



General Certificate of Education (A-level)
June 2011

Mathematics

MFP4

(Specification 6360)

Further Pure 4

Final

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
✓ or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
-x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1(a)	$\det \mathbf{A} = 5p - 1$ $\det \mathbf{B} = p^2 - 10p - 11$	B1 M1A1	3	M1A0 if num error(s) made
(b)	Use of $\det(\mathbf{AB}) = \det \mathbf{A} \det \mathbf{B}$ Finding three values of p $p = \frac{1}{5}, 11, -1$	B1 M1 A1F	3	PI Allow correct factors here ft numerical errors in (a)
	Total		6	
2	$\begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{bmatrix} \& \begin{bmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{bmatrix}$ Mult'n of these in the correct order Use of addition formulae $\begin{bmatrix} \cos(2\alpha + \beta) & \sin(2\alpha + \beta) \\ \sin(2\alpha + \beta) & -\cos(2\alpha + \beta) \end{bmatrix}$ Reflection in $y = x \tan(\alpha + \frac{1}{2}\beta)$	B1 B1 M1 A1F A1F A1F	 6	used or written down at least two entries correct At least once ft only for use of clockwise rot'n and/or mult'n in wrong order ft as above ft as above
	Total		6	
3(a)	Vector product attempted $\mathbf{p} \times \mathbf{q} = \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix} \times \begin{bmatrix} 7 \\ -2 \\ 4 \end{bmatrix} = \begin{bmatrix} 30 \\ 45 \\ -30 \end{bmatrix}$ $\dots = 15 \begin{bmatrix} 2 \\ 3 \\ -2 \end{bmatrix}, \text{ so } t = -2$	M1 A1 A1	 3	
(b)	Scalar triple product attempted $\mathbf{p} \times \mathbf{q} \bullet \mathbf{r} = 15 \begin{bmatrix} 2 \\ 3 \\ -2 \end{bmatrix} \bullet \begin{bmatrix} 2 \\ 3 \\ t \end{bmatrix} = 15(13 - 2t)$ $\dots = 0, \text{ so } t = 6\frac{1}{2}$ ALT: $5\mathbf{p} + \mathbf{q} = 6\mathbf{r}$ $\dots \Rightarrow t = 6\frac{1}{2}$	M1 A1 A1 B2,0 B1	 3	OE, eg determinant or any correct linear relationship
	Total		6	

Q	Solution	Marks	Total	Comments
4(a)	$\begin{vmatrix} 2 & 1 & 3 \\ 5 & -2 & a+1 \\ a & 2 & 4 \end{vmatrix} = a^2 + 3a - 10$ Equating to 0 and solving quadratic in a $a = 2, -5$	M1 A1 m1 A1	4	Attempt at det of coeff matrix Correct (accept unsimplified) SC: B1 for verifying $a = 2$ B1 for verifying $a = -5$
(b)	$\begin{aligned} 2x + y + 3z &= 3 \\ 5x - 2y + 3z &= 3 \\ 2x + 2y + 4z &= b \end{aligned}$ Eliminations leading to two equations in two variables Further elimination leading to value of b $b = 4$	B1 M1 m1 A1	4	
	ALT: Finding two variables in terms of third Substituting into third equation $b = 4$	M1 m1 A1		eg $y = x$ and $z = 1 - x$
			8	
5(a)	(i) Characteristic eqn $\lambda^2 - 9\lambda + 14 = 0$ $\lambda = 2, 7$ Substituting back for at least one eval evecs $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ (ii) $\mathbf{U} = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$, $\mathbf{D} = \begin{bmatrix} 2 & 0 \\ 0 & 7 \end{bmatrix}$ $\mathbf{U}^{-1} = \frac{1}{5} \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$	M1A1 A1 m1 A1A1 B1FB1F B1F B1F	6	M1A0 if num error(s) made for $\lambda = 2$, $-x + 3y = 0$ or for $\lambda = 7$, $-2x + y = 0$ or non-zero multiples
(b)	(i) evals of $\mathbf{M}^3$ are λ^3, μ^3 since $\mathbf{M}^3 = \mathbf{U} \mathbf{D}^3 \mathbf{U}^{-1}$ (ii) evecs of $\mathbf{M}^3$ are $\mathbf{v}_1$ and $\mathbf{v}_2$	B1 E1 B1	2 1	Columns of $\mathbf{U}$ and $\mathbf{D}$ are interchangeable, but must match; ft wrong answers in (i) 1/det $\mathbf{U}$ adjoint matrix; ft incorrect $\mathbf{U}$ (provided $\det \neq 0$)
			13	

Q	Solution	Marks	Total	Comments
6 (a)(i)	$\mathbf{r} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ 3 \\ 6 \end{bmatrix}$	B2,1	2	Any correct vector line equation; B1 if one vector correct, or if both correct but equation not in correct form
	(ii) $\mathbf{r} = \begin{bmatrix} 1 & 4 & -3 \\ 2 & -1 & 0 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1+2\lambda \\ 2+3\lambda \\ 3+6\lambda \end{bmatrix} = \begin{bmatrix} -4\lambda \\ \lambda \\ -\lambda \end{bmatrix}$	M1 A1 A1		Attempt at multiplication At least one entry correct All three correct
	Clear and valid explanation that this is a line through O	E1	4	
	(b)(i) $\begin{bmatrix} 1 & 4 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} p \\ \frac{1}{2}p + k \end{bmatrix} = \begin{bmatrix} 3p + 4k \\ \frac{3}{2}p - k \end{bmatrix}$ Answer satisfies $y = \frac{1}{2}x - 3k$	B1 M1A1 A1	4	For LHS For RHS
	(ii) Equal gradients, hence parallel	E1F		ft if previous answer is of the form $y = \frac{1}{2}x + c$
	Distance = $ k - c \cos \theta$ with $\tan \theta = \frac{1}{2}$... = $\frac{8k}{\sqrt{5}}$	M1 A1	3	Allow incorrect value of c here Allow 3.58k
			13	
7 (a)(i)	Appropriate row/column operation	M1		eg $R_1' = R_1 + R_3$, $R_3' = R_3 + R_1$ or $C_3' = C_3 - nC_2$
	$\Delta = \begin{vmatrix} n^2 + n + 1 & 0 & 0 \\ 0 & 1 & n \\ 1 & -(n+1) & 1 \end{vmatrix}$... = $(n^2 + n + 1) \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & n \\ 1 & -(n+1) & 1 \end{vmatrix}$	A1	2	Factor correctly extracted
	(ii) Expanding remaining determinant $\Delta = (n^2 + n + 1)^2$	M1 A1	2	OE
	(b) $\Delta = (n^2 + n)^2 + 2n^2 + 2n + 1$... = $(n^2 + n)^2 + (n + 1)^2 + n^2$	B1 B1	2	Accept unsimplified
	(c) Setting $n = 10$ $111^2 = 12321 = 110^2 + 11^2 + 10^2$	M1 A1	2	
			8	

Q	Solution	Marks	Total	Comments
8(a)	Use of $\sin$ or $\cos \theta = \frac{\text{scalar product}}{\text{product of moduli}}$	M1		using $\begin{bmatrix} 3 \\ -2 \\ 6 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$
	Numerator = 16, denominator = 21	B1B1		Allow numerator $\sqrt{185}$
	$\sin \theta = \frac{16}{21} \Rightarrow \theta \approx 49.6^\circ$	A1	4	Allow AWRT 49.6
(b)	$\begin{bmatrix} 2\lambda + 1 \\ \lambda + 2 \\ 2\lambda - 7 \end{bmatrix} \bullet \begin{bmatrix} 3 \\ -2 \\ 6 \end{bmatrix} = 37$	M1		
	$6\lambda + 3 - 2\lambda - 4 + 12\lambda - 42 = 37$	m1		with attempt to solve
	$\dots \Rightarrow \lambda = 5$ giving $P = (11, 7, 3)$	A1 B1F	4	ft wrong value of λ
(c)(i)	Use of the vectors $\begin{bmatrix} 3 \\ -2 \\ 6 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$	M1		
	Vector product attempted	m1		OE
	Required vector is $\begin{bmatrix} -10 \\ 6 \\ 7 \end{bmatrix}$	A1	3	Or a non-zero multiple
(ii)	$\mathbf{a} = \begin{bmatrix} 11 \\ 7 \\ 3 \end{bmatrix}$	B1F		ft wrong answer in (b)
	$\mathbf{b} = \begin{bmatrix} -10 \\ 6 \\ 7 \end{bmatrix} \times \begin{bmatrix} 3 \\ -2 \\ 6 \end{bmatrix} = \begin{bmatrix} 50 \\ 81 \\ 2 \end{bmatrix}$	M1A1F		Or a non-zero multiple; ft wrong answer to (c)(i)
	Fully correct equation for L'	A1	4	
			15	
	TOTAL		75	