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General Certificate of Education

Mathematics 6360

MFP3 Further Pure 3

Mark Scheme

2006 examination - January series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key To Mark Scheme And Abbreviations Used In Marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–x EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MFP3

Q	Solution	Marks	Total	Comments
1(a)	$(m+1)^2 = -1$	M1		Completing sq or formula
	$m = -1 \pm i$	A1	2	
(b)(i)	CF is $e^{-x}(A\cos x + B\sin x)$	M1		If m is real give M0
	{or $e^{-x}A\cos(x+B)$ **but not** $Ae^{(-1+i)x} + Be^{(-1-i)x}$}	A1√		On wrong a's and b's but roots must be complex.
	{P.Int.} try $y = px + q$	M1		OE
	$2p + 2(px+q) = 4x$	A1		
	$p = 2,\ \ q = -2$	A1√		On one slip
	GS $y = e^{-x}(A\cos x + B\sin x) + 2x - 2$	B1√	6	Their CF + their PI with two arbitrary constants.
(ii)	$x=0,\ y=1 \Rightarrow A = 3$	B1√		Provided an M1gained in (b)(i)
	$y'(x) = -e^{-x}(A\cos x + B\sin x) +$	M1		Product rule used
	$+ e^{-x}(-A\sin x + B\cos x) + 2$	A1√		
	$y'(0) = 2 \Rightarrow 2 = -A+B+2 \Rightarrow B = 3$	A1√		Slips
	$y = 3e^{-x}(\cos x + \sin x) + 2x - 2$		4	
	Total		**12**	
2(a)	$\int xe^{-2x}dx = -\frac{1}{2}xe^{-2x} - \int -\frac{1}{2}e^{-2x}dx$	M1 A1		Reasonable attempt at parts
	$= -\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}\ \{+c\}$	A1√		Condone absence of $+c$
	$\int_0^a xe^{-2x}dx = -\frac{1}{2}ae^{-2a} - \frac{1}{4}e^{-2a} - (0 - \frac{1}{4})$	M1		F(a) – F(0)
	$= \frac{1}{4} - \frac{1}{2}ae^{-2a} - \frac{1}{4}e^{-2a}$	A1	5	
(b)	$\lim_{a\to\infty} a^k e^{-2a} = 0$	B1	1	
(c)	$\int_0^\infty xe^{-2x}dx =$			
	$= \lim_{a\to\infty} \{\frac{1}{4} - \frac{1}{2}ae^{-2a} - \frac{1}{4}e^{-2a}\}$	M1		If this line oe is missing then 0/2
	$= \frac{1}{4} - 0 - 0 = \frac{1}{4}$	A1√	2	On candidate's "1/4" in part (a). B1 must have been earned
	Total		**8**	

MFP3

Q	Solution	Marks	Total	Comments
3(a)	$y = x^3 - x \Rightarrow y'(x) = 3x^2 - 1$	B1		Accept general cubic.
	$\frac{dy}{dx} + \frac{2xy}{x^2-1} = 3x^2 - 1 + \frac{2x(x^3 - x)}{x^2 - 1}$	M1		Substitution into LHS of DE
	$= 3x^2 - 1 + \frac{2x^2(x^2-1)}{x^2-1} = 5x^2 - 1$	A1	3	Completion. If using general cubic all unknown constants must be found
(b)	$\frac{d}{dx}\left[(x^2-1)y\right] = 2xy + (x^2-1)\frac{dy}{dx}$	M1A1		
	Differentiating $(x^2-1)y = c$ wrt x leads to $2xy + (x^2-1)\frac{dy}{dx} = 0$			**SC** Differentiated but not implicitly give max of 1/3 for complete solution
	$\Rightarrow y = \frac{c}{x^2-1}$ is a soln. of $\frac{dy}{dx} + \frac{2xy}{x^2-1} = 0$	A1	3	Be generous
(c)	$\Rightarrow y = \frac{c}{x^2-1}$ is a soln with one arb. constant of $\frac{dy}{dx} + \frac{2xy}{x^2-1} = 0$			
	$\Rightarrow y = \frac{c}{x^2-1}$ is a CF of the DE			
	GS is CF + PI	M1		Must be using 'hence'; CF and PI functions of x only
	$y = \frac{c}{x^2-1} + x^3 - x$	A1	2	CSO Must have explicitly considered the link between one arbitrary constant and the GS of a first order differential equation.
	Total		**8**	

MFP3

Q	Solution	Marks	Total	Comments
4(a)	$\ln(1-x) = -x - \frac{1}{2}x^2 - \frac{1}{3}x^3 - \frac{1}{4}x^4 \ldots$	B1	1	
(b)(i)	$\mathrm{f}(x) = \mathrm{e}^{\sin x} \Rightarrow \mathrm{f}(0) = 1$	B1		
	$\mathrm{f}'(x) = \cos x\,\mathrm{e}^{\sin x}$ $\Rightarrow \mathrm{f}'(0) = 1$	M1A1		
	$\mathrm{f}''(x) = -\sin x\,\mathrm{e}^{\sin x} + \cos^2 x\,\mathrm{e}^{\sin x}$ $\mathrm{f}''(0) = 1$	M1A1		Product rule used
	Maclaurin $\mathrm{f}(x) = \mathrm{f}(0) + x\mathrm{f}'(0) + \frac{x^2}{2}\mathrm{f}''(0)$			
	so 1st three terms are $1 + x + \frac{1}{2}x^2$	A1	6	**CSO AG**
(ii)	$\mathrm{f}'''(x) = \cos x(\cos^2 x - \sin x)\,\mathrm{e}^{\sin x} +$ $+\{2\cos x(-\sin x) - \cos x\}\,\mathrm{e}^{\sin x}$	M1A1		
	$\mathrm{f}'''(0) = 0$ so the coefficient of x^3 in the series is zero	A1	3	**CSO AG** **SC** for (b): Use of series expansions….max of 4/9
(c)	$\sin x \approx x$.	B1		Ignore higher power terms in sin$x$ expansion
	$\dfrac{\mathrm{e}^{\sin x} - 1 + \ln(1-x)}{x^2 \sin x} = \dfrac{-\frac{1}{3}x^3 + o(x^4)}{x^3}$	M1 A1		Series from (a) & (b) used Numerator kx^3 (+…)
	$= \dfrac{-\frac{1}{3} + o(x)}{1 + o(x^2)}$			Condone if this step is missing
	$\lim\limits_{x \to 0} \dfrac{\mathrm{e}^{\sin x} - 1 + \ln(1-x)}{x^2 \sin x} = -\dfrac{1}{3}$	A1√	4	On candidate's x^3 coefficient in (a) provided lower powers cancel
	Total		**14**	

MFP3

Q	Solution	Marks	Total	Comments
5(a)(i)	$y(1.1) = y(1) + 0.1[1\ln 1+1/1]$	M1A1		
	$= 1+0.1 = 1.1$	A1	3	
(ii)	$y(1.2) = y(1) + 2(0.1)[f(1.1, y(1.1)]$	M1A1		
	$\ldots. = 1+2(0.1)[1.1\ln 1.1+(1.1)/1.1]$	A1√		On answer to (a)(i)
	$\ldots. = 1+0.2\times 1.104841198\ldots.$			
	$\ldots. = 1.22096824 = 1.221$ to 3dp	A1	4	CAO
(b)(i)	IF is $e^{\int -\frac{1}{x}\,dx}$	M1		Condone $e^{\int \frac{1}{x}dx}$ for M mark
	$= e^{-\ln x}$	A1		
	$= e^{\ln x^{-1}} = x^{-1} = \dfrac{1}{x}$	A1	3	**AG** (be convinced) (b)(i) Solutions using the printed answer must be convincing before any marks are awarded
(ii)	$\dfrac{d}{dx}\left(\dfrac{y}{x}\right) = \ln x$	M1A1		
	$\dfrac{y}{x} = \int \ln x\,dx = x\ln x - \int x\left(\dfrac{1}{x}\right)dx$	M1		Integration by parts for $x^k \ln x$
	$\dfrac{y}{x} = x\ln x - x + c$	A1		Condone missing c.
	$y(1) = 1 \Rightarrow 1 = \ln 1 - 1 + c$	m1		Dependent on at least one of the two previous M marks
	$\Rightarrow c = 2 \Rightarrow y = x^2\ln x - x^2 + 2x$	A1	6	OE eg $\dfrac{y}{x} = x\ln x - x + 2$
(iii)	$y(1.2) = 1.222543\ldots = 1.223$ to 3dp	B1	1	
	Total		**17**	

MFP3

Q	Solution	Marks	Total	Comments
6(a)	$x^2+y^2-12y+36=36$	M1 M1		Use of $y=r\sin\theta$ ($x=r\cos\theta$ PI) Use of $x^2+y^2=r^2$
	$r^2-12r\sin\theta+36=36$	m1		
	$\Rightarrow r=12\sin\theta$	A1	4	**CSO AG**
(b)	Area = $\frac{1}{2}\int(2\sin\theta+5)^2\,\mathrm{d}\theta$.	M1		Use of $\frac{1}{2}\int r^2\,\mathrm{d}\theta$.
	.. = $\frac{1}{2}\int_0^{2\pi}(4\sin^2\theta+20\sin\theta+25)\mathrm{d}\theta$	B1 B1		Correct expn. of $(2\sin\theta+5)^2$ Correct limits
	$=\frac{1}{2}\int_0^{2\pi}(2(1-\cos 2\theta)+20\sin\theta+25)\mathrm{d}\theta$	M1		Attempt to write $\sin^2\theta$ in terms of $\cos 2\theta$.
	$=\frac{1}{2}\left[27\theta-\sin 2\theta-20\cos\theta\right]_0^{2\pi}$	A1√		Correct integration ft wrong coeffs
	$=27\pi$.	A1	6	CSO
(c)	At intersection $12\sin\theta=2\sin\theta+5$	M1		OE eg $r=6(r-5)$
	$\Rightarrow \sin\theta=\frac{5}{10}$	A1		OE eg $r=6$
	Points $\left(6,\frac{\pi}{6}\right)$ and $\left(6,\frac{5\pi}{6}\right)$	A1		OE
	OPMQ is a rhombus of side 6			Or two equilateral triangles of side 6
	Area = $6\times6\times\sin\frac{2\pi}{3}$ oe	M1 A1		Any valid complete method to find the area (or half area) of quadrilateral.
	$=18\sqrt{3}$	A1	6	Accept unsimplified surd
	Total		**16**	
	Total		**75**	

Extra notes:

The SC for Q4

$$e^{\sin x}=1+\left(x-\frac{x^3}{3!}\ldots\right)+\frac{1}{2!}\left(x-\frac{x^3}{3!}\ldots\right)^2+\frac{1}{3!}\left(x-\frac{x^3}{3!}\ldots\right)^3\ldots$$

M1 for 1st 3 terms ignoring any higher powers than those shown.

A1 for all 4 terms (could be treated separately ie last term often only comes into (b)(ii)

$= 1 + x - \frac{x^3}{6} + \frac{1}{2}\left(x^2 - ....\right) + \frac{1}{6}\left(x^3 - ....\right)$

$= 1 + x + \frac{1}{2}x^2$ **A1 (be convinced…..ignore any powers of *x* above power 2)**

Coefficient of x^3: $-\frac{x^3}{6} + \frac{1}{6}x^3 = 0$ **A1 (be convinced…..ignore any powers of *x* above power 3)**

Quite often the 2nd A mark is awarded before the 1st A1