General Certificate of Education
June 2006
Advanced Level Examination

MATHEMATICS **MFP2**
Unit Further Pure 2

Monday 19 June 2006 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP2.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 (a) Given that

$$\frac{r^2+r-1}{r(r+1)} = A + B\left(\frac{1}{r} - \frac{1}{r+1}\right)$$

find the values of A and B. *(3 marks)*

(b) Hence find the value of

$$\sum_{r=1}^{99} \frac{r^2+r-1}{r(r+1)}$$

(4 marks)

2 A curve has parametric equations

$$x = t - \frac{1}{3}t^3, \quad y = t^2$$

(a) Show that

$$\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2 = (1+t^2)^2$$

(3 marks)

(b) The arc of the curve between $t = 1$ and $t = 2$ is rotated through 2π radians about the x-axis.

Show that S, the surface area generated, is given by $S = k\pi$, where k is a rational number to be found. *(5 marks)*

3 The curve C has equation

$$y = \cosh x - 3\sinh x$$

(a) (i) The line $y = -1$ meets C at the point $(k, -1)$.

Show that

$$e^{2k} - e^{k} - 2 = 0$$ *(3 marks)*

(ii) Hence find k, giving your answer in the form $\ln a$. *(4 marks)*

(b) (i) Find the x-coordinate of the point where the curve C intersects the x-axis, giving your answer in the form $p\ln a$. *(4 marks)*

(ii) Show that C has no stationary points. *(3 marks)*

(iii) Show that there is exactly one point on C for which $\dfrac{d^2y}{dx^2} = 0$. *(1 mark)*

4 (a) On one Argand diagram, sketch the locus of points satisfying:

(i) $|z - 3 + 2i| = 4$; *(3 marks)*

(ii) $\arg(z - 1) = -\frac{1}{4}\pi$. *(3 marks)*

(b) Indicate on your sketch the set of points satisfying both

$$|z - 3 + 2i| \leqslant 4$$

and $$\arg(z - 1) = -\frac{1}{4}\pi$$ *(1 mark)*

Turn over for the next question

5 The cubic equation

$$z^3 - 4\mathrm{i}z^2 + qz - (4 - 2\mathrm{i}) = 0$$

where q is a complex number, has roots α, β and γ.

(a) Write down the value of:

(i) $\alpha + \beta + \gamma$; *(1 mark)*

(ii) $\alpha\beta\gamma$. *(1 mark)*

(b) Given that $\alpha = \beta + \gamma$, show that:

(i) $\alpha = 2\mathrm{i}$; *(1 mark)*

(ii) $\beta\gamma = -(1 + 2\mathrm{i})$; *(2 marks)*

(iii) $q = -(5 + 2\mathrm{i})$. *(3 marks)*

(c) Show that β and γ are the roots of the equation

$$z^2 - 2\mathrm{i}z - (1 + 2\mathrm{i}) = 0$$ *(2 marks)*

(d) Given that β is real, find β and γ. *(3 marks)*

6 (a) The function f is given by

$$\mathrm{f}(n) = 15^n - 8^{n-2}$$

Express

$$\mathrm{f}(n+1) - 8\mathrm{f}(n)$$

in the form $k \times 15^n$. *(4 marks)*

(b) Prove by induction that $15^n - 8^{n-2}$ is a multiple of 7 for all integers $n \geqslant 2$. *(4 marks)*

7 (a) Find the six roots of the equation $z^6 = 1$, giving your answers in the form $\mathrm{e}^{\mathrm{i}\phi}$, where $-\pi < \phi \leqslant \pi$. *(3 marks)*

(b) It is given that $w = \mathrm{e}^{\mathrm{i}\theta}$, where $\theta \neq n\pi$.

(i) Show that $\dfrac{w^2 - 1}{w} = 2\mathrm{i}\sin\theta$. *(2 marks)*

(ii) Show that $\dfrac{w}{w^2 - 1} = -\dfrac{\mathrm{i}}{2\sin\theta}$. *(2 marks)*

(iii) Show that $\dfrac{2\mathrm{i}}{w^2 - 1} = \cot\theta - \mathrm{i}$. *(3 marks)*

(iv) Given that $z = \cot\theta - \mathrm{i}$, show that $z + 2\mathrm{i} = zw^2$. *(2 marks)*

(c) (i) Explain why the equation

$$(z + 2\mathrm{i})^6 = z^6$$

has five roots. *(1 mark)*

(ii) Find the five roots of the equation

$$(z + 2\mathrm{i})^6 = z^6$$

giving your answers in the form $a + \mathrm{i}b$. *(4 marks)*

END OF QUESTIONS

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