General Certificate of Education
January 2006
Advanced Level Examination

MATHEMATICS **MFP3**
Unit Further Pure 3

ASSESSMENT and QUALIFICATIONS ALLIANCE

Friday 27 January 2006 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP3.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 (a) Find the roots of the equation $m^2 + 2m + 2 = 0$ in the form $a + \mathrm{i}b$. *(2 marks)*

(b) (i) Find the general solution of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2\frac{\mathrm{d}y}{\mathrm{d}x} + 2y = 4x$$

(6 marks)

(ii) Hence express y in terms of x, given that $y = 1$ and $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2$ when $x = 0$. *(4 marks)*

2 (a) Find $\displaystyle\int_0^a x\mathrm{e}^{-2x}\,\mathrm{d}x$, where $a > 0$. *(5 marks)*

(b) Write down the value of $\displaystyle\lim_{a\to\infty} a^k \mathrm{e}^{-2a}$, where k is a positive constant. *(1 mark)*

(c) Hence find $\displaystyle\int_0^\infty x\mathrm{e}^{-2x}\,\mathrm{d}x$. *(2 marks)*

3 (a) Show that $y = x^3 - x$ is a particular integral of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{2xy}{x^2 - 1} = 5x^2 - 1$$

(3 marks)

(b) By differentiating $(x^2 - 1)y = c$ implicitly, where y is a function of x and c is a constant, show that $y = \dfrac{c}{x^2 - 1}$ is a solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{2xy}{x^2 - 1} = 0$$

(3 marks)

(c) Hence find the general solution of

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{2xy}{x^2 - 1} = 5x^2 - 1$$

(2 marks)

4 (a) Use the series expansion

$$\ln(1+x) = x - \tfrac{1}{2}x^2 + \tfrac{1}{3}x^3 - \tfrac{1}{4}x^4 + \ldots$$

to write down the first four terms in the expansion, in ascending powers of x, of $\ln(1-x)$. *(1 mark)*

(b) The function f is defined by

$$\mathrm{f}(x) = \mathrm{e}^{\sin x}$$

Use Maclaurin's theorem to show that when $\mathrm{f}(x)$ is expanded in ascending powers of x:

(i) the first three terms are

$$1 + x + \tfrac{1}{2}x^2$$ *(6 marks)*

(ii) the coefficient of x^3 is zero. *(3 marks)*

(c) Find

$$\lim_{x \to 0} \frac{\mathrm{e}^{\sin x} - 1 + \ln(1-x)}{x^2 \sin x}$$ *(4 marks)*

Turn over for the next question

5 (a) The function $y(x)$ satisfies the differential equation

$$\frac{dy}{dx} = f(x, y)$$

where $$f(x, y) = x \ln x + \frac{y}{x}$$

and $$y(1) = 1$$

(i) Use the Euler formula

$$y_{r+1} = y_r + h\,f(x_r, y_r)$$

with $h = 0.1$, to obtain an approximation to $y(1.1)$. *(3 marks)*

(ii) Use the formula

$$y_{r+1} = y_{r-1} + 2h\,f(x_r, y_r)$$

with your answer to part (a)(i) to obtain an approximation to $y(1.2)$, giving your answer to three decimal places. *(4 marks)*

(b) (i) Show that $\frac{1}{x}$ is an integrating factor for the first-order differential equation

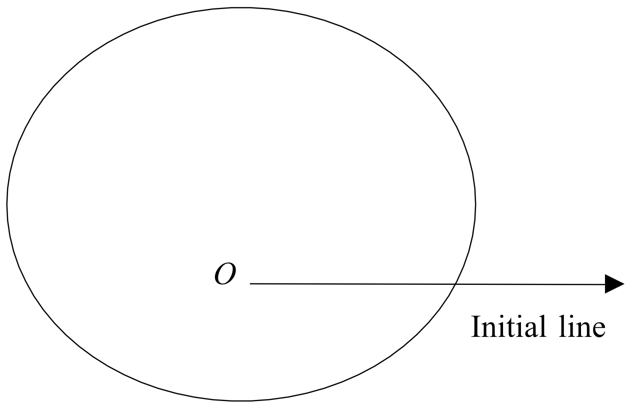
$$\frac{dy}{dx} - \frac{1}{x}y = x \ln x$$ *(3 marks)*

(ii) Solve this differential equation, given that $y = 1$ when $x = 1$. *(6 marks)*

(iii) Calculate the value of y when $x = 1.2$, giving your answer to three decimal places. *(1 mark)*

6 (a) A circle C_1 has cartesian equation $x^2 + (y - 6)^2 = 36$. Show that the polar equation of C_1 is $r = 12\sin\theta$. *(4 marks)*

(b) A curve C_2 with polar equation $r = 2\sin\theta + 5,\ 0 \leqslant \theta \leqslant 2\pi$ is shown in the diagram.



Calculate the area bounded by C_2. *(6 marks)*

(c) The circle C_1 intersects the curve C_2 at the points P and Q. Find, in surd form, the area of the quadrilateral $OPMQ$, where M is the centre of the circle and O is the pole. *(6 marks)*

END OF QUESTIONS

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