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ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MFP4 Further Pure 4

Mark Scheme

2006 examination – January series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key To Mark Scheme And Abbreviations Used In Marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

$\surd$ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MFP4

Q	Solution	Marks	Total	Comments
1	Rotation about the y-axis through an angle of 53.13°	B1 B1 B1	 3	 Ignore direction Accept $\cos^{-1} 0.6$ etc.
	Total		**3**	
2(a)	Attempt at either determinant	M1		
	det $\mathbf{P} = k - 2$ det $\mathbf{Q} = 3k - 28$	A1 A1	3	
(b)	Use of det ($\mathbf{PQ}$) = (det $\mathbf{P}$)(det $\mathbf{Q}$)	M1		
	Creating a quadratic	dM1		From $(k-2)(3k-28) = 16$
	$3k^2 - 34k + 40 = 0$	A1√		
	$k = \frac{4}{3}$ or 10	B1	4	CAO
	Alternative (b)			
	$\mathbf{PQ} = \begin{bmatrix} 28 & 2k+15 & 3 \\ 7k+2 & 2k+4 & 2k+2 \\ 48 & 3k+26 & 6 \end{bmatrix}$	B1		
	Det ($\mathbf{PQ}$) = $3k^2 - 34k + 56$	B1√		
	Equating to 16 and solving	M1		
	$k = \frac{4}{3}$ or 10	A1	4	CAO
	Total		**7**	
3(a)	(i) $\mathbf{n} = (3\mathbf{j} + \mathbf{k}) \times (4\mathbf{i} - \mathbf{j}) = \mathbf{i} + 4\mathbf{j} - 12\mathbf{k}$	M1A1	2	
	(ii) $d = (2\mathbf{i} + 5\mathbf{j} + \mathbf{k}) \cdot (\mathbf{i} + 4\mathbf{j} - 12\mathbf{k}) = 10$	M1 A1√	2	Their **n**
(b)	Either substituting.			
	r = $\begin{bmatrix} 2+3t \\ 2 \\ 5-t \end{bmatrix}$ into $\left(\mathbf{r} - \begin{bmatrix} -1 \\ 2 \\ 6 \end{bmatrix}\right) \times \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$	M1		
	And showing result is **0**	A1	2	
	Or			
	Explaining p.v. and d.v. of line	M1 A1	2	Give the M1 for either

MFP4 (cont)

Q	Solution	Marks	Total	Comments
3(c)	Subst^g. $\mathbf{r} = \begin{bmatrix} 2+3t \\ 2 \\ 5-t \end{bmatrix}$ into their plane eqn.	M1		
	Solving a linear eqn. in t	dM1		$2 + 3t + 8 - 60 + 12t = 10$
	t = 4	A1√		
	Pt. of i/sctn. at $14\mathbf{i} + 2\mathbf{j} + \mathbf{k}$	B1	4	CAO
	Alternative (a) (i)			
	$\begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix} \bullet \begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0$ and/or $\begin{bmatrix} 4 \\ -1 \\ 0 \end{bmatrix} \bullet \begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0$	M1		
	$\Rightarrow c = -3b, b = 4a$ i.e. $a(\mathbf{i} + 4\mathbf{j} - 12\mathbf{k})$	A1	2	
	First alternative (c)			Uses (b)'s given form for L and the original form for $\prod$
	Setting $\begin{bmatrix} 2+3t \\ 2 \\ 5-t \end{bmatrix} = \begin{bmatrix} 2+4\mu \\ 5+3\lambda-\mu \\ 1+\lambda \end{bmatrix}$	M1		Must make an attempt to solve
	$\mu = \frac{3}{4}t$, $\mu = 3 + 3\lambda$, $\lambda = 4 - t$	A1		
	Eliminating λ and μ (e.g.) $\Rightarrow$ t = 4	dM1		
	Pt. of i/sctn. at $14\mathbf{i} + 2\mathbf{j} + \mathbf{k}$	A1	4	CAO
3(c)	**Second alternative (c)**			Uses other given form for L
	$\left(\begin{bmatrix} 2+4\mu \\ 5+3\lambda-\mu \\ 1+\lambda \end{bmatrix} - \begin{bmatrix} -1 \\ 2 \\ 6 \end{bmatrix}\right) \times \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$ gives			
	$\begin{bmatrix} 3+4\mu \\ 3+3\lambda-\mu \\ \lambda-5 \end{bmatrix} \times \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -3-3\lambda+\mu \\ 3\lambda+4\mu-12 \\ 3\mu-9\lambda-9 \end{bmatrix}$	M1 A1		
	Equating to **0** and solving	M1		$\Rightarrow \lambda = 0, \mu = 3$
	Pt. of i/sctn. at $14\mathbf{i} + 2\mathbf{j} + \mathbf{k}$	A1	4	CAO
	Total		**10**	

MFP4 (cont)

Q	Solution	Marks	Total	Comments
4(a)(i)	det **M** = 15 + 2 + 4 + 12 – 1 + 10 =42	M1 A1	 2	
(ii)	Since answer is non-zero, lin. Indt.	B1	1	With explanation/reason
(b)(i)	**b . c** = 8 – 3 – 5 = 0	M1 A1	2	
(ii)	$\mathbf{b}\times\mathbf{c} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 3 & -1 \\ 4 & -1 & 5 \end{vmatrix} = \begin{bmatrix} 14 \\ -14 \\ -14 \end{bmatrix} = 14\mathbf{a}$	M1 A1	 2	i.e. $m = 14$
(iii)	**a**, **b**, **c** (mutually) perpendicular	B1	1	Or equivalent
(c)	$V = \lvert \det \mathbf{M} \rvert = 42$ **Or** $V = \mathbf{a} \,.\, \mathbf{b}\times\mathbf{c} = \begin{bmatrix} 14 \\ -14 \\ -14 \end{bmatrix} . \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} = 42$ **Or** $V = OA.OB.OC = \sqrt{3}\,.\sqrt{14}\,.\sqrt{42} = 42$	M1 A1 M1 A1 M1 A1√	 2	
	Total		**10**	
5(a)	For an *invariant line*, all points on the line have image points also on the line. For a *line of invariant points*, all points on the line map onto themselves.	 B1	 1	Clearly explained
(b)	Det = 3 The s.f.. of **area** enlargement under T	B1 B1	 2	Must mention area
5(c)	Setting $x' = x$ and $y' = y$ and solving $\Rightarrow\ y = x$ **Or** Char. Eqn. is $\lambda^2 - 4\lambda + 3 = 0$ $\lambda = 1$ gives l.o.i.p.s and $y = x$	M1 A1 M1 A1	 2	$x = 2x - y$ and $y = -x + 2y$
(d)(i)	$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ -x + c \end{bmatrix}$ $= \begin{bmatrix} 3x - c \\ -3x + 2c \end{bmatrix}$	M1 A1	 2	
(ii)	$y' = -x' + c$ also $\Rightarrow\ y = -x + c$ invariant for all c	M1 A1	 2	
(e)	Stretch, s.f. 3, perp^r. to $y = x$ (or ‖ to $y = -x$)	M1 A1 A1	 3	
	Total		**12**	

MFP4 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$\Delta = \begin{vmatrix} 0 & 0 & 1 \\ a-c & b-c & c \\ b(c-a) & a(c-b) & ab \end{vmatrix}$	M1		By $C_1' = C_1 - C_3$ and $C_2' = C_2 - C_3$
	$= (a-c)(b-c)\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & c \\ -b & -a & ab \end{vmatrix}$	A1 A1		1^{st} and 2^{nd} factors
	$= (a-c)(b-c)(b-a)$	M1		By expanding
	$= (a-b)(b-c)(c-a)$	A1	5	**Answer given**
(b)(i)	Noting $a = b = 3$, $c = 5$ in coefft. matrix $\Rightarrow$ $\Delta = 0$ since factor $(a-b) = 0$	M1		Or starting from scratch
	Hence no unique soln. to system	A1	2	Clearly explained
(ii)	$3R_2 + R_3 = 24R_1$	M1		
	giving consistency iff $24p - 3q - r = 0$	A1	2	**Answer given**
(iii)	$x + y + z = 1$ $3x + 3y + 5z = 8$ $5x + 5y + 3z = 0$			
	(2) $- 3 \times$ **(1)** or **(3)** $- 5 \times$ **(1)** $\Rightarrow$ $z = 2\frac{1}{2}$	M1 A1		
	Then $x + y = -1\frac{1}{2}$	B1		
	Setting $x = \lambda$ (e.g.) $\Rightarrow$ $y = -1\frac{1}{2} - \lambda$	M1		Parametrisation
	All correct, any form	A1	5	CAO
	<u>Special Case</u> ruling for those who don't attempt to parametrise but who show this *is* the full solution: **4 + B0**			
	First alternative (a)			
	Setting $a = b$ (etc.) $\Rightarrow$ $\Delta = 0$ since $C_1 = C_2$	M1		
	$\Rightarrow$ $(a-b)$ a factor, by the factor theorem	A1		
	By cyclic symmetry, $(b-c)$ and $(c-a)$ are also factors	M1 A1		
	Checking sign/coefft, = (+) 1	A1	5	
	Second alternative (a)			
	Expanding the determinant	M1		
	$\Delta = ab^2 + a^2c + bc^2 - b^2c - ac^2 - a^2b$	A1		
	Multiplying out $(a-b)(b-c)(c-a\}$	M1		No fudging, or jumping
	$= abc + ab^2 + a^2c + bc^2 - b^2c - ac^2 - a^2b - abc$	A1		straight to the answer
	Fully correct working to show the 2 things =	A1	5	allowed
	Total		**14**	

MFP4 (Cont)

Q	Solution	Marks	Total	Comments
7(a)	$\mathbf{M}\begin{bmatrix}1\\1\\2\end{bmatrix} = \begin{bmatrix}2\\2\\4\end{bmatrix}, \mathbf{M}\begin{bmatrix}0\\1\\1\end{bmatrix} = \begin{bmatrix}0\\-2\\-2\end{bmatrix}$	M1 A1 A1		Either/both attempted Correct
	$\Rightarrow \lambda_U = 2 \Rightarrow \lambda_V = -2$	B1 B1√	5	If appropriate
(b)	$\mathbf{M}\begin{bmatrix}a\\b\\c\end{bmatrix} = \begin{bmatrix}a-b+c\\3a-3b+c\\3a-5b+3c\end{bmatrix} = \begin{bmatrix}a\\b\\c\end{bmatrix}$	M1 A1 dM1		Attempt Correct image Equating to original + solving
	$\Rightarrow$ b = c and $3a + \text{c} = 4b$	A1 A1		
	Evec. is any non-zero multiple of $\begin{bmatrix}1\\1\\1\end{bmatrix}$	A1	6	
(c)(i)	$\begin{bmatrix}1\\2\\3\end{bmatrix} = \mathbf{u} + \mathbf{v}$ or $\begin{bmatrix}1\\1\\2\end{bmatrix} + \begin{bmatrix}0\\1\\1\end{bmatrix}$	B1	1	
(ii)	$\mathbf{M}^n\begin{bmatrix}1\\2\\3\end{bmatrix} = \mathbf{M}^n(\mathbf{u} + \mathbf{v}) = \mathbf{M}^n\mathbf{u} + \mathbf{M}^n\mathbf{v}$	M1 A1		
	$= 2^n\ \mathbf{u} + (-2)^n\ \mathbf{v}$	M1 A1	4	Replacing **M**'s by evals $\lambda = 2$ and $\mu = -2$
(iii)	$\mathbf{M}^n\begin{bmatrix}1\\2\\3\end{bmatrix} = \begin{bmatrix}2^n\\2^n\\2\times 2^n\end{bmatrix} - \begin{bmatrix}0\\2^n\\2^n\end{bmatrix}$	M1 B1		Since n is odd (clearly stated)
	$= \begin{bmatrix}2^n\\0\\2^n\end{bmatrix}$	A1	3	

MFP4 (cont)

Q	Solution	Marks	Total	Comments
7	**Alternative (a) and (b)**			
	Char. Eqn. is $k^3 - k^2 - 4k + 4 = 0$	M1 A1 A1 A1		Attempt. One each correct (non-leading) coefft.
	Factorisation $(k-1)(k-2)(k+2)$	M1		Attempt
	$k = 1, \pm 2$	A1	6	CAO
	Subst^{g}. back any eval.	M1		
	Solving relevant system(s)	dM1		
	$k = 1$: $\begin{bmatrix}1\\1\\1\end{bmatrix}$; $k = 2$: $\begin{bmatrix}1\\1\\2\end{bmatrix}$; $k = -2$: $\begin{bmatrix}0\\1\\1\end{bmatrix}$	A1 A1 A1	5	Any (non-zero) multiple will do for $k = 1$. For $k = \pm 2$, they must have the evecs. given in the qn. visibly displayed.
	Total		**19**	
	Total		**75**	