Version 1.0

General Certificate of Education (A-level)
June 2012

Mathematics MFP2

(Specification 6360)

Further Pure 2

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

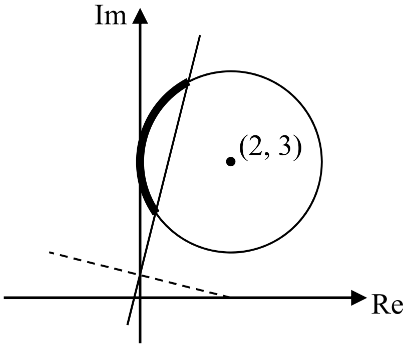
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MFP2

Q	Solution	Marks	Total	Comments
1(a)	Sketch of $y = \cosh x$	B1	1	approximately correct with minimum point above the *x*-axis, symmetrical about *y*-axis
(b)	Attempt to factorise	M1		or complete square or use (correct unsimplified) formula
	$(3\cosh x - 5)(2\cosh x + 1) = 0$	A1		
	$\cosh x \neq -\frac{1}{2}$	E1		indicated or stated (not merely neglected)
	$x = \ln\left(\frac{5}{3} + \sqrt{\frac{25}{9} - 1}\right)$	M1		evidence of use of formula. Must see –1 or equivalent
		A1F		ft incorrect factorisation
	$= \pm \ln 3$	A1F	6	A1 for $\pm$
	Alternative:			
	$3\left(\frac{e^x + e^{-x}}{2}\right) = 5$			
	$3e^{2x} - 10e^x + 3 = 0$	(M1)		
	$(3e^x - 1)(e^x - 3) = 0$	(A1F)		Correct factors
	$x = \ln\frac{1}{3}$ or $\ln 3$	(A1F)		for both
	NB if $\cosh x = \frac{e^x + e^{-x}}{2}$ used initially, M0 until quartic in e^x is factorised			M1 for e^x–3 is a factor A1 if correct M1 for $3e^x$–1is a factor A1 if correct A1 for $x = \pm \ln 3$ E1 for showing remaining quadratic has no real roots
	Total		**7**	

MFP2

Q	Solution	Marks	Total	Comments
2(a)				
(i)	Circle	B1		Convex loop
	Correct centre	B1		Some indication of position of centre
	Touching Im axis	B1	3	
(ii)	Straight line well to left of centre	B1		$\frac{1}{2}$ line through $(0, \frac{1}{2})$ B0
	through $(0, \frac{1}{2})$	B1		Point approximately between 0 and 1
	$\perp$ to line joining $(-2,1)$ and $(2,0)$	B1	3	
	NB 0/3 for line parallel to *x*-axis 0/3 for line joining the two points (–2, 1) and (2,0) 0/3 for line joining (0,0) to centre of circle			
(b)	Minor arc indicated	B1F	1	ft incorrect position of line or circle
	Total		**7**	

MFP2

Q	Solution	Marks	Total	Comments
3(a)	Attempt to put LHS over common denominator	M1		
	$\frac{2^{r+1}(r+1)-2^r(r+2)}{(r+1)(r+2)}$	A1		any form
	$=\frac{r(2^{r+1}-2^r)}{(r+1)(r+2)}$			
	$=\frac{r2^r}{(r+1)(r+2)}$ must see $r2^{r+1}=2r2^r$	A1	3	clearly shown as AG
(b)	$\frac{2^2}{3}-\frac{2}{2}$			
	$\frac{2^3}{4}-\frac{2^2}{3}$			
	………	M1		3 rows indicated (PI)
	$\frac{2^{31}}{32}-\frac{2^{30}}{31}$			
	$S_{30}=\frac{2^{31}}{32}-1$ or $S_n=\frac{2^{n+1}}{n+2}-1$	A1		
	$=2^{26}-1$	A1	3	CAO
	Total		**6**	
4(a)(i)	$\alpha+\beta+\gamma=0$	B1	1	
(ii)	$\alpha\beta\gamma=-q$	B1	1	
(b)	$\alpha^3+p\alpha+q=0$	M1		
	$\sum\alpha^3+p\sum\alpha+3q=0$	m1		
	$\alpha^3+\beta^3+\gamma^3=3\alpha\beta\gamma$	A1	3	AG
	Alternative to (b)			
	Use of			
	$\left(\sum\alpha\right)^3=\left(\sum\alpha^3\right)+6\alpha\beta\gamma+3\left(\sum\alpha\sum\alpha\beta-3\alpha\beta\gamma\right)$	(M1)		
	Substitution of $\sum\alpha=0$	(m1)		
	Result	(A1)		
(c)(i)	$\beta=4-7\mathrm{i},\ \gamma=-8$	B1,B1	2	
(ii)	Attempt at either p or q	M1		
	$p=1$	A1F		
	$q=520$	A1F	3	ft incorrect roots provided p and q are real
(d)	Replace z by $\frac{1}{z}$ in cubic equation	M1		or $\sum\frac{1}{\alpha}=-\frac{p}{q},\sum\frac{1}{\alpha\beta}=0,\frac{1}{\alpha\beta\gamma}=-\frac{1}{q}$
		A1F		ft on incorrect p and/or q
	$520z^3+z^2+1=0$ coefficients must be integers	A1	3	CAO
	Total		**13**	

MFP2

Q	Solution	Marks	Total	Comments
5(a)	$\frac{1}{x} = \cos y$ or $\frac{1}{y} = \cos x$	M1		
	$y = \cos^{-1}\frac{1}{x}$ ie result	A1	2	CSO
(b)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\sec^{-1} x\right) = \frac{\mathrm{d}}{\mathrm{d}x}\left(\cos^{-1}\frac{1}{x}\right)$	M1		
	$= -\frac{1}{\sqrt{1-\frac{1}{x^2}}}$ if in terms of u A0	A1		
	$\times\left(-\frac{1}{x^2}\right)$	A1		
	$= \frac{1}{\sqrt{x^4 - x^2}}$	A1	4	clearly shown (AG)
	Alternative			
	$\cos y = \frac{1}{x}$			Use of $\sec y = x$ M0
	$-\sin y\ \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-1}{x^2}$	(M1) (A1)		
	Substitute for $\sin y$	(A1)		
	Result	(A1)		
	Total		**6**	

MFP2

Q	Solution	Marks	Total	Comments
6(a)	Use of $\cosh 2x = 2\cosh^2 x - 1$	M1		or $\cosh 4x = 2\cosh^2 2x - 1$
	$\text{RHS} = \frac{1}{2}\cosh 2x + \frac{1}{2}\cosh^2 2x$	A1		
	$= \frac{1}{4}(1 + 2\cosh 2x + \cosh 4x)$	A1	3	
	If substituted for both cosh 4x and cosh 2x in LHS M1 only, until corrected If RHS is put in terms of e^x M1 for correct substitution A1 for correct expansion A1 for correct result			
(b)	$\frac{dy}{dx} = 2\cosh x \sinh x = \sinh 2x$	M1A1		allow A1 for $1 + \left(\frac{dy}{dx}\right)^2 = 1 - 4\cosh^2 x + 4\cosh^4 x$ Incorrect form for $\cosh^2 x$ in terms of cosh 2x M1 only
	Or $y = \left(\frac{e^x + e^{-x}}{2}\right)^2 = \frac{e^{2x} + 2 + e^{-2x}}{4}$ $\frac{dy}{dx} = \frac{2e^{2x} - 2e^x}{4}$	(M1)		
	$= \sinh 2x$	(A1)		
	$1 + \left(\frac{dy}{dx}\right)^2 = 1 + \sinh^2 2x = \cosh^2 2x$	A1	3	AG
(c)	$S = 2\pi\int_{(0)}^{(\ln 2)} \cosh^2 x \cosh 2x \, dx$	M1A1		allow even if limits missing
	$= 2\pi\int_0^{\ln 2} \frac{1}{4}(1 + 2\cosh 2x + \cosh 4x)\, dx$	m1		
	$= \frac{2\pi}{4}\left[x + \frac{2\sinh 2x}{2} + \frac{\sinh 4x}{4}\right]$	A1		Integrated correctly
	Correct use of limits	m1		
	$a = 128$, $b = 495$	A1,A1	7	accept correct answers written down with no working. Only one A1 if 2π not used
	Total		**13**	

MFP2

Q	Solution	Marks	Total	Comments
7(a)	Assume true for $n = k$			
	Then $\sum_{r=1}^{k+1} \frac{2r+1}{r^2(r+1)^2}$			
	$= 1 - \frac{1}{(k+1)^2} + \frac{2k+3}{(k+1)^2(k+2)^2}$	M1A1		M1A0 if no LHS
	$= 1 - \frac{1}{(k+1)^2}\left(1 - \frac{2k+3}{(k+2)^2}\right)$	m1		attempt to factorise or put over a common denominator
	$= 1 - \frac{1}{(k+1)^2}\left(\frac{k^2+2k+1}{(k+2)^2}\right)$	A1		any correct combination starting 1–
	$= 1 - \frac{1}{(k+2)^2}$	A1		
	True for $n = 1$ LHS = RHS = $\frac{3}{4}$	B1		
	Method of induction set out properly	E1	7	must score all 6 previous marks for this mark
(b)	$(n+1)^2 > 10^5$ or $\frac{1}{(n+1)^2} > 10^{-5}$	M1		Condone equals
	$n+1 > 316.2$			
	$n > 315.2$			
	$n = 316$	A1	2	
	Total		**9**	

MFP2

Q	Solution	Marks	Total	Comments
8(a)	Use of $(\cos\theta + \mathrm{i}\sin\theta)^n = \cos n\theta + \mathrm{i}\sin n\theta$	M1		Stated or used
	$\cos(-n\theta) + \mathrm{i}\sin(-n\theta) = \cos n\theta - \mathrm{i}\sin n\theta$	A1		allow $\frac{2}{3}$ if this line is assumed allow if complex conjugate used
	$z^n + \dfrac{1}{z^n} = 2\cos n\theta$	A1	3	AG
(b)(i)	$z^8 + 4z^4 + 6 + 4z^{-4} + z^{-8}$	B1	1	allow in retrospect
(ii)	$z^2 + \dfrac{1}{z^2} = 2\cos 2\theta$ used	B1		Can be implied from (b)(i)
	$(2\cos 2\theta)^4 = 2\cos 8\theta + 8\cos 4\theta + 6$	M1A1		M1 for RHS A1 for whole line
	$\cos^4 2\theta = \frac{1}{8}\cos 8\theta + \frac{1}{2}\cos 4\theta + \frac{3}{8}$	A1F	4	ft coefficients on previous line
	Alternative to (b)(ii) $\cos^4 2\theta = \left(\dfrac{1+\cos 4\theta}{2}\right)^2$	(M1) (A1)		
	$\cos^2 4\theta = \frac{1}{2}(1+\cos 8\theta)$ Final result	(B1) (A1)		
(c)	$8\cos^4 2\theta = \cos 8\theta + 5 \to \cos 4\theta = \frac{1}{2}$	M1 A1F		ft provided simplifies to $\cos 4\theta = p$
	$k = \frac{1}{12}, \frac{5}{12}, \frac{7}{12}, \frac{11}{12}$	A1	3	CAO
(d)	$\int_0^{\frac{\pi}{2}} \cos^4 2\theta \, \mathrm{d}\theta =$			
	$\left[\dfrac{\sin 8\theta}{64} + \dfrac{\sin 4\theta}{8} + \dfrac{3}{8}\theta\right]_0^{\frac{\pi}{2}}$	M1 A1F		ie their $\cos^4 2\theta$
	$= \dfrac{3\pi}{16}$	A1	3	AG
	Total		**14**	
	TOTAL		**75**	