

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										



General Certificate of Education
Advanced Level Examination
January 2011

Mathematics

MFP3

Unit Further Pure 3

Monday 24 January 2011 9.00 am to 10.30 am

For this paper you must have:

- the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed

- 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
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4	
5	
6	
7	
8	
TOTAL	



J A N 1 1 M F P 3 0 1

Answer **all** questions in the spaces provided.

1 The function $y(x)$ satisfies the differential equation

$$\frac{dy}{dx} = f(x, y)$$

where $f(x, y) = x + \sqrt{y}$

and $y(3) = 4$

Use the improved Euler formula

$$y_{r+1} = y_r + \frac{1}{2}(k_1 + k_2)$$

where $k_1 = hf(x_r, y_r)$ and $k_2 = hf(x_r + h, y_r + k_1)$ and $h = 0.1$, to obtain an approximation to $y(3.1)$, giving your answer to three decimal places. (5 marks)

QUESTION
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QUESTION
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Turn over ►



- 2 (a)** Find the values of the constants p and q for which $p \sin x + q \cos x$ is a particular integral of the differential equation

$$\frac{dy}{dx} + 5y = 13 \cos x \quad (3 \text{ marks})$$

- (b)** Hence find the general solution of this differential equation. (3 marks)

QUESTION
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QUESTION
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Turn over ►



0 5

(a) Find the cartesian equation of C , giving your answer in the form $y^2 = f(x)$.
(5 marks)

(b) The straight line with polar equation $4r = 3 \sec \theta$ intersects the curve C at the points P and Q . Find the length of PQ . (4 marks)

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QUESTION
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4

By using an integrating factor, find the solution of the differential equation

$$\frac{dy}{dx} - \frac{2}{x}y = 2x^3e^{2x}$$

given that $y = e^4$ when $x = 2$. Give your answer in the form $y = f(x)$. (9 marks)QUESTION
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5 (a) Write $\frac{4}{4x+1} - \frac{3}{3x+2}$ in the form $\frac{C}{(4x+1)(3x+2)}$, where C is a constant. (1 mark)

(b) Evaluate the improper integral

$$\int_1^{\infty} \frac{10}{(4x+1)(3x+2)} dx$$

showing the limiting process used and giving your answer in the form $\ln k$, where k is a constant. (6 marks)

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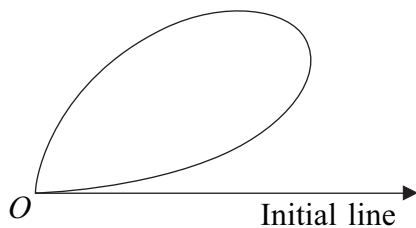


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- 6 The diagram shows a sketch of a curve C .



The polar equation of the curve is

$$r = 2 \sin 2\theta \sqrt{\cos \theta}, \quad 0 \leq \theta \leq \frac{\pi}{2}$$

Show that the area of the region bounded by C is $\frac{16}{15}$. (7 marks)

QUESTION
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QUESTION
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Turn over ►



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8 (a) Given that $x = e^t$ and that y is a function of x , show that

$$x \frac{dy}{dx} = \frac{dy}{dt} \quad (2 \text{ marks})$$

(b) Hence show that the substitution $x = e^t$ transforms the differential equation

$$x^2 \frac{d^2 y}{dx^2} - 3x \frac{dy}{dx} + 4y = 2 \ln x$$

into

$$\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 4y = 2t \quad (5 \text{ marks})$$

(c) Find the general solution of the differential equation

$$\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 4y = 2t \quad (6 \text{ marks})$$

(d) Hence solve the differential equation $x^2 \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} + 4y = 2 \ln x$, given that $y = \frac{3}{2}$ and $\frac{dy}{dx} = \frac{1}{2}$ when $x = 1$.

QUESTION	PART	REFERENCE
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