

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										



General Certificate of Education
Advanced Level Examination
January 2013

Mathematics

MFP4

Unit Further Pure 4

Wednesday 30 January 2013 9.00 am to 10.30 am

For this paper you must have:

- the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed

- 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	



J A N 1 3 M F P 4 0 1

Answer **all** questions.

Answer each question in the space provided for that question.

1 Two planes have equations

$$x + 2y + 2z = 5 \quad \text{and} \quad px + 3y = 10$$

where p is a non-zero constant.

Given that the acute angle, θ , between the planes is such that $\cos \theta = \frac{2}{3}$, find the value of p . (5 marks)

QUESTION
PART
REFERENCE

Answer space for question 1



QUESTION
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REFERENCE

Answer space for question 1

Answer space for question 1

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2 It is given that **A** and **B** are 3×3 matrices such that

$$\det(\mathbf{AB}) = 24 \quad \text{and} \quad \det(\mathbf{A}^{-1}) = -3$$

(a) State the value of $\det \mathbf{A}$. (1 mark)

(b) A three-dimensional shape S , with volume 20 cm^3 , is transformed using matrix **B**.

Find the volume of the image of S . (3 marks)

QUESTION
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Answer space for question 2



Answer space for question 2

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3 (a) Expand and simplify, as far as possible,

$$(\mathbf{a} - 4\mathbf{b}) \times (\mathbf{a} + 3\mathbf{b})$$

where $\mathbf{a}$ and $\mathbf{b}$ are vectors.

(3 marks)

(b) Given that $\mathbf{a}$ and $\mathbf{b}$ are perpendicular, deduce that

$$|(\mathbf{a} - 4\mathbf{b}) \times (\mathbf{a} + 3\mathbf{b})| = \lambda |\mathbf{a}| |\mathbf{b}|$$

where λ is an integer.

(2 marks)

QUESTION
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Answer space for question 3



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4

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

- (a)** Given that $\mathbf{A}^2 = \begin{bmatrix} p & -2 & -4 \\ 5 & 6 & 4 \\ 10 & q & 9 \end{bmatrix}$, find the value of p and the value of q . (2 marks)

- (b)** Given that $\mathbf{A}^3 - 6\mathbf{A}^2 + 11\mathbf{A} - 6\mathbf{I} = \mathbf{0}$, prove that

$$\mathbf{A}^{-1} = \frac{1}{6}(\mathbf{A}^2 - 6\mathbf{A} + 11\mathbf{I}) \quad (2 \text{ marks})$$

- (c) Given that $\mathbf{A}^{-1} = \frac{1}{6} \begin{bmatrix} r & -2 & 2 \\ -1 & 5 & -2 \\ -2 & s & 2 \end{bmatrix}$, find the value of r and the value of s . (2 marks)

- (d)** Hence, or otherwise, find the solution of the system of equations

$$x \quad - \quad z = k$$

$$x + 2y + z = 5$$

$$2x + 2y + 3z = 7$$

giving your answers in terms of k . (3 marks)

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QUESTION
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Answer space for question 4

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QUESTION
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5 (a)

By direct expansion, or otherwise, show that the value of

$$\begin{vmatrix} -2 & 1 & 2k \\ -1 & 1 & k+1 \\ 2 & k-1 & 1 \end{vmatrix}$$

is independent of k .

*(4 marks)***(b)**

State, with a reason, whether the vectors

$$\begin{bmatrix} -2 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \text{ and } \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}$$

are linearly dependent or linearly independent.

*(2 marks)***(c) (i)**

State, with a reason, whether the equations

$$-2x + y + 6z = 1$$

$$-x + y + 4z = 0$$

$$2x + 2y + z = -1$$

are consistent or inconsistent.

(2 marks)

(ii) The three equations given in part **(c)(i)** are the Cartesian equations of three planes.

State the geometrical configuration of these three planes.

(1 mark)

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6 The linear transformations T_1 and T_2 are represented by the matrices

$$\mathbf{M}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \text{ and } \mathbf{M}_2 = \begin{bmatrix} \frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ -\frac{\sqrt{3}}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

respectively.

(a) Give a full geometrical description of the transformations:

(i) T_1 ; (2 marks)

(ii) T_2 . (3 marks)

(b) Find the matrix which represents the transformation T_1 followed by T_2 . (2 marks)

(c) The linear transformation T_3 is represented by the matrix

$$\mathbf{M}_3 = \begin{bmatrix} k & 2 & -1 \\ 1 & 1 & 1 \\ 3 & 4 & 1 \end{bmatrix}$$

where k is a constant.

For one particular value of k , T_3 has a line L of invariant points.

(i) Find k .

(ii) Find the Cartesian equations of L in the form $\frac{x}{p} = \frac{y}{q} = \frac{z}{r}$. (7 marks)

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7 The matrix $\mathbf{M}$ is defined by

$$\mathbf{M} = \begin{bmatrix} -a & 0 & a \\ 0 & 6 & 0 \\ a & 0 & 2 \end{bmatrix}$$

where a is a real number. The distinct eigenvalues of $\mathbf{M}$ are λ_1 , λ_2 and λ_3 with corresponding eigenvectors $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$.

(a) Given that $\mathbf{v}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, find λ_1 . (2 marks)

(b) Given that $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$, find the value of a . (3 marks)

(c) Given that $\lambda_3 = -6$, find a possible eigenvector $\mathbf{v}_3$. (3 marks)

(d) The matrix $\mathbf{M}$ can be expressed as $\mathbf{UDU}^{-1}$, where $\mathbf{D}$ is a diagonal matrix.

Write down possible matrices $\mathbf{D}$ and $\mathbf{U}$. (3 marks)

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8 The four vertices of a parallelogram $ABCD$ have coordinates

$$A(1, 0, 2), B(3, -1, 5), C(7, 2, 4) \text{ and } D(5, 3, 1)$$

(a) (i) Find $\overrightarrow{AB} \times \overrightarrow{AD}$. (3 marks)

(ii) Show that the area of the parallelogram is $p\sqrt{10}$, where p is an integer to be found. (2 marks)

(b) The diagonals AC and BD of the parallelogram meet at the point M . The line L passes through M and is perpendicular to the plane $ABCD$.

Find an equation for the line L , giving your answer in the form $(\mathbf{r} - \mathbf{u}) \times \mathbf{v} = \mathbf{0}$. (4 marks)

(c) The plane Π is parallel to the plane $ABCD$ and passes through the point $Q(6, 5, 17)$.

(i) Find the coordinates of the point of intersection of the line L with the plane Π . (6 marks)

(ii) One face of a parallelepiped is $ABCD$ and the opposite face lies in the plane Π .

Find the volume of the parallelepiped. (3 marks)

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Answer space for question 8

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QUESTION
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Answer space for question 8

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END OF QUESTIONS



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