



# **General Certificate of Education**

## **Mathematics 6360**

**MFP3**

**Further Pure 3**

## **Mark Scheme**

*2008 examination - January series*

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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**Key to mark scheme and abbreviations used in marking**

M	mark is for method		
m or dM	mark is dependent on one or more M marks and is for method		
A	mark is dependent on M or m marks and is for accuracy		
B	mark is independent of M or m marks and is for method and accuracy		
E	mark is for explanation		
✓ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
−x EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

**No Method Shown**

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

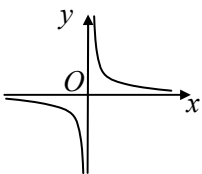
**MFP3**

<b>Q</b>	<b>Solution</b>	<b>Marks</b>	<b>Total</b>	<b>Comments</b>
<b>1(a)</b>	$y(2.1) = y(2) + 0.1[2^2 - 1^2]$ $= 1 + 0.1 \times 3 = 1.3$	M1A1 A1	3	
<b>(b)</b>	$y(2.2) = y(2) + 2(0.1)[f(2.1, y(2.1))]$  .... $= 1 + 2(0.1)[2.1^2 - 1.3^2]$  .... $= 1 + 0.2 \times 2.72 = 1.544$	M1  A1✓  A1	  3	  Ft on cand's answer to (a)  CAO
<b>Total</b>			<b>6</b>	
<b>2(a)</b>	Area $= \frac{1}{2} \int (1 + \tan \theta)^2 d\theta$  .... $= \frac{1}{2} \int (1 + 2 \tan \theta + \tan^2 \theta) d\theta$  $= \frac{1}{2} \int (\sec^2 \theta + 2 \tan \theta) d\theta$  $= \frac{1}{2} [\tan \theta + 2 \ln(\sec \theta)] \Big _0^{\frac{\pi}{3}}$  $= \frac{1}{2} [(\sqrt{3} + 2 \ln 2) - 0] = \frac{\sqrt{3}}{2} + \ln 2$	M1  B1  M1  A1✓ B1✓  A1	      6	Use of $\frac{1}{2} \int r^2 d\theta$  Correct expansion of $(1 + \tan \theta)^2$  $1 + \tan^2 \theta = \sec^2 \theta$ used  Integrating $p \sec^2 \theta$ correctly Integrating $q \tan \theta$ correctly  Completion. AG CSO be convinced
<b>(b)</b>	$OP = 1$ ; $OQ = 1 + \tan \frac{\pi}{3}$ Shaded area = 'answer (a)' $- \frac{1}{2} OP \times OQ \times \sin\left(\frac{\pi}{3}\right)$  $= \frac{\sqrt{3}}{2} + \ln 2 - \frac{\sqrt{3}}{4}(1 + \sqrt{3})$  $= \frac{\sqrt{3}}{4} + \ln 2 - \frac{3}{4}$	B1  M1  A1	   3	Both needed. Accept 2.73 for $OQ$   ACF. Condone 0.376... if exact 'value' for area of triangle seen
<b>Total</b>			<b>9</b>	

**MFP3 (cont)**

<b>Q</b>	<b>Solution</b>	<b>Marks</b>	<b>Total</b>	<b>Comments</b>
<b>3(a)</b>	$(m+2)^2 = -1$ $m = -2 \pm i$  CF is $e^{-2x}(A \cos x + B \sin x)$ {or $e^{-x}A \cos(x+B)$ but not $Ae^{(-1+i)x} + Be^{(-1-i)x}$ }  PI try $y = p \Rightarrow 5p = 5$ PI is $y = 1$  GS $y = e^{-2x}(A \cos x + B \sin x) + 1$	M1 A1  M1 A1✓  B1  B1✓	6	Completing sq or formula  If $m$ is real give M0 Ft on wrong $a$ 's and $b$ 's but roots must be complex  Their CF + their PI with two arbitrary constants.
<b>(b)</b>	$x=0, y=2 \Rightarrow A=1$ $y'(x) = -2e^{-2x}(A \cos x + B \sin x) + e^{-2x}(-A \sin x + B \cos x)$  $y'(0) = 3 \Rightarrow 3 = -2A + B \Rightarrow B = 5$ $y = e^{-2x}(\cos x + 5 \sin x) + 1$	B1✓ M1 A1✓  A1✓	4	Provided previous B1✓ awarded Product rule used  Ft on one slip
<b>Total</b>			<b>10</b>	
<b>4(a)</b>	The interval of integration is infinite	E1	1	OE
<b>(b)</b>	$\int x e^{-3x} dx = -\frac{1}{3} x e^{-3x} - \int -\frac{1}{3} e^{-3x} dx$  $= -\frac{1}{3} x e^{-3x} - \frac{1}{9} e^{-3x} \{+c\}$	M1 A1  A1✓	3	Reasonable attempt at parts  Condone absence of $+c$
<b>(c)</b>	$I = \int_1^{\infty} x e^{-3x} dx = \lim_{a \rightarrow \infty} \int_1^a x e^{-3x} dx$  $\lim_{a \rightarrow \infty} \left\{ -\frac{1}{3} a e^{-3a} - \frac{1}{9} e^{-3a} \right\} - \left[ -\frac{4}{9} e^{-3} \right]$  $\lim_{a \rightarrow \infty} a e^{-3a} = 0$  $I = \frac{4}{9} e^{-3}$	M1  M1  A1	3	F(a) – F(1) with an indication of limit ' $a \rightarrow \infty$ '  For statement with limit/limiting process shown
<b>Total</b>			<b>7</b>	

**MFP3 (cont)**

Q	Solution	Marks	Total	Comments
5	<p>IF is <math>e^{\int \frac{4x}{x^2+1} dx}</math></p> <p><math>= e^{2\ln(x^2+1)}</math></p> <p><math>= e^{\ln(x^2+1)^2} = (x^2+1)^2</math></p> <p><math>\frac{d}{dx}(y(x^2+1)^2) = x(x^2+1)^2</math></p> <p><math>y(x^2+1)^2 = \int x(x^2+1)^2 dx</math></p> <p><math>y(x^2+1)^2 = \frac{1}{6}(x^2+1)^3 + c</math></p> <p><math>y(0) = 1 \Rightarrow c = \frac{5}{6}</math></p> <p><math>y = \frac{1}{6}(x^2+1) + \frac{5}{6(x^2+1)^2}</math></p>	<p>M1</p> <p>A1</p> <p>A1✓</p> <p>M1</p> <p>A1✓</p> <p>M1</p> <p>A1</p> <p>m1</p> <p>A1</p>	9	<p>Ft on <math>e^{p\ln(x^2+1)}</math></p> <p>LHS as <math>d/dx(y \times \text{cand's IF})</math> PI and also RHS of form <math>kx(x^2+1)^p</math></p> <p>Use of suitable substitution to find RHS or reaching <math>k(x^2+1)^3</math> OE</p> <p>Condone missing <math>c</math></p> <p>Accept other forms of <math>f(x)</math></p> <p>eg <math>y = \frac{\left(\frac{x^6}{6} + \frac{2x^4}{4} + \frac{x^2}{2} + 1\right)}{(x^2+1)^2}</math></p>
<b>Total</b>			<b>9</b>	
6(a)	<p><math>r^2 \sin \theta \cos \theta = 8</math></p> <p><math>x = r \cos \theta \quad y = r \sin \theta</math></p> <p><math>xy = 4 \quad , \quad y = \frac{4}{x}</math></p>	<p>M1</p> <p>M1</p> <p>A1</p>	3	<p><math>\sin 2\theta = 2 \sin \theta \cos \theta</math> used</p> <p>Either <u>one</u> stated or used</p> <p>Either OE eg <math>y = \frac{8}{2x}</math></p>
(b)		B1	1	
(c)	<p><math>r = 2 \sec \theta</math> is <math>x = 2</math></p> <p>Sub <math>x = 2</math> in <math>xy = 4 \Rightarrow 2y = 4</math></p> <p>In cartesian, <math>A(2, 2)</math></p> <p><math>\Rightarrow \tan \theta = \frac{y}{x} = 1 \Rightarrow \theta = \frac{\pi}{4}</math></p> <p><math>\Rightarrow r = \sqrt{x^2 + y^2} = \sqrt{8}</math></p> <p><math>\theta = \frac{\pi}{4} ; r = \sqrt{8}</math></p> <p>Altn2: Eliminating <math>r</math> to reach eqn. in <math>\cos \theta</math> and <math>\sin \theta</math> only (M1) <math>\theta = \frac{\pi}{4}</math> (A1)</p> <p>Substitution <math>r = 2 \sec \left(\frac{\pi}{4}\right)</math> (m1)</p> <p><math>r = \sqrt{8}</math> (A1) OE surd</p>	<p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p>	4	<p>Used either <math>\tan \theta = \frac{y}{x}</math> or <math>r = \sqrt{x^2 + y^2}</math></p> <p><math>r</math> must be given in surd form</p> <p>Altn3: <math>r \sin \theta = 2</math> (B1)</p> <p>Solving <math>r \cos \theta = 2</math> and <math>r \sin \theta = 2</math> simultaneously (M1)</p> <p><math>\tan \theta = 1</math> or <math>r^2 = 2^2 + 2^2</math> (M1)</p> <p><math>\theta = \frac{\pi}{4} ; r = \sqrt{8}</math> (A1) need both</p>
<b>Total</b>			<b>8</b>	

**MFP3 (cont)**

<b>Q</b>	<b>Solution</b>	<b>Marks</b>	<b>Total</b>	<b>Comments</b>
<b>7(a)(i)</b>	$\ln(1+2x) = 2x - 2x^2 + \frac{8}{3}x^3 \dots$	M1 A1	2	Use of expansion of $\ln(1+x)$ Simplified 'numerators'.
<b>(ii)</b>	$-\frac{1}{2} < x \leq \frac{1}{2}$	B1	1	
<b>(b)(i)</b>	$y = \ln \cos x \Rightarrow y'(x) = \frac{1}{\cos x}(-\sin x)$ $y''(x) = -\sec^2 x$ $y'''(x) = -2\sec x (\sec x \tan x)$ $\{y'''(x) = -2\tan x (\sec^2 x)\}$	M1 A1 M1		ACF Chain rule OE
<b>(ii)</b>	$y''''(x) = -2[\sec^2 x (\sec^2 x) + \tan x (2\sec x (\sec x \tan x))]$ $y''''(0) = -2[(1)^2 + 0] = -2$	M1 A1		Product rule OE ACF
<b>(iii)</b>	$\ln \cos x \approx 0 + 0 + \frac{x^2}{2}(-1) + 0 + \frac{x^4}{4!}(-2)$ $\approx -\frac{x^2}{2} - \frac{x^4}{12}$	M1 A1	3 2	Ft a slip CSO throughout part (b). AG
<b>(c)</b>	Limit = $\lim_{x \rightarrow 0} \left[ \frac{x \ln(1+2x)}{x^2 - \ln \cos x} \right]$ = $\lim_{x \rightarrow 0} \left[ \frac{x(2x - 2x^2 + \dots)}{x^2 - \left(-\frac{x^2}{2} - \frac{x^4}{12} \dots\right)} \right]$ Limit = $\lim_{x \rightarrow 0} \frac{2x^2 - o(x^3)}{1.5x^2 + o(x^4)}$ = $\lim_{x \rightarrow 0} \frac{2 - o(x)}{1.5 + o(x^2)} = \frac{4}{3}$	M1 A1		Using earlier expansions The notation $o(x^n)$ can be replaced by a term of the form $kx^n$ Need to see stage, division by $x^2$
	<b>Total</b>		<b>15</b>	

**MFP3 (cont)**

<b>Q</b>	<b>Solution</b>	<b>Marks</b>	<b>Total</b>	<b>Comments</b>
<b>8(a)(i)</b>	$\frac{dx}{dt} = e^t \quad \{=x\}$ $x \frac{dy}{dx} = x \frac{dy}{dt} \frac{dt}{dx}$ $= x \frac{dy}{dt} \frac{1}{x} = \frac{dy}{dt}$	B1 M1 A1	3	Chain rule Completion. AG
<b>(ii)</b>	$\frac{d^2 y}{dt^2} = \frac{d}{dt} \left( x \frac{dy}{dx} \right) =$ $= \frac{dx}{dt} \frac{dy}{dx} + x \frac{d}{dt} \left( \frac{dy}{dx} \right)$ $\dots = \frac{dy}{dt} + x \frac{dx}{dt} \frac{d}{dx} \left( \frac{dy}{dx} \right)$ $\dots = \frac{dy}{dt} + x^2 \left( \frac{d^2 y}{dx^2} \right)$ $\Rightarrow x^2 \frac{d^2 y}{dx^2} = \frac{d^2 y}{dt^2} - \frac{dy}{dt}$	M1 M1 A1	3	Product rule Condone leaving in this form AG
<b>(b)</b>	$x^2 \frac{d^2 y}{dx^2} - 6x \frac{dy}{dx} + 6y = 0$ $\Rightarrow \frac{d^2 y}{dt^2} - 7 \frac{dy}{dt} + 6y = 0$ Auxl eqn $m^2 - 7m + 6 = 0$ $(m - 6)(m - 1) = 0$ $m = 1$ and $6$ $y = Ae^{6t} + Be^t$ $y = Ax^6 + Bx$	M1 m1 A1 M1 A1✓	5	Using results in (a) to reach DE of this form PI PI Must be solving the 'correct' DE. (Give M1A0 for $y = Ae^{6x} + Be^x$ ) Ft a minor slip only if previous A0 and all three method marks gained
	<b>Total</b>		<b>11</b>	
	<b>TOTAL</b>		<b>75</b>	