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General Certificate of Education (A-level)
January 2011

Mathematics MFP2

(Specification 6360)

Further Pure 2

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

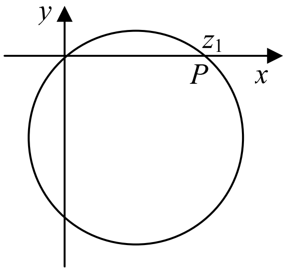
Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MFP2

Q	Solution	Marks	Total	Comments
1(a)	 Circle	B1		
	correct centre	B1		
	through (0, 0)	B1	3	
(b)(i)	z_1 correctly chosen	B1F	1	ft if circle encloses (0, 0)
(ii)	$\lvert z_1 \rvert = 8$	B1F	1	ft if centre misplotted
	Total		**5**	
2(a)	$u_r - u_{r-1} =$			
	$\frac{1}{6}r(r+1)(4r+11) - \frac{1}{6}(r-1)r(4r+7)$	M1		
	Correct expansion in any form, eg			
	$\frac{1}{6}r(4r^2+15r+11-4r^2-3r+7)$	A1		
	$= r(2r+3)$	A1	3	AG
(b)	Attempt to use method of differences	M1		
	$S_{100} = u_{100} - u_0$	A1		
	$= 691850$	A1	3	CAO
	Total		**6**	
3(a)	$(1+\mathrm{i})^2 = 2\mathrm{i}$ or $(1+\mathrm{i}) = \sqrt{2}\,\mathrm{e}^{\frac{\pi \mathrm{i}}{4}}$	B1		
	$2\mathrm{i}(1+\mathrm{i}) = 2\mathrm{i} - 2$	B1	2	AG **Alternative method:** $(1+\mathrm{i})^3 = 1+3\mathrm{i}+3\mathrm{i}^2+\mathrm{i}^3$ B1 $= 2\mathrm{i} - 2$ B1
(b)(i)	Substitute $z = 1+\mathrm{i}$	M1		
	Correct expansion	A1		allow for correctly picking out either the real or the imaginary parts
	$k = -5$	A1	3	
(ii)	$\beta + \gamma = 5 + \mathrm{i} - \alpha = 4$	B1	1	AG
(iii)	$\alpha\beta\gamma = 5(1+\mathrm{i})$	M1		allow if sign error
	$\beta\gamma = 5$	A1F		ft incorrect k
	$z^2 - 4z + 5 = 0$	M1		
	Use of formula or $(z-2)^2 = -1$	A1F		No ft for real roots if error in k
	$z = 2 \pm \mathrm{i}$ NB allow marks for (b) in whatever order they appear	A1F	5	
	Total		**11**	

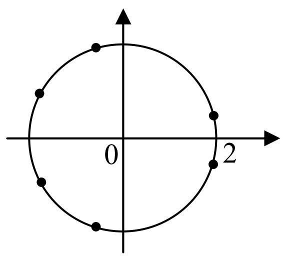
MFP2 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}=12\sinh x-8\cosh x-1$	B1		The B1 and M1 could be in reverse order if put in terms of e first
	$12\frac{\left(\mathrm{e}^{x}-\mathrm{e}^{-x}\right)}{2}-8\frac{\left(\mathrm{e}^{x}+\mathrm{e}^{-x}\right)}{2}-1=0$	M1		M0 if $\sinh x$ and $\cosh x$ in terms of e^{x} are interchanged
	$2\mathrm{e}^{2x}-\mathrm{e}^{x}-10=0$	A1F		ft slips of sign
	$\left(2\mathrm{e}^{x}-5\right)\left(\mathrm{e}^{x}+2\right)=0$	M1A1F		ft provided quadratic factorises
	$\mathrm{e}^{x}\neq-2$	E1		some indication of rejection needed
	$x=\ln\frac{5}{2}$ one stationary point	A1F	7	Condone $\mathrm{e}^{x}=\frac{5}{2}$ with statement provided quadratic factorises **Special Case** If $\frac{\mathrm{d}y}{\mathrm{d}x}=12\sinh x-8\cosh x$ B0 For substitution in terms of e^{x} M1 leading to $\mathrm{e}^{2x}=5$ A1 Then M0
(b)	$b=12\frac{\left(\frac{5}{2}+\frac{2}{5}\right)}{2}-8\frac{\left(\frac{5}{2}-\frac{2}{5}\right)}{2}-\ln\frac{5}{2}$	M1A1F		for substitution into original equation
	$=\frac{174}{10}-\frac{84}{10}-\ln\frac{5}{2}$	A1		CAO
	$=9-a$	A1	4	AG; accept $b=9-a$
	Total		**11**	
5(a)	$\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{2}\left(1-x^{2}\right)^{-\frac{1}{2}}$	B1		
	$\times(-2x)$	B1	2	
(b)	$\int\sin^{-1}x\,\mathrm{d}x=x\sin^{-1}x-\int\frac{x}{\sqrt{1-x^{2}}}\,\mathrm{d}x$	M1 A1A1		A1 for each part of the integration by parts
	$\int-\frac{x}{\sqrt{1-x^{2}}}\,\mathrm{d}x=\sqrt{1-x^{2}}$ used	A1F		ft sign error in $\frac{\mathrm{d}u}{\mathrm{d}x}$
	$\frac{\sqrt{3}}{2}\frac{\pi}{3}+\sqrt{1-\frac{3}{4}}-1$	m1		substitution of limits
	$\frac{1}{6}\sqrt{3}\,\pi-\frac{1}{2}$	A1	6	CAO
	Total		**8**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$\frac{dx}{dt} = \sec t - \cos t$	B1,B1		use of FB for $\sec t$; if done from first principles, allow B1 when $\sec t$ is arrived at
	Use of $1 - \cos^2 t = \sin^2 t$	M1		
	$\frac{dx}{dt} = \sin t \tan t$	A1	4	AG
(b)	$\dot{x}^2 + \dot{y}^2 = \sin^2 t \tan^2 t + \sin^2 t$	M1A1		sign error in $\frac{dy}{dt}$ A0
	Use of $1 + \tan^2 t = \sec^2 t$	m1		
	$\sqrt{\dot{x}^2 + \dot{y}^2} = \tan t$	A1F		ft sign error in $\frac{dy}{dt}$
	$\int_0^{\frac{\pi}{3}} \tan t \, dt = [\ln \sec t]_0^{\frac{\pi}{3}}$	A1F		ft sign error in $\frac{dy}{dt}$
	$= \ln 2$	A1	6	CAO
	Total		**10**	
7(a)	$f(k+1) - 5f(k)$			
	$= 12^{k+1} + 2 \times 5^k - 5(12^k + 2 \times 5^{k-1})$	M1		
	$= 12^{k+1} + 2 \times 5^k - 5 \times 12^k - 2 \times 5^k$	A1		for expansion of bracket $5 \times 5^{k-1} = 5^k$ used
	$= 12 \times 12^k - 5 \times 12^k = 7 \times 12^k$	A1	3	clearly shown
(b)	Assume $f(k) = M(7)$			
	Then $f(k+1) = 5f(k) + M(7)$	M1		Not merely a repetition of part (a)
	$= M(7)$	A1		clearly shown
	$f(1) = 12 + 2 = 14 = M(7)$	B1		
	Correct inductive process	E1	4	(award only if all 3 previous marks earned)
	Total		**7**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
8(a)(i)	$4\left(1+\mathrm{i}\sqrt{3}\right)=8\left(\frac{1}{2}+\mathrm{i}\frac{\sqrt{3}}{2}\right)$	M1		for either $4\left(1+\mathrm{i}\sqrt{3}\right)$ or $4\left(1-\mathrm{i}\sqrt{3}\right)$ used
	$=8\mathrm{e}^{\frac{\pi\mathrm{i}}{3}}$	A1		If either r or θ is incorrect but the same value in both (i) and (ii) allow A1 but for θ only if it is given as $\frac{\pi}{6}$
(ii)	$4\left(1-\mathrm{i}\sqrt{3}\right)=8\mathrm{e}^{\frac{-\pi\mathrm{i}}{3}}$	A1	3	
(b)	$z^3-4=\pm\sqrt{-48}$	M1		taking square root
	$z^3=4\pm4\sqrt{3}\,\mathrm{i}$	A1	2	AG
(c)(i)	$z=2\mathrm{e}^{\frac{\frac{\pi\mathrm{i}}{3}+2k\pi\mathrm{i}}{3}}$ or $z=2\mathrm{e}^{\frac{\frac{-\pi\mathrm{i}}{3}+2k\pi\mathrm{i}}{3}}$	B1F M1		for the 2; ft incorrect 8, but no decimals for either, PI
	$z=2\mathrm{e}^{\frac{\pi\mathrm{i}}{9}},2\mathrm{e}^{\frac{7\pi\mathrm{i}}{9}},2\mathrm{e}^{\frac{5\pi\mathrm{i}}{9}}$ $=2\mathrm{e}^{\frac{-\pi\mathrm{i}}{9}},2\mathrm{e}^{\frac{-7\pi\mathrm{i}}{9}},2\mathrm{e}^{\frac{-5\pi\mathrm{i}}{9}}$	A3,2,1F	5	Allow A1 for any 2 roots not +/– each other Allow A2 for any 3 roots not +/– each other Allow A3 for all 6 correct roots Deduct A1 for each incorrect root in the interval; ignore roots outside the interval ft incorrect r
(ii)	 Radius 2	B1F		clearly indicated; ft incorrect r allow B1 for 3 correct points condone lines
	Plotting roots	B2,1	3	
(d)(i)	Sum of roots = 0 as coefficient of $z^5=0$	E1	1	OE
(ii)	Use of, say, $\frac{1}{2}\left(\mathrm{e}^{\frac{\pi\mathrm{i}}{9}}+\mathrm{e}^{\frac{-\pi\mathrm{i}}{9}}\right)=\cos\frac{\pi}{9}$	M1		
	$\cos\frac{3\pi}{9}=\frac{1}{2}$ used	A1		
	$\cos\frac{\pi}{9}+\cos\frac{3\pi}{9}+\cos\frac{5\pi}{9}+\cos\frac{7\pi}{9}=\frac{1}{2}$	A1	3	AG
	Total		**17**	
	TOTAL		**75**	