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Other Names					
Candidate Signature					



General Certificate of Education Advanced Level Examination January 2012

# **Mathematics**

MFP2

**Unit Further Pure 2** 

Friday 20 January 2012 1.30 pm to 3.00 pm

### For this paper you must have:

• the blue AQA booklet of formulae and statistical tables. You may use a graphics calculator.

#### Time allowed

• 1 hour 30 minutes

## Instructions

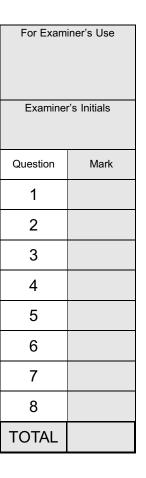
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer all questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost
- Do all rough work in this book. Cross through any work that you do not want to be marked.

## Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

#### **Advice**

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.





2

			Answ	er <b>all</b> ques	stions in th	ne spaces	provided			
1 (a	)	Show, by n	neans of a	sketch, th	at the cur	ves with e	equations			
					$y = \sinh$	X				
		and			$y = \operatorname{sech}$	1X				
		have exactly	y one poi	nt of inters	section.				(4	marks)
(b		Find the $x$ -of form $a \ln b$ .		of this po	int of inte	ersection,	giving yo	ur answer i		marks)
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2 (a	)	Draw on an Argand diagram the locus $L$ of points satisfying the equation $\arg z = \frac{\pi}{6}$ .  (1 mark)
(b	) (i)	A circle $C$ , of radius 6, has its centre lying on $L$ and touches the line $Re(z)=0$ . Draw $C$ on your Argand diagram from part (a). (2 marks)
	(ii)	Find the equation of C, giving your answer in the form $ z - z_0  = k$ . (3 marks)
	(iii)	The complex number $z_1$ lies on $C$ and is such that $\arg z_1$ has its least possible value. Find $\arg z_1$ , giving your answer in the form $p\pi$ , where $-1 . (2 marks)$
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3	Α	curve	has	cartesian	equation
J	$\boldsymbol{\Gamma}$	curve	mas	Cartesian	cquation

$$y = \frac{1}{2} \ln(\tanh x)$$

(a) Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\sinh 2x} \tag{4 marks}$$

(b) The points A and B on the curve have x-coordinates  $\ln 2$  and  $\ln 4$  respectively. Find the arc length AB, giving your answer in the form  $p \ln q$ , where p and q are rational numbers. (8 marks)

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4 The sequence  $u_1$ ,  $u_2$ ,  $u_3$ , ... is defined by

$$u_1 = \frac{3}{4} \qquad u_{n+1} = \frac{3}{4 - u_n}$$

Prove by induction that, for all  $n \ge 1$ ,

$$u_n = \frac{3^{n+1} - 3}{3^{n+1} - 1} \tag{6 marks}$$

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$\frac{\left(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}\right)^p}{\left(\cos\frac{\pi}{12} - i\sin\frac{\pi}{12}\right)^q} = i \qquad (7 \text{ marks})$	5	Find the smallest positive integer values of $p$ and $q$ for which	
PART		$\frac{\left(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}\right)^p}{\left(\cos\frac{\pi}{12} - i\sin\frac{\pi}{12}\right)^q} = i$ (7 marks)	5)
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- 6 (a) Express  $7 + 4x 2x^2$  in the form  $a b(x c)^2$ , where a, b and c are integers.

  (2 marks)
  - (b) By means of a suitable substitution, or otherwise, find the exact value of

$$\int_{1}^{\frac{5}{2}} \frac{\mathrm{d}x}{\sqrt{7 + 4x - 2x^2}}$$
 (6 marks)

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7 The numbers  $\alpha$ ,  $\beta$  and  $\gamma$  satisfy the equations

$$\alpha^{2} + \beta^{2} + \gamma^{2} = -10 - 12i$$
  

$$\alpha\beta + \beta\gamma + \gamma\alpha = 5 + 6i$$

(a) Show that  $\alpha + \beta + \gamma = 0$ .

(2 marks)

**(b)** The numbers  $\alpha$ ,  $\beta$  and  $\gamma$  are also the roots of the equation

$$z^3 + pz^2 + qz + r = 0$$

Write down the value of p and the value of q.

(2 marks)

- (c) It is also given that  $\alpha = 3i$ .
  - (i) Find the value of r.

(3 marks)

(ii) Show that  $\beta$  and  $\gamma$  are the roots of the equation

$$z^2 + 3iz - 4 + 6i = 0$$

(2 marks)

(iii) Given that  $\beta$  is real, find the values of  $\beta$  and  $\gamma$ .

(3 marks)

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- Write down the five roots of the equation  $z^5=1$ , giving your answers in the form  $e^{i\theta}$ , where  $-\pi < \theta \leqslant \pi$ .
  - **(b)** Hence find the four linear factors of

$$z^4 + z^3 + z^2 + z + 1$$
 (3 marks)

(c) Deduce that

$$z^{2} + z + 1 + z^{-1} + z^{-2} = \left(z - 2\cos\frac{2\pi}{5} + z^{-1}\right) \left(z - 2\cos\frac{4\pi}{5} + z^{-1}\right)$$
 (4 marks)

(d) Use the substitution  $z + z^{-1} = w$  to show that  $\cos \frac{2\pi}{5} = \frac{\sqrt{5} - 1}{4}$ . (6 marks)

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