{bmatrix}$ is an eigenvector of $\mathbf{M}$ and state its corresponding eigenvalue. *(3 marks)*

(ii) For each of the other two eigenvalues of $\mathbf{M}$, find a corresponding eigenvector. *(7 marks)*

(c) The transformation T has matrix $\mathbf{M}$. Write down cartesian equations for any one of the invariant lines of T. *(2 marks)*

END OF QUESTIONS