



General Certificate of Education (A-level)
January 2012

Mathematics

MFP4

(Specification 6360)

Further Pure 4

Final

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
✓ or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
-x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MFP4

Q	Solution	Marks	Total	Comments
1	Use of $ab \cos \theta = \mathbf{a} \cdot \mathbf{b} = 21$ $\Rightarrow \cos \theta = \frac{7}{5\sqrt{2}}$ $\Rightarrow \sin \theta = \frac{1}{5\sqrt{2}}$ Use of $ \mathbf{a} \times \mathbf{b} = ab \sin \theta = 3$	M1 A1 B1 ft M1 A1	5	FT exact only CSO
	Total		5	
2(a)	Reflection in $x = z$	M1 A1	2	
(b)	Rotation about the y -axis Through $\cos^{-1} 0.6$ ($\approx 53.13^\circ$)	M1 A1 A1	3	Ignore direction
	Total		5	
3(a)	Char. Eqn. is $\lambda^2 - 8\lambda - 9 = 0$ Quadratic solved to get two roots $\Rightarrow \lambda = 9, -1$ Subst ^g . back λ (at least once) : $\lambda = 9 \Rightarrow -x + y = 0$ $\Rightarrow \lambda = 9$ has evecs. $\alpha \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\lambda = -1 \Rightarrow x + y = 0$ $\Rightarrow \lambda = -1$ has evecs. $\beta \begin{bmatrix} 1 \\ -1 \end{bmatrix}$	M1 dM1 A1 M1 A1 A1	6	Attempted any $\alpha \neq 0$ any $\beta \neq 0$
(b)	(0, 0)	B1	1	
	Total		7	

MFP4 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$\mathbf{X} \mathbf{X}^T = \begin{bmatrix} 3 & x \\ -1 & 7 \end{bmatrix} \begin{bmatrix} 3 & -1 \\ x & 7 \end{bmatrix}$ $= \begin{bmatrix} x^2 + 9 & 7x - 3 \\ 7x - 3 & 50 \end{bmatrix}$	M1	2	Attempted multn. with $\mathbf{X}^T$ correct
(b)	$\mathbf{X}^T \mathbf{X} = \begin{bmatrix} 3 & -1 \\ x & 7 \end{bmatrix} \begin{bmatrix} 3 & x \\ -1 & 7 \end{bmatrix}$ $= \begin{bmatrix} 10 & 3x - 7 \\ 3x - 7 & x^2 + 49 \end{bmatrix}$ $\mathbf{X} \mathbf{X}^T - \mathbf{X}^T \mathbf{X} = \begin{bmatrix} x^2 - 1 & 4x + 4 \\ 4x + 4 & 1 - x^2 \end{bmatrix}$ $\text{Det}(\mathbf{X} \mathbf{X}^T - \mathbf{X}^T \mathbf{X}) = \begin{vmatrix} x^2 - 1 & 4x + 4 \\ 4x + 4 & 1 - x^2 \end{vmatrix}$ $= (x + 1)^2 \begin{vmatrix} x - 1 & 4 \\ 4 & 1 - x \end{vmatrix}$ $= -(x + 1)^2 \{(x - 1)^2 + 16\} \leq 0$ for all real x	M1		Good attempt
		M1		Good attempt
		M1		Good attempt to factorise/expand the determinant
		E1	4	Explained/demonstrated fully
(c)	$x = -1$	B1	1	CSO
	Total		7	

MFP4 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$\begin{vmatrix} 2 & n & 1 \\ 3 & -1 & n \\ -1 & 7 & 1 \end{vmatrix}$ Expanding the det. of the coefft. mtx. Setting it = 0 Obtaining & solving a quadratic eqn. in n $0 = n^2 + 17n - 18 = (n + 18)(n - 1)$ $\Rightarrow n = 1, -18$	M1 M1 M1 A1	4	CSO
(b)	$n = 1$ gives $2x + y + z = 5$ $3x - y + z = 1$ $-x + 7y + z = 1$ Eliminating one variable from a pair of equations, twice e.g. ② - ① $\Rightarrow x - 2y = -4$ and ② - ③ $\Rightarrow 4x - 8y = 0$ Inconsistency clearly demonstrated from fully correct working 3 planes have no common intersection (or form a Δ^r prism)	B1 M1 A1 ft A1 ft E1 B1 ft	6	ft their chosen integer n Also ft “3 planes meet in a common line” or “3 planes form a sheaf” if consistency conclusion made
	Total		10	

MFP4 (cont)

Q	Solution	Marks	Total	Comments
6(a)	Use of scalar product on $\begin{pmatrix} 2 \\ 1 \\ 7 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 1 \\ -4 \end{pmatrix}$	M1		
	Sc.Prod. = ± 21	B1		
	Moduli $\sqrt{54}$ and $\sqrt{26}$ correct	B1		Accept 7.348... & 5.099...
	AWRT 56°	A1	4	From correct working
(b)	$2x + y + 7t = 10$ and $3x + y - 4t = 7$ noted or used	M1		
	Eliminating (say) y to get x as a fn. of t $x = 11t - 3$	M1 A1		CAO
	Subst ⁿ . back for y $y = 16 - 29t$	M1 A1		CAO
	$\frac{x+3}{11} = \frac{y-16}{-29} = z \quad (=t)$	B1 ft	6	
(c)	Attempt at either $\begin{pmatrix} \lambda + 20 \\ 9\lambda - 1 \\ 4\lambda + 7 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 1 \\ 7 \end{pmatrix} = 10$ or	M1		
	$\begin{pmatrix} \lambda + 20 \\ 9\lambda - 1 \\ 4\lambda + 7 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ 1 \\ -4 \end{pmatrix} = 7$			
	Solving either $40 + 2\lambda - 1 + 9\lambda + 49 + 28\lambda = 10$ or $60 + 3\lambda - 1 + 9\lambda - 28 - 16\lambda = 7$	M1		
	$\lambda_1 = -2$ $\lambda_2 = 6$	A1A1		
	$P = (18, -19, -1)$ and $Q = (26, 53, 31)$			
	$PQ =$ $\sqrt{8^2 + 72^2 + 32^2} = \sqrt{6272} = 56\sqrt{2}$	M1A1	6	NB P, Q not required: $d = \lambda_1 - \lambda_2 \times \mathbf{i} + 9\mathbf{j} + 4\mathbf{k} $ $= 8 \times 7\sqrt{2} = 56\sqrt{2}$ M1 A1
	Total		16	

MFP4 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$\begin{bmatrix} c & s \\ -s & c \end{bmatrix}$ $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c & s \\ -s & c \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}$ $= \begin{bmatrix} cX + sY \\ -sX + cY \end{bmatrix}$	B1 M1 A1	3	
(b)(i)	$(cX + sY)^2 - 6(cX + sY)(-sX + cY) - 7(-sX + cY)^2 = 8$ $(c^2X^2 + 2csXY + s^2Y^2) - 6([c^2 - s^2]XY + sc[Y^2 - X^2]) - 7(s^2X^2 - 2csXY + c^2Y^2) = 8$ $p = c^2 + 6sc - 7s^2$ $q = 16cs - 6(c^2 - s^2)$ $r = s^2 - 6sc - 7c^2$	M1 A1 A1 A1	4	Substn. for x & y in eqn. <i>and</i> multiplying out AG
(ii)	Factorising: $3s^2 + 8sc - 3c^2 = (3s - c)(s + 3c) = 0$ Deducing a tan value $\tan \theta = \frac{1}{3}$ (θ acute) $\cos \theta = \frac{3}{\sqrt{10}}, \sin \theta = \frac{1}{\sqrt{10}}$ Subst^g. sensible values back for p and r $2X^2 - 8Y^2 = 8$ $\frac{X^2}{2^2} - \frac{Y^2}{1^2} = 1$	M1A1 M1 A1 A1 M1 A1	8	Or by double angles Both CSO
(iii)	Since C' is a hyperbola, and it is just C rotated, it follows that C is a hyperbola	E1	1	
	Total		16	

MFP4 (cont)

Q	Solution	Marks	Total	Comments
8	For considering $\begin{vmatrix} 1 & 2n & n-1 \\ n & 2n^2+n & n^2-1 \\ n^2 & -1 & 1-n^2 \end{vmatrix}$ $= (n-1) \begin{vmatrix} 1 & 2n & 1 \\ n & 2n^2+n & n+1 \\ n^2 & -1 & -1-n \end{vmatrix}$ $= (n-1) \begin{vmatrix} 1 & 2n & 1 \\ n & 2n^2+n & n+1 \\ n(n+1) & (n+1)(2n-1) & 0 \end{vmatrix}$ $R_3' = R_3 + R_2$ $= (n-1)(n+1) \begin{vmatrix} 1 & 2n & 1 \\ n & 2n^2+n & n+1 \\ n & 2n-1 & 0 \end{vmatrix}$ $= (n-1)(n+1) \{2n^3 + 2n^2 + 2n^2 - n - 2n^3 - n^2 - 2n^2 - n + 1\}$ OR $= (n-1)(n+1) \begin{vmatrix} 1 & 2n & 1 \\ 0 & n & 1 \\ n & 2n-1 & 0 \end{vmatrix}$ $R_2' = R_2 - nR_1 =$ $(n-1)(n+1) \{2n^2 - n^2 - 2n + 1\}$ $= (n-1)(n+1)(n-1)^2$ $n = -1$	B1 M1A1 M1 M1A1 M1 A1 B1	9	Or by scalar triple product For 1st factor Row ops. for 2nd factor Full method for remaining factors CSO Note: Expanding straightaway scores B1 M1 and then A1 for $n^4 - 2n^3 + 2n - 1$. Thereafter, M1 A1 for 1st factor, M1 for 2nd factor attempted and M1 for full method for remaining factors plus A1 and B1 cso at the end, as above.
	Total		9	
	TOTAL		75	