General Certificate of Education

Mathematics 6360

MFP4 Further Pure 4

Mark Scheme

2009 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MFP4

Q	Solution	Marks	Total	Comments
1(a)	$\begin{bmatrix} 1 & 4 & 2 \\ -1 & 2 & 6 \end{bmatrix}\begin{bmatrix} k & 1 \\ 2 & -1 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} k+14 & -1 \\ 22-k & 3 \end{bmatrix}$	M1 A1 A1	3	**PQ** a 2×2 matrix At least one element in C_1 correct All correct
(b)	Det(**PQ**) = $3k + 42 + 22 - k$ $= 2k + 64 = 0$ $k = -32$	M1 A1	2	Det of a square matrix attempted and equated to zero ft in 2×2 case only (linear eqns.)
	Total		**5**	
2(a)(i)	$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	B2	2	
(ii)	$\mathbf{B} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	B1	1	
(b)(i)	$\mathbf{R} = \mathbf{BA} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	M1 A1 A1	3	Product correct way around Most correct; all correct ft ft
(ii)	Reflection in $x = 0$ (or y–z plane)	M1 A1	2	M for correct **R**
	<u>Note 1:</u> For $\mathbf{R} = \mathbf{AB} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	(B1)		If all correct, ft their **A**, **B**
	Reflection in $y = 0$ (or x–z plane)	(M1) (A1)		Full ft, M for correct **R**
	<u>Note 2:</u> 90^{o} rotation in –ve sense gives $\mathbf{B} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	(B1)		**A** as before
	$\mathbf{R} = \mathbf{BA} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ Reflection in $y = 0$ (or x–z plane)	(M1) (A1) (A1) (M1) (A1)		Full ft (incl. Note 1 possibility – Reflection in $x = 0$ (or y–z plane))
	Total		**8**	

MFP4 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$\mathbf{n} = (3\mathbf{i} + \mathbf{j} + 2\mathbf{k}) \times (4\mathbf{i} - \mathbf{j} + \mathbf{k})$	M1		
	$= 3\mathbf{i} + 5\mathbf{j} - 7\mathbf{k}$	A1		cao
	$d = (2\mathbf{i} + \mathbf{j} + \mathbf{k}) \bullet (\text{their } \mathbf{n}) = 4$	M1 A1	4	ft
(b)	$\begin{bmatrix} 7+10t \\ 1+t \\ 4+5t \end{bmatrix}$ subst[d]. into their plane eqn.	M1		(In at least the LHS of it)
	$21 + 30t + 5 + 5t - 28 - 35t = 4$	dM1		Linear "eqn." in t created (LHS)
	Since $-2 \neq 4$, no intersection	A1		Explained or stated. N.B. can ft other d's (except -2) but if $\mathbf{n}$ is wrong also the t won't vanish, so no ft then
	Line parallel to plane	B1		May be independently asserted
	OR			
	$\begin{bmatrix} 3 \\ 5 \\ -7 \end{bmatrix} \bullet \begin{bmatrix} 10 \\ 1 \\ 5 \end{bmatrix} = 0$	(M1) (A1) (B1)		 For showing line not in plane
	Line perp[r]. to nml. $\Rightarrow$ line // to plane	(B1)		
	OR			
	$\begin{bmatrix} 7+10t \\ 1+t \\ 4+5t \end{bmatrix}$ equated to $\begin{bmatrix} 2+3\lambda+4\mu \\ 1+\lambda-\mu \\ 4+2\lambda+\mu \end{bmatrix}$	(M1)		Incl. starting to do something
	Eliminating λ, μ to get linear eqn. in t	(dM1)		
	Since $-2 \neq 4$, no intersection	(A1)		Explained or stated
	Line parallel to plane	(B1)	4	May be independently asserted
	Total		**8**	

MFP4 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$3 \times [1] - [2] \Rightarrow 5x - 4y + 14z = 16$	M2 A1		Or eliminating (say) y twice to get two lots of $7x - 2z = 28$
	Giving no unique soln. *and* consistent	E1		
	For those who just show $\Delta = 0$ to conclude that there is no unique soln.	(M1) (A1)		and save the other M1 A1 for demonstrating consistency
	OR			
	Solving e.g. in [1] & [2]: $\frac{x-4}{2} = \frac{y-1}{27} = \frac{z}{7} = \lambda$	(M1) (A1)		
	Subst[g]. in [3] for x, y, z in terms of λ	(M1)		$5(2\lambda + 4) - 4(1 + 27\lambda) + 14(7\lambda)$
	Showing LHS = RHS = 16	(A1)		
	OR			
	$\left(\begin{array}{ccc\|c} 3 & -1 & 3 & 11 \\ 4 & 1 & -5 & 17 \\ 5 & -4 & 14 & 16 \end{array}\right) \rightarrow \left(\begin{array}{ccc\|c} 3 & -1 & 3 & 1 \\ 1 & 2 & -8 & 6 \\ -1 & -2 & 8 & -6 \end{array}\right)$	(M1) (A1) (A1)		$R_2' = R_2 - R_1$ $R_3' = R_3 - 2R_1$
	$R_2' = -R_3' \Rightarrow$ no unique soln. and consistency	(E1)		
	OR			
	Showing $\Delta = 0 \Rightarrow$ no unique soln.	(M1) (A1)		
	Attempt at each of $\Delta_x = \begin{vmatrix} 11 & -1 & 3 \\ 17 & 1 & -5 \\ 16 & -4 & 14 \end{vmatrix}$, $\Delta_y = \begin{vmatrix} 3 & 11 & 3 \\ 4 & 17 & -5 \\ 5 & 16 & 14 \end{vmatrix}$ and $\Delta_z = \begin{vmatrix} 3 & -1 & 11 \\ 4 & 1 & 17 \\ 5 & -4 & 16 \end{vmatrix}$	(M1)		
	Each shown = 0 and this $\Rightarrow$ consistency	(A1)	4	
(b)	Setting $x' = x,\ y' = y,\ z' = z$	M1		
	$2 = -y + 3z$ $-12 = 2x + 5y - 4z$ $30 = 4x + 11y + 3z$	A1		Or equivalent
	E.g. $\left.\begin{array}{l} 2 = 3z - y \\ 54 = 11z + y \end{array}\right\}$ by (3) – 2 × (2)	M1 A1		Reducing to 2×2 system; Correctly ft their system
	$z = 4,\ \ y = 10$	M1 A1		Solving ; correctly
	$x = -23$	M1 A1	8	Subst[g]. back to find 3rd coord.
	OR Other methods for solving a 3×3 system will be constructed should they arise			
	Total		**12**	

MFP4 (cont)

Q	Solution	Marks	Total	Comments
5(a)(i)	$\mathbf{a}\bullet\mathbf{b}\times\mathbf{c} = \begin{vmatrix} 2 & 1 & 4 \\ 3 & 2 & 5 \\ 1 & -1 & 5 \end{vmatrix} = 0$	M1 A1	2	Legitimately shown to be zero
(ii)	$\overrightarrow{AB} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\overrightarrow{AC} = \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$, $\overrightarrow{AD} = \begin{bmatrix} 3 \\ 4 \\ 7 \end{bmatrix}$	M1 A1		At least two correct
	Attempt at $\overrightarrow{AB} \bullet \overrightarrow{AC} \times \overrightarrow{AD}$	M1		Any order (+/–), some Sc.Trip.Pr.
	$V = 6$	A1	4	cao and not –ve
(b)(i)	$\overrightarrow{BD} = \begin{bmatrix} 2 \\ 3 \\ 6 \end{bmatrix}$; i.e. $2 : 3 : 6$	M1 A1	2	
(ii)	$\sqrt{2^2 + 3^2 + 6^2} = 7$	M1		
	DCs are $\frac{2}{7}, \frac{3}{7}, \frac{6}{7}$	A1	2	ft
	Total		**10**	
6(a)	Det(**M**) = 1 $\Rightarrow$ Area invariant under T	B1 B1	2	2nd B1 ft ref. “area”
(b)	Char. Eqn. $\lambda^2 - 2\lambda + 1 = 0$	M1		
	$\Rightarrow \lambda = 1$ (twice)	A1		
	Subst^{g}. their λ back to find an evec:	M1		
	$\alpha \begin{bmatrix} 2 \\ 1 \end{bmatrix}$	A1		Any (non-zero) α
	(Since $\lambda = 1$) this represents a line of inv. pts.	B1	5	ft if $\lambda \neq 1$
(c)	$\begin{bmatrix} -1 & 4 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} x \\ \frac{1}{2}x + k \end{bmatrix} = \begin{bmatrix} x + 4k \\ \frac{1}{2}x + 3k \end{bmatrix}$	M1 A1		
	Verifying that $y' = \frac{1}{2}x' + k$	A1	3	Be convinced AG
(d)	Inv. line (or parallel to) $y = \frac{1}{2}x$	B1		
	Mapping (e.g.) (1, 0) to $(-1, -1)$ Give 0 + 0 if called any other kind of transformation	B1	2	Any pt. not on $y = \frac{1}{2}x$ and its image
	Total		**12**	

MFP4 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	$\mathbf{D} = \begin{bmatrix} 3 & 0 \\ 0 & -3 \end{bmatrix}$ $\quad \mathbf{U} = \begin{bmatrix} 1 & 1 \\ 2 & 4 \end{bmatrix}$	B1 B1		**D**, **U** (alt. choices ok)
	$\mathbf{U}^{-1} = \frac{1}{2}\begin{bmatrix} 4 & -1 \\ -2 & 1 \end{bmatrix}$	B1 B1	4	ft 1st B1 provided det $\neq 0$ ft 2nd B1 in non-trivial cases
(ii)	$\mathbf{M} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} 4 & -1 \\ -2 & 1 \end{bmatrix}$	M1		Some attempt at mtx. multn.
	$= \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 12 & -3 \\ 6 & -3 \end{bmatrix}$ or	A1		First multn. correct ft
	$\frac{1}{2}\begin{bmatrix} 3 & -3 \\ 6 & -12 \end{bmatrix}\begin{bmatrix} 4 & -1 \\ -2 & 1 \end{bmatrix}$			
	$= \begin{bmatrix} 9 & -3 \\ 24 & -9 \end{bmatrix}$	A1	3	Ft missing $\frac{1}{2}$ only
(b)(i)	When n even, $\mathbf{D}^n = \begin{bmatrix} 3^n & 0 \\ 0 & 3^n \end{bmatrix}$	M1		Incl. use in mtx. multn. of form $\mathbf{U}\,\mathbf{D}^n\,\mathbf{U}^{-1}$
	$\mathbf{M}^n = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 4.3^n & -3^n \\ -2.3^n & 3^n \end{bmatrix}$ or $\frac{1}{2}\begin{bmatrix} 3^n & 3^n \\ 2.3^n & 4.3^n \end{bmatrix}\begin{bmatrix} 4 & -1 \\ -2 & 1 \end{bmatrix}$ correct	A1		Correct ft
	Showing $\mathbf{M}^n = 3^n\,\mathbf{I}$ legitimately	A1	3	
(ii)	When n odd, $\mathbf{D}^n = \begin{bmatrix} 3^n & 0 \\ 0 & -3^n \end{bmatrix}$	M1		Incl. use in mtx. multn. of form $\mathbf{U}\,\mathbf{D}^n\,\mathbf{U}^{-1}$
	$\mathbf{M}^n = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 4.3^n & -3^n \\ 2.3^n & -3^n \end{bmatrix}$ or $\frac{1}{2}\begin{bmatrix} 3^n & -3^n \\ 2.3^n & -4.3^n \end{bmatrix}\begin{bmatrix} 4 & -1 \\ -2 & 1 \end{bmatrix}$ correct	A1		Correct ft
	Showing $\mathbf{M}^n = 3^{n-1}\,\mathbf{M}$ legitimately	A1	3	
	Total		**13**	
8(a)	Det(**M**) = $a^3 + b^3 + c^3 - 3abc$	M1 A1	2	Good attempt; correct
(b)	$\begin{bmatrix} ad+bf+ce & ae+bd+cf & af+be+cd \\ af+be+cd & ad+bf+ce & ae+bd+cf \\ ae+bd+cf & af+be+cd & ad+bf+ce \end{bmatrix}$	M1 A1 A1	3	At least 5 correct; all 9 correct
(c)	Use of det(**MN**) = det(**M**) det(**N**)	M1		
	$x = ad + bf + ce$, $y = ae + bd + cf$ and $z = af + be + cd$	A1	2	All correctly identified Give B1 (SC) if just this with no explanation why
	Total		**7**	
	TOTAL		**75**	