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General Certificate of Education (A-level)
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Mathematics MFP4

(Specification 6360)

Further Pure 4

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MFP4

Q	Solution	Marks	Total	Comments
1(a)	$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ x & y & z \\ x+y+z & y+z+x & z+x+y \end{vmatrix}$	M1		e.g. $R_3{}' = R_3 + R_2$
	$= (x+y+z)\begin{vmatrix} 1 & 2 & 3 \\ x & y & z \\ 1 & 1 & 1 \end{vmatrix}$	A1	2	
(b)	Expanding remaining det.	M1		
	$\Delta = (x+y+z)(x-2y+z)$	A1	2	
	Total		**4**	
2	$c = \lvert \mathbf{a} \times \mathbf{b} \rvert = ab\sin\theta$	B1		
	$d = \lvert \mathbf{a} \bullet \mathbf{b} \rvert = ab\,\lvert\cos\theta\rvert$	B1		Condone lack of | - |
	$c^2 + d^2 = a^2b^2(\cos^2\theta + \sin^2\theta) = a^2b^2$	B1	3	Legitimately shown
	Total		**3**	
3(a)	$\begin{vmatrix} t & 2 & 3 \\ 2 & 3 & -t \\ 3 & 5 & t+1 \end{vmatrix} = 8t^2 - 7t - 1 = 0$	M1 M1		Attempt at det. of coefft. mtx. (or equivalent) Equating to zero and solving a quadratic eqn. in t
	$t = 1,\ -\frac{1}{8}$	A1	3	
(b)	$t = 1 \Rightarrow \begin{matrix} x + 2y + 3z = a \\ 2x + 3y - z = b \\ 3x + 5y + 2z = c \end{matrix}$	B1√		FT any integer value found
	E.g. ① + ② – ③ $\Rightarrow a + b = c$	M1 A1	3	
	Total		**6**	
4(a)	**(i)** $\mathbf{X}^2 = \begin{bmatrix} 23 & 1 & -3 \\ 2 & 24 & 3 \\ -4 & 2 & 19 \end{bmatrix}$	M1 A1		≥ 5 correct for the M All 9 correct for the A
	$\mathbf{X}^2 - \mathbf{X} = 20\mathbf{I}$ i.e. $k = 20$	A1	3	Shown legitimately
	(ii) Mult^{g}. $\mathbf{X}^2 - \mathbf{X} = 20\mathbf{I}$ by $\mathbf{X}^{-1}$	M1		
	Re-arranging $\mathbf{X} - \mathbf{I} = 20\,\mathbf{X}^{-1}$ to get $\mathbf{X}^{-1} = \frac{1}{20}(\mathbf{X} - \mathbf{I})$	A1	2	Legitimately
(b)	$\mathbf{X}^{-1} = \frac{1}{20}\begin{bmatrix} 2 & 1 & -3 \\ 2 & 3 & 3 \\ -4 & 2 & -2 \end{bmatrix}$	B1		Noted or used
	$(\mathbf{XY})^{-1} = \mathbf{Y}^{-1}\mathbf{X}^{-1}$	M1		Incl. attempt at the multn.
	$\begin{bmatrix} 6 & 3 & -9 \\ 2 & -1 & 1 \\ 2 & 3 & 3 \end{bmatrix}$	A1	3	
	Total		**8**	

MFP4(cont)

Q	Solution	Marks	Total	Comments
5(a)	$\Pi_1 : 6x + 2y + 9z = 5$ $\Pi_2 : 10x - y - 11z = 4$	B1	1	Both
(b)	**Method 1**			
	E.g. $\Pi_1 + 2\,\Pi_2 \Rightarrow 13(2x - z = 1)$	M1		
	$\frac{x}{1} = \frac{z+1}{2} = \lambda$	M1 A1		Parametrisation attempt
	Using 1st two to find 3rd in terms of λ: $x = \lambda,\ z = 2\lambda - 1 \Rightarrow y = 7 - 12\lambda$	M1		
	$\mathbf{r} = \begin{bmatrix} 0 \\ 7 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ -12 \\ 2 \end{bmatrix}$	A1	5	Any correct vector eqn.form
	Method 2			
	$(0, 7, -1)$ or $\left(\frac{7}{12},\ 0,\ \frac{1}{6}\right)$ or $\left(\frac{1}{2},\ 1,\ 0\right)$	(M1A1)		Finding a pt. on the line
	$\begin{bmatrix} 6 \\ 2 \\ 9 \end{bmatrix} \times \begin{bmatrix} 10 \\ -1 \\ -11 \end{bmatrix} = (\pm)13 \begin{bmatrix} 1 \\ -12 \\ 2 \end{bmatrix}$	(M1) (A1)		Finding a d.v.
	$\mathbf{r} = \begin{bmatrix} 0 \\ 7 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ -12 \\ 2 \end{bmatrix}$	(A1)		Any correct vector eqn.form
(c)	Substg. $x = \lambda,\ y = 7 - 12\lambda,\ z = 2\lambda - 1$ into $5x + 3y + 11z = 28$	M1		
	Solving a linear eqn. in λ:	M1		
	$\lambda = -2$	A1		CAO
	$(-2, 31, -5)$	B1√	4	FT their λ in their line eqn.
(d)	$\mathbf{n} = \begin{bmatrix} 1 \\ -12 \\ 2 \end{bmatrix}$	B1√		FT when used as part of a plane eqn., *not* a line
	$d = \begin{bmatrix} 4 \\ 1 \\ 9 \end{bmatrix} \bullet \begin{bmatrix} 1 \\ -12 \\ 2 \end{bmatrix} = 10$	M1		Attempted
	$\mathbf{r} \bullet \begin{bmatrix} 1 \\ -12 \\ 2 \end{bmatrix} = 10$	A1√	3	Any correct vector eqn. form FT
	Alt. $\mathbf{r} = \begin{bmatrix} 4 \\ 1 \\ 9 \end{bmatrix} + \lambda \begin{bmatrix} 6 \\ 2 \\ 9 \end{bmatrix} + \begin{bmatrix} 10 \\ -1 \\ -11 \end{bmatrix}$	(M1) (A1) (A1)		Plane eqn. attempt Point + at least 1 d.v. All correct
	Total		**13**	

MFP4(cont)

Q	Solution	Marks	Total	Comments
6(a)	$\begin{bmatrix}1\\1\\-1\end{bmatrix}\bullet\begin{bmatrix}12\\15\\16\end{bmatrix}=12+15-16=11$ shown	B1	1	
(b) (i)	eqn., or use, of line incorporating p.v. $\begin{bmatrix}1\\1\\-1\end{bmatrix}$ and d.v. $\begin{bmatrix}12\\15\\16\end{bmatrix}$	M1		
	$\frac{x-1}{12}=\frac{y-1}{15}=\frac{z+1}{16}$	A1	2	
(ii)	$\sqrt{12^2+15^2+16^2}=25$	B1√		FT
	$\frac{12}{25}, \frac{15}{25}, \frac{16}{25}$	B1√	2	FT
(c)	Use of $\mathbf{r}=\begin{bmatrix}1\\1\\-1\end{bmatrix}+\lambda\begin{bmatrix}12\\15\\16\end{bmatrix}$ with $\lambda=\pm 2$	M1		
	(25, 31, 31) and (– 23, – 29, – 33)	A1 A1	3	
(d)	<u>**Method 1**</u>			
	$PQ=5$	B1		
	Area $\Delta PMN=\frac{1}{2}\times PQ\times MN=250$	M1 A1	3	(Since Q = midpt.MN)
	<u>**Method 2**</u>			
	$\overrightarrow{PM}=\begin{bmatrix}20\\30\\35\end{bmatrix}$, $\overrightarrow{PN}=\begin{bmatrix}-28\\-30\\-29\end{bmatrix}$	(M1)		Attempted
	$\overrightarrow{PM}\times\overrightarrow{PN}=(\pm)20\begin{bmatrix}9\\-20\\12\end{bmatrix}$ attempted within an area formula	(M1)		$\vert\vert$gm. or Δ
	Area $\Delta PMN=250$	(A1)		CAO
	Total		**11**	

MFP4(cont)

Q	Solution	Marks	Total	Comments
7(a)	Attempt at Char.Eqn.	M1		
	$\lambda^3 - 9\lambda^2 + 24\lambda - 16 = 0$	A3,2,1		One each following coefft.
	Attempt at (at least) one linear factor	M1		
	$(\lambda - 1)(\lambda - 4)^2 = 0$	A1		
	$\lambda = 1, 4, 4$	A1	7	
(b)	$\lambda = 1 \Rightarrow \begin{matrix} 2x - y + z = 0 \\ -x + 2y + z = 0 \\ x + y + 2z = 0 \end{matrix} \Rightarrow \alpha\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$	M1 A1		Subst^{g}. back and solving
	$\lambda = 4 \Rightarrow x + y - z = 0$	B1		
	Choosing any two independent vectors satisfying this	M1		
	E.g.s: $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$	A1	5	
(c)	$\alpha\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \equiv$ a line of invariant points	M1 A1		Invariant line LoIPs
	e.g. $\alpha\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + \beta\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \equiv$ an invariant plane	B1	3	
	Total		**15**	

MFP4(cont)

Q	Solution	Marks	Total	Comments
8(a)	Det(**M**) = – 1	B1		
	Magnitude = 1 $\Rightarrow$ area invariant	B1$\checkmark$		FT area s.f.
	– ve sign $\Rightarrow$ cyclic order of vertices is reversed OR "reflection" involved	B1	3	
(b)	**Method 1**			
	Char. Eqn.: $\lambda^2 - 1 = 0 \Rightarrow \lambda = \pm 1$	M1 A1		Finding and solving attempt
	Subst^g. back: $\lambda = 1 \Rightarrow y = \frac{1}{2}x$	M1 A1		
	and $\lambda = -1 \Rightarrow y = \frac{1}{4}x$	A1	5	
	Method 2			
	$\begin{bmatrix} -3 & 8 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} x \\ mx \end{bmatrix} = \begin{bmatrix} (8m-3)x \\ (3m-1)x \end{bmatrix}$	(M1)		Attempted
	Use of $y' = mx'$: $3m - 1 = 8m^2 - 3m$	(M1)		
	Solving a quadratic eqn. in $m = \frac{1}{4}, \frac{1}{2}$	(M1A1)		From $(4m - 1)(2m - 1) = 0$
	$p = \frac{1}{2}$ and $q = \frac{1}{4}$	(A1)		
(c)	**(i)** $p = \frac{1}{2} = \tan\theta$			
	$\Rightarrow \cos 2\theta = \frac{3}{5}$ and $\sin 2\theta = \frac{4}{5}$	M1		For these attempted and used in a reflection matrix
	$\mathbf{R} = \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & -\frac{3}{5} \end{bmatrix}$	A1	2	
	(ii) Use $\begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & -\frac{3}{5} \end{bmatrix}\mathbf{S} = \begin{bmatrix} -3 & 8 \\ -1 & 3 \end{bmatrix}$	M1		FT their **R**
	S found using inverse matrix	M1		Or equivalent method
	$= \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & -\frac{3}{5} \end{bmatrix}\begin{bmatrix} -3 & 8 \\ -1 & 3 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -13 & 36 \\ -9 & 23 \end{bmatrix}$	A1		
	Shear, parallel to $y = \frac{1}{2}x$	B1		CAO
	mapping (e.g.) $(1, 1) \rightarrow (4.6, 2.8)$	B1$\checkmark$	5	FT any pt. and its image
	Total		**15**	
	TOTAL		**75**	