

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										



General Certificate of Education
Advanced Level Examination
June 2012

Mathematics

MFP4

Unit Further Pure 4

Friday 22 June 2012 1.30 pm to 3.00 pm

For this paper you must have:

- the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed

- 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	



J U N 1 2 M F P 4 0 1

Answer **all** questions.

Answer each question in the space provided for that question.

1 Find the value of the constant p for which the vectors

$$\mathbf{u} = 3\mathbf{i} + 2\mathbf{j} + p\mathbf{k}, \quad \mathbf{v} = 7\mathbf{i} - \mathbf{j} + 6\mathbf{k} \quad \text{and} \quad \mathbf{w} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$$

are linearly dependent.

(3 marks)

QUESTION
PART
REFERENCE

Answer space for question 1



2 A line has vector equation $\left(\mathbf{r} - \begin{bmatrix} 3 \\ -2 \\ 6 \end{bmatrix} \right) \times \begin{bmatrix} 4 \\ 7 \\ -4 \end{bmatrix} = \mathbf{0}$.

(a) Determine the direction cosines of this line. (3 marks)

(b) Explain the geometrical significance of the direction cosines in relation to the line. (1 mark)

QUESTION
PART
REFERENCE

Answer space for question 2

Turn over ►



3

$$\text{Let } \Delta = \begin{vmatrix} yz & xz & xy \\ x & y & z \\ x^2 & -y^2 & z^2 \end{vmatrix}.$$

- (a) Show that $(y + z)$ is a factor of Δ . (2 marks)
- (b) Factorise Δ as completely as possible. (4 marks)

QUESTION
PART
REFERENCE

Answer space for question 3



Answer space for question 3

This image shows a blank sheet of white paper designed for writing. It features a series of evenly spaced horizontal blue lines across its entire width. A single vertical red line runs down the left side, creating a narrow margin. The paper is otherwise completely empty, with no text or markings.

4 The lines L_1 and L_2 have equations

$$\mathbf{r} = \begin{bmatrix} 7 \\ -25 \\ 9 \end{bmatrix} + \alpha \begin{bmatrix} 3 \\ -4 \\ 7 \end{bmatrix} \quad \text{and} \quad \mathbf{r} = \begin{bmatrix} 7 \\ 19 \\ -2 \end{bmatrix} + \beta \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$$

respectively.

(a) Determine a vector, \mathbf{n} , which is perpendicular to both lines. (2 marks)

(b) (i) The point A on L_1 and the point B on L_2 are such that $\overrightarrow{AB} = \lambda \mathbf{n}$ for some constant λ .

Show that

$$3\alpha - 2\beta + 2\lambda = 0$$

$$4\alpha - 2\beta - 5\lambda = -44$$

$$7\alpha - 3\beta + 2\lambda = -11 \quad (3 \text{ marks})$$

(ii) Find the position vectors of A and B . (3 marks)

(iii) Deduce the shortest distance between L_1 and L_2 . (2 marks)

QUESTION
PART
REFERENCE

Answer space for question 4



QUESTION
PART
REFERENCE

Answer space for question 4

Handwriting practice area with horizontal dotted lines.

Turn over ►



QUESTION
PART
REFERENCE

Answer space for question 4

Handwriting practice area with horizontal dotted lines.



Answer space for question 4

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5 The matrix $\mathbf{M} = \begin{bmatrix} -11 & 9 \\ -16 & 13 \end{bmatrix}$ represents the plane transformation T.

- (a) (i) Determine the eigenvalue, and a corresponding eigenvector, of \mathbf{M} . (4 marks)
- (ii) Hence write down the value of m for which $y = mx$ is the invariant line of T which passes through the origin, and explain why it is actually a line of invariant points. (2 marks)
- (iii) Show that, for this value of m , all lines with equations $y = mx + c$ are invariant lines of T. (3 marks)
- (b) Given that T is a shear, give a full geometrical description of this transformation. (2 marks)
- (c) Give a full geometrical description of the plane transformation represented by the matrix \mathbf{M}^{-1} . (2 marks)

QUESTION
PART
REFERENCE

Answer space for question 5



Answer space for question 5

[illegible]

Answer space for question 5

[illegible]

Answer space for question 5

This image shows a full page of primary-ruled paper. It features a vertical solid line on the left side, creating a narrow margin. The rest of the page is filled with horizontal dashed lines, providing a guide for letter height and placement. There are no markings or text on the page.

6 The planes Π_1 , Π_2 and Π_3 have cartesian equations

$$2x + y - z = 3$$

$$3x - 2y + z = 5$$

$$12x - y - z = 40$$

respectively.

- (a) Find, in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{d}$, a vector equation for the line L which is the intersection of Π_1 and Π_2 . (5 marks)
- (b) (i) Determine whether L meets Π_3 , and use your answer to decide whether the system given by the equations of these three planes is consistent or inconsistent. (3 marks)
- (ii) Describe geometrically the arrangement of the three planes. (1 mark)
- (c) (i) Find the coordinates of a common point of Π_2 and Π_3 . (3 marks)
- (ii) Deduce a vector equation for the line of intersection of Π_2 and Π_3 . (1 mark)

QUESTION
PART
REFERENCE

Answer space for question 6



Answer space for question 6

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QUESTION
PART
REFERENCE

Answer space for question 6

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Answer space for question 6

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7 The matrix $\mathbf{A} = \begin{bmatrix} k & 1 & 2 \\ 2 & k & 1 \\ 1 & 2 & k \end{bmatrix}$, where k is a real constant.

(a) (i) Show that there is a value of k for which

$$\mathbf{A}\mathbf{A}^T = m\mathbf{I}$$

where m is a rational number to be determined and \mathbf{I} is the 3×3 identity matrix.

(6 marks)

(ii) Deduce the inverse matrix, \mathbf{A}^{-1} , of \mathbf{A} for this value of k .

(1 mark)

(b) (i) Find $\det \mathbf{A}$ in terms of k .

(2 marks)

(ii) In the case when \mathbf{A} is singular, find the integer value of k and show that there are no other possible real values of k .

(3 marks)

(iii) Find the value of k for which $\lambda = 7$ is a real eigenvalue of \mathbf{A} .

(2 marks)

QUESTION
PART
REFERENCE

Answer space for question 7



Answer space for question 7

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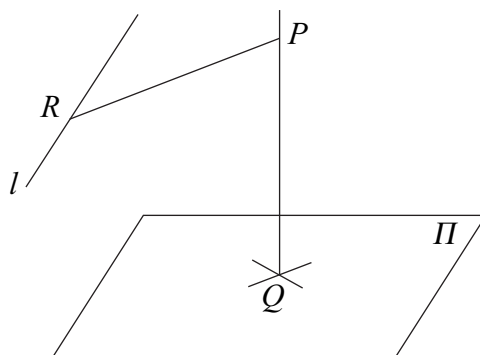
Answer space for question 7

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Answer space for question 7

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- 8** The point Q has position vector $\mathbf{q} = \begin{bmatrix} 7 \\ 4 \\ 6 \end{bmatrix}$, the plane Π has equation $\mathbf{r} \cdot \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} = 36$,
and the line l has equation $\mathbf{r} = \begin{bmatrix} 20 \\ -8 \\ 1 \end{bmatrix} + \mu \begin{bmatrix} -7 \\ 5 \\ 3 \end{bmatrix}$.
- (a)** Show that Q lies in Π . (1 mark)
- (b)** Show also that l is parallel to Π . (2 marks)
- (c)** The diagram shows the point P , which lies on the normal to Π that passes through Q . The point R is the point on l which is closest to P , and $PQ = PR$.



Determine the coordinates of P .

(9 marks)

QUESTION	PART	REFERENCE
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Answer space for question 8

[illegible]

Answer space for question 8

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Answer space for question 8

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