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ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MFP4 Further Pure 4

Mark Scheme

2008 examination – June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MFP4

Q	Solution	Marks	Total	Comments
1	Attempt at char eqn $\lambda^2 - 7\lambda - 144 = 0$	M1		Any suitable method Ignore missing "= 0"
	Solving quadratic to find evals	M1		Any method
	$\lambda = 16$ or -9	A1		CAO
	$\lambda = 16 \Rightarrow -9x + 12y = 0 \Rightarrow y = \frac{3}{4}x$	M1		Either λ substituted back
	$\Rightarrow$ evecs $\alpha\begin{bmatrix}4\\3\end{bmatrix}$	A1		CAO (for any non-zero α)
	$\lambda = -9 \Rightarrow 16x + 12y = 0 \Rightarrow y = -\frac{4}{3}x$			
	$\Rightarrow$ evecs $\beta\begin{bmatrix}3\\-4\end{bmatrix}$	A1	6	CAO (for any non-zero β)
	Total		**6**	
2(a)(i)	$\mathbf{a}\times\mathbf{b} = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\1 & 2 & 3\\2 & 1 & 2\end{vmatrix} = \begin{bmatrix}1\\4\\-3\end{bmatrix}$	M1 A1	 2	Genuine vector product attempt CAO
(ii)	$(\mathbf{a}\times\mathbf{b})\bullet\mathbf{c} = \begin{bmatrix}1\\4\\-3\end{bmatrix}\bullet\begin{bmatrix}-2\\t\\6\end{bmatrix} = 4t - 20$	M1 A1	 2	Must get a scalar answer ft
(iii)	$(\mathbf{a}\times\mathbf{b})\times\mathbf{c} = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\1 & 4 & -3\\-2 & t & 6\end{vmatrix} = \begin{bmatrix}3t+24\\0\\t+8\end{bmatrix}$	M1 A1	 2	Either using (a)(i) or starting again CAO
(b)	$(\mathbf{a}\times\mathbf{b})\bullet\mathbf{c} = 0 \Rightarrow t = 5$	M1A1	2	ft from (a)(ii)
(c)	$(\mathbf{a}\times\mathbf{b})\times\mathbf{c} = \mathbf{0}$ or $\mathbf{c}$ = mult. of $(\mathbf{a}\times\mathbf{b})$	M1		Use of any non-zero row to find some value of t
	$\Rightarrow t = -8$	A1	2	CAO – allow unseen check
	Total		**10**	
3(a)	Det $\mathbf{A} = k + 3 + 12 - 4 - 9 - k = 2$	M1 A1	 2	 CAO
(b)	$\mathbf{A}^{-1} = \frac{1}{\text{Det A}}(\text{adj }\mathbf{A})$	B1 M1		Correct use of the determinant (any value) Attempt at matrix of cofactors
	$= \frac{1}{2}\begin{bmatrix}k-9 & 3-k & 2\\12-k & k-4 & -2\\-1 & 1 & 0\end{bmatrix}$	M1 A1 A1	 5	Use of transposition **and** signs At least 5 entries correct (even if 2nd M1 not earned) CAO – ft det only
	Total		**7**	

MFP4 (cont)

Q	Solution	Marks	Total	Comments
4(a)	Use of $\cos\theta = \dfrac{\text{scalar product}}{\text{product of moduli}}$	M1		**Must** be $\begin{bmatrix}2\\1\\4\end{bmatrix}$ and $\begin{bmatrix}5\\1\\-1\end{bmatrix}$ "$\sin\theta =$" scores M0 at this stage
	Numerator = 7 Denominator = $\sqrt{21}\sqrt{27}$	B1,B1		Allow denominator unsimplified
	$\theta = 72.9^{\circ}$	A1	4	CAO (but A0 if candidate proceeds to find its complement)
(b)(i)	$a + 4b = 7$ and $a - b = 12$	B1		At least one correctly stated
		M1		Solving simultaneously
	$a = 11$ and $b = -1$	A1	3	CAO
(ii)	$\begin{bmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\5 & 1 & -1\\2 & 1 & 4\end{bmatrix} = \begin{bmatrix}5\\-22\\3\end{bmatrix}$	M1		For any valid, complete method for finding a suitable direction **vector**, eg finding a 2nd common point, eg (2½ , 0, ½) or (1⅔, 3⅔, 0), and then **dv** = difference
		A1	2	CAO
(iii)	$\mathbf{r} = \begin{bmatrix}0\\11\\-1\end{bmatrix} + \lambda\begin{bmatrix}5\\-22\\3\end{bmatrix}$	M1		**Must** be a line equation and use their (b)(ii)
	or other equivalent line form eg $(\mathbf{r} - \mathbf{a}) \times \mathbf{d} = \mathbf{0}$	A1	2	ft their (b)(i) point, or any other correct point on the line A0 if no **r** = or l = etc
	Total		**11**	
5(a)	$y = 0$ (or "x-axis") and $y = x$	B1,B1		or $\mathbf{r} = a\begin{bmatrix}1\\0\end{bmatrix}$, $\mathbf{r} = b\begin{bmatrix}1\\1\end{bmatrix}$
	$y = 0$ is a line of invariant points since $\lambda = 1$	B1	3	Allow if proven from $(x', y') = (x, y)$ or ft from their line corresponding to $\lambda = 1$
(b)(i)	$\mathbf{D} = \begin{bmatrix}1 & 0\\0 & 2\end{bmatrix}$, $\mathbf{U} = \begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}$,	B1,B1	2	ft **U** from **D**
(ii)	$\mathbf{U}^{-1} = \begin{bmatrix}1 & -1\\0 & 1\end{bmatrix}$	B1		ft from **U** (provided non-singular)
	$\mathbf{M} = \begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 2\end{bmatrix}\begin{bmatrix}1 & -1\\0 & 1\end{bmatrix}$	M1		Attempt
	$= \begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\begin{bmatrix}1 & -1\\0 & 2\end{bmatrix}$ or $\begin{bmatrix}1 & 2\\0 & 2\end{bmatrix}\begin{bmatrix}1 & -1\\0 & 1\end{bmatrix}$	A1		ft first multiplication
	$= \begin{bmatrix}1 & 1\\0 & 2\end{bmatrix}$	A1	4	CAO NMS $\Rightarrow$ 0

MFP4 (cont)

Q	Solution	Marks	Total	Comments
(iii)	$\mathbf{D}^n = \begin{bmatrix} 1 & 0 \\ 0 & 2^n \end{bmatrix}$	B1		Noted or used
	$\mathbf{M}^n = \mathbf{U}\,\mathbf{D}^n\,\mathbf{U}^{-1}$	M1		Used; **must** actually do some multiplying
	$= \begin{bmatrix} 1 & 2^n - 1 \\ 0 & 2^n \end{bmatrix}$	A1	3	
	Total		**12**	
6(a)	eg **(2) – (1)** $\Rightarrow x + 7z = -3$	M1A1		Eliminating first variable
	(3) $- 2 \times$ **(2)** $\Rightarrow x + 8z = -2$	A1		
	Solving 2×2 system	M1		
	$x = -10$, $y = 19$, $z = 1$	A1	5	
(b)(i)	$\begin{vmatrix} 1 & 1 & -3 \\ 2 & 1 & 4 \\ 5 & 2 & a \end{vmatrix} = 15 - a$	B1		Determinant
	Setting = to zero and solving for a	M1		**Must** get a numerical answer
	$a = 15$	A1	3	ft
(ii)	$x + y - 3z = b$ $2x + y + 4z = 3$ $5x + 2y + 15z = 4$			
	eg **(2) – (1)** $\Rightarrow x + 7z = 3 - b$	M1A1		Eliminating first variable
	(3) $- 2 \times$ **(2)** $\Rightarrow x + 7z = -2$	A1		
	Equating the two RHSs	M1		
	$b = 5$	A1	5	CAO
				NB Eliminating x : $-y + 10z = 3 - 2b$ $-3y + 30z = 4 - 5b$ $-y + 10z = -7$
				NB Eliminating z : $10x + 7y = 4b + 9$ $10x + 7y = 5b + 4$ $10x + 7y = 29$
	Total		**13**	
6(a)	**Alternate Schemes** **Cramer's Rule**			
	$\Delta = \begin{vmatrix} 1 & 1 & -3 \\ 2 & 1 & 4 \\ 5 & 2 & 16 \end{vmatrix}$, $\Delta_x = \begin{vmatrix} 6 & 1 & -3 \\ 3 & 1 & 4 \\ 4 & 2 & 16 \end{vmatrix}$, $\Delta_y = \begin{vmatrix} 1 & 6 & -3 \\ 2 & 3 & 4 \\ 5 & 4 & 16 \end{vmatrix}$, $\Delta_z = \begin{vmatrix} 1 & 1 & 6 \\ 2 & 1 & 3 \\ 5 & 2 & 4 \end{vmatrix}$	M1		Attempt at any two
	$= -1, 10, -19$ and -1 respectively	A1		Any one correct
	$x = \dfrac{\Delta_x}{\Delta}$, $y = \dfrac{\Delta_y}{\Delta}$, $z = \dfrac{\Delta_z}{\Delta}$	M1		At least one attempted numerically
	$x = -10$, $y = 19$, $z = 1$	A1		Any 2 correct ft
		A1	(5)	All 3 correct CAO

MFP4 (cont)

Q	Solution	Marks	Total	Comments
6(a)	**Inverse matrix method** $C^{-1} = \frac{1}{-1}\begin{bmatrix} 8 & -22 & 7 \\ -12 & 31 & -10 \\ -1 & 3 & -1 \end{bmatrix}$	M1 A1		M0 if no inverse matrix is given
	$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = C^{-1}\begin{bmatrix} 6 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} -10 \\ 19 \\ 1 \end{bmatrix}$	M1 A1 A1	(5)	Any 2 correct ft All 3 correct CAO
6(all)	$\left[\begin{array}{ccc\|c} 1 & 1 & -3 & b \\ 2 & 1 & 4 & 3 \\ 5 & 2 & a & 4 \end{array}\right]$			
	$\to \left[\begin{array}{ccc\|c} 1 & 1 & -3 & b \\ 0 & -1 & 10 & 3-2b \\ 0 & -3 & a+15 & 4-5b \end{array}\right]$			$R_2' = R_2 - 2R_1$ $R_3' = R_3 - 5R_1$
	$\to \left[\begin{array}{ccc\|c} 1 & 1 & -3 & b \\ 0 & 1 & -10 & 2b-3 \\ 0 & 0 & a-15 & b-5 \end{array}\right]$		(4)	$R_3' = R_3 + 3R_2$
	(b)(i) For non-unique solutions, $a = 15$		(2)	
	(ii) For consistency, $4 - 5b = 3(3 - 2b) \Rightarrow b = 5$		(2)	
	(a) When $a = 16, b = 6$ $\left[\begin{array}{ccc\|c} 1 & 1 & -3 & 6 \\ 0 & 1 & -10 & 9 \\ 0 & 0 & 1 & 1 \end{array}\right]$ $\Rightarrow z = 1,\ y = 19,\ x = -10$		(5)	
7(a)(i)	det **W** = 0 Transformed shapes have zero volume	B1 B1	2	Or equivalent statement ft volume statements
(ii)	Char eqn is $\lambda^3 - 4\lambda^2 + 4\lambda = 0$ Solving the cubic eqn $\lambda = 0, 2, (2)$	M1A3 M1 A1	 6	One A mark for each coefficient (not the λ^3)

MFP4 (cont)

Q	Solution	Marks	Total	Comments
7(a)(ii)	**Alternative:**			
	Det $(\mathbf{W} - \lambda\mathbf{I}) = \begin{vmatrix} 3-\lambda & -1 & 1 \\ 2 & -\lambda & 2 \\ -1 & 1 & 1-\lambda \end{vmatrix}$			
	$= \begin{vmatrix} 2-\lambda & 0 & 2-\lambda \\ 2 & -\lambda & 2 \\ -1 & 1 & 1-\lambda \end{vmatrix}$	M1		Use of R/C ops. $R_1' \to R_1 + R_3$
	$= (2-\lambda)\begin{vmatrix} 1 & 0 & 1 \\ 2 & -\lambda & 2 \\ -1 & 1 & 1-\lambda \end{vmatrix}$	A1		Factor of $(2-\lambda)$ correctly extracted
	$= (2-\lambda)\begin{vmatrix} 1 & 0 & 0 \\ 2 & -\lambda & 0 \\ -1 & 1 & 2-\lambda \end{vmatrix}$			$C_1' \to C_3 - C_1$
	$= (2-\lambda)^2\begin{vmatrix} 1 & 0 & 0 \\ 2 & -\lambda & 0 \\ -1 & 1 & 1 \end{vmatrix}$	A1A1		
	$= (2-\lambda)^2(-\lambda)$	M1		Complete factorisation attempt
	giving eigenvalues 0 and 2 (twice)	A1	(6)	
(b)(i)	$x - y + z = 0$	B1	1	
(ii)	$\begin{bmatrix} 3 & -1 & 1 \\ 2 & 0 & 2 \\ -1 & 1 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 3a-b+c \\ 2a+2c \\ -a+b+c \end{bmatrix}$	M1A1		
	$x' - y' + z' = 3a - b + c - 2a - 2c - a + b + c$	M1		
	$= 0 \Rightarrow P'$ in H also	A1	4	Shown carefully
	Total		**13**	
8	Expanding fully: $\Delta = x^3 + y^3 + z^3 - 3xyz$	B1		
	Using row/column operations: eg $R_1' = R_1 + (R_2 + R_3)$	M1		
	$\Rightarrow \Delta = (x+y+z)\begin{vmatrix} 1 & 1 & 1 \\ z & x & y \\ y & z & x \end{vmatrix}$	A1	3	With conclusion that $(x + y + z)$ is a factor of the required expression when $k = 3$
	NB Any line of argument that leads correctly from $(x + y + z)\,f(x, y, z)$ to $x^3 + y^3 + z^3 - 3xyz$ scores full marks			
	Total		**3**	
	TOTAL		**75**	