AQA

General Certificate of Education
Advanced Level Examination
June 2011

Mathematics　　　　MFP2

Unit Further Pure 2

Monday 13 June 2011　9.00 am to 10.30 am

For this paper you must have:
• the blue AQA booklet of formulae and statistical tables.
You may use a graphics calculator.

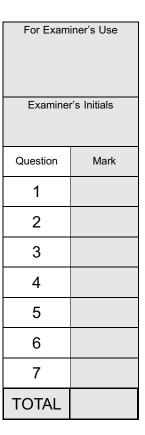## Time allowed
• 1 hour 30 minutes

Instructions
• Use black ink or black ball-point pen. Pencil should only be used for drawing.
• Fill in the boxes at the top of this page.
• Answer **all** questions.
• Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
• You must answer the questions in the spaces provided. Do not write outside the box around each page.
• Show all necessary working; otherwise marks for method may be lost.
• Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
• The marks for questions are shown in brackets.
• The maximum mark for this paper is 75.

Advice
• Unless stated otherwise, you may quote formulae, without proof, from the booklet.

MFP2

Answer **all** questions in the spaces provided.

1 (a) Draw on the same Argand diagram:

(i) the locus of points for which

$$|z - 2 - 5i| = 5$$

(3 marks)

(ii) the locus of points for which

$$\arg(z + 2i) = \frac{\pi}{4}$$

(3 marks)

(b) Indicate on your diagram the set of points satisfying both

$$|z - 2 - 5i| \leqslant 5$$

and

$$\arg(z + 2i) = \frac{\pi}{4}$$

(2 marks)

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2 (a) Use the definitions of $\cosh\theta$ and $\sinh\theta$ in terms of e^θ to show that

$$\cosh x \cosh y - \sinh x \sinh y = \cosh(x - y)$$

(4 marks)

(b) It is given that x satisfies the equation

$$\cosh(x - \ln 2) = \sinh x$$

(i) Show that $\tanh x = \frac{5}{7}$.

(4 marks)

(ii) Express x in the form $\frac{1}{2}\ln a$.

(2 marks)

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3 (a) Show that

$$(r+1)! - (r-1)! = (r^2 + r - 1)(r-1)!$$ *(2 marks)*

(b) Hence show that

$$\sum_{r=1}^{n}(r^2 + r - 1)(r-1)! = (n+2)n! - 2$$ *(4 marks)*

QUESTION PART REFERENCE	

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4   The cubic equation

$$z^3 - 2z^2 + k = 0 \qquad (k \neq 0)$$

has roots α, β and γ.

(a) (i) Write down the values of $\alpha + \beta + \gamma$ and $\alpha\beta + \beta\gamma + \gamma\alpha$. *(2 marks)*

(ii) Show that $\alpha^2 + \beta^2 + \gamma^2 = 4$. *(2 marks)*

(iii) Explain why $\alpha^3 - 2\alpha^2 + k = 0$. *(1 mark)*

(iv) Show that $\alpha^3 + \beta^3 + \gamma^3 = 8 - 3k$. *(2 marks)*

(b) Given that $\alpha^4 + \beta^4 + \gamma^4 = 0$:

(i) show that $k = 2$; *(4 marks)*

(ii) find the value of $\alpha^5 + \beta^5 + \gamma^5$. *(3 marks)*

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5 (a)   The arc of the curve $y^2 = x^2 + 8$ between the points where $x = 0$ and $x = 6$ is rotated through 2π radians about the x-axis. Show that the area S of the curved surface formed is given by

$$S = 2\sqrt{2}\pi \int_0^6 \sqrt{x^2 + 4}\, dx$$

(5 marks)

(b)   By means of the substitution $x = 2\sinh\theta$, show that

$$S = \pi(24\sqrt{5} + 4\sqrt{2}\sinh^{-1} 3)$$

(8 marks)

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6 (a) Show that

$$(k+1)(4(k+1)^2 - 1) = 4k^3 + 12k^2 + 11k + 3$$ *(2 marks)*

(b) Prove by induction that, for all integers $n \geqslant 1$,

$$1^2 + 3^2 + 5^2 + ... + (2n-1)^2 = \tfrac{1}{3}n(4n^2 - 1)$$ *(6 marks)*

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7 (a) (i) Use de Moivre's Theorem to show that

$$\cos 5\theta = \cos^5 \theta - 10 \cos^3 \theta \sin^2 \theta + 5 \cos \theta \sin^4 \theta$$

and find a similar expression for $\sin 5\theta$. *(5 marks)*

(ii) Deduce that

$$\tan 5\theta = \frac{\tan \theta (5 - 10 \tan^2 \theta + \tan^4 \theta)}{1 - 10 \tan^2 \theta + 5 \tan^4 \theta}$$ *(3 marks)*

(b) Explain why $t = \tan \dfrac{\pi}{5}$ is a root of the equation

$$t^4 - 10t^2 + 5 = 0$$

and write down the three other roots of this equation in trigonometrical form.
(3 marks)

(c) Deduce that

$$\tan \frac{\pi}{5} \tan \frac{2\pi}{5} = \sqrt{5}$$ *(5 marks)*

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END OF QUESTIONS