

General Certificate of Education
June 2008
Advanced Subsidiary Examination



MATHEMATICS
Unit Further Pure 1

MFP1

Monday 16 June 2008 1.30 pm to 3.00 pm

For this paper you must have:

- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables
- an insert for use in Questions 4 and 8 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- Fill in the boxes at the top of the insert.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The equation

$$x^2 + x + 5 = 0$$

has roots α and β .

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. (2 marks)

(b) Find the value of $\alpha^2 + \beta^2$. (2 marks)

(c) Show that $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = -\frac{9}{5}$. (2 marks)

(d) Find a quadratic equation, with integer coefficients, which has roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$. (2 marks)

2 It is given that $z = x + iy$, where x and y are real numbers.

(a) Find, in terms of x and y , the real and imaginary parts of

$$3iz + 2z^*$$

where z^* is the complex conjugate of z . (3 marks)

(b) Find the complex number z such that

$$3iz + 2z^* = 7 + 8i$$
 (3 marks)

3 For each of the following improper integrals, find the value of the integral **or** explain briefly why it does not have a value:

(a) $\int_9^\infty \frac{1}{\sqrt{x}} \, dx$; (3 marks)

(b) $\int_9^\infty \frac{1}{x\sqrt{x}} \, dx$. (4 marks)

4 [Figure 1 and Figure 2, printed on the insert, are provided for use in this question.]

The variables x and y are related by an equation of the form

$$y = ax + \frac{b}{x+2}$$

where a and b are constants.

- (a) The variables X and Y are defined by $X = x(x+2)$, $Y = y(x+2)$.

Show that $Y = aX + b$. (2 marks)

- (b) The following approximate values of x and y have been found:

x	1	2	3	4
y	0.40	1.43	2.40	3.35

- (i) Complete the table in Figure 1, showing values of X and Y . (2 marks)

- (ii) Draw on Figure 2 a linear graph relating X and Y . (2 marks)

- (iii) Estimate the values of a and b . (3 marks)

- 5 (a) Find, in **radians**, the general solution of the equation

$$\cos\left(\frac{x}{2} + \frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}$$

giving your answer in terms of π . (5 marks)

- (b) Hence find the smallest **positive** value of x which satisfies this equation. (2 marks)

- 6 The matrices \mathbf{A} and \mathbf{B} are given by

$$\mathbf{A} = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$$

- (a) Calculate the matrix \mathbf{AB} . (2 marks)

- (b) Show that \mathbf{A}^2 is of the form $k\mathbf{I}$, where k is an integer and \mathbf{I} is the 2×2 identity matrix. (2 marks)

- (c) Show that $(\mathbf{AB})^2 \neq \mathbf{A}^2\mathbf{B}^2$. (3 marks)

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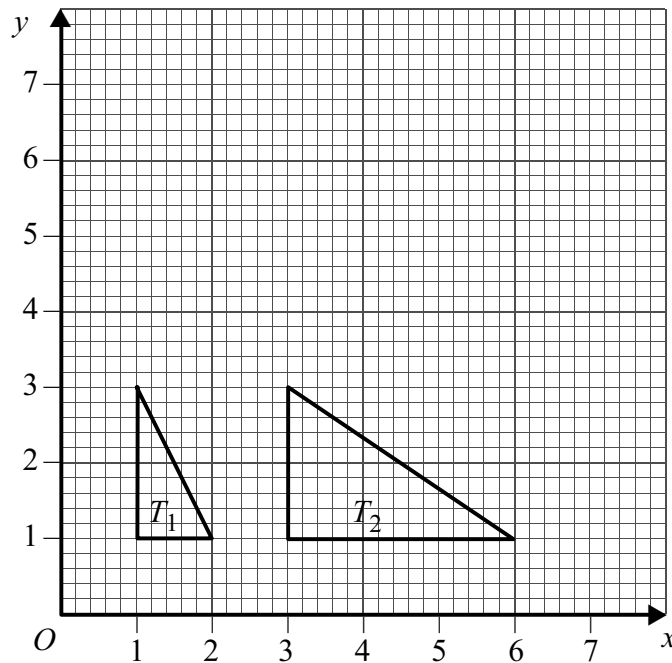
7 A curve C has equation

$$y = 7 + \frac{1}{x+1}$$

- (a) Define the translation which transforms the curve with equation $y = \frac{1}{x}$ onto the curve C . (2 marks)
- (b) (i) Write down the equations of the two asymptotes of C . (2 marks)
- (ii) Find the coordinates of the points where the curve C intersects the coordinate axes. (3 marks)
- (c) Sketch the curve C and its two asymptotes. (3 marks)

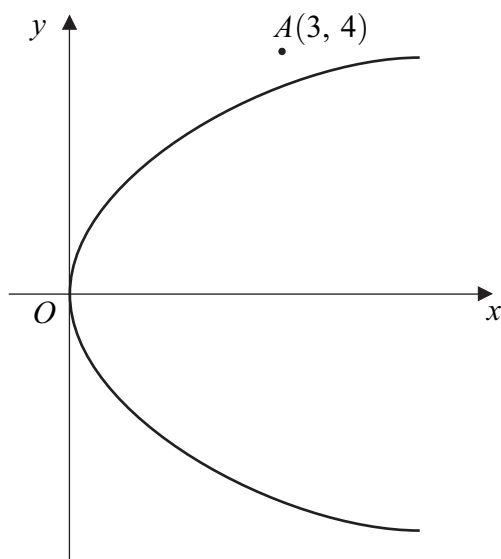
8 [Figure 3, printed on the insert, is provided for use in this question.]

The diagram shows two triangles, T_1 and T_2 .



- (a) Find the matrix of the stretch which maps T_1 to T_2 . (2 marks)
- (b) The triangle T_2 is reflected in the line $y = x$ to give a third triangle, T_3 .
On Figure 3, draw the triangle T_3 . (2 marks)
- (c) Find the matrix of the transformation which maps T_1 to T_3 . (3 marks)

- 9 The diagram shows the parabola $y^2 = 4x$ and the point A with coordinates $(3, 4)$.



- (a) Find an equation of the straight line having gradient m and passing through the point $A(3, 4)$. *(2 marks)*
- (b) Show that, if this straight line intersects the parabola, then the y -coordinates of the points of intersection satisfy the equation

$$my^2 - 4y + (16 - 12m) = 0 \quad \text{span style="float: right;">*(3 marks)*$$

- (c) By considering the discriminant of the equation in part (b), find the equations of the two tangents to the parabola which pass through A .

(No credit will be given for solutions based on differentiation.) *(5 marks)*

- (d) Find the coordinates of the points at which these tangents touch the parabola. *(4 marks)*

END OF QUESTIONS

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Surname						Other Names					
Centre Number						Candidate Number					
Candidate Signature											

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Insert

Insert for use in **Questions 4 and 8**.

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

Turn over for Figure 1

Turn over ►

Figure 1 (for use in Question 4)

x	1	2	3	4
y	0.40	1.43	2.40	3.35
X	3			
Y	1.20			

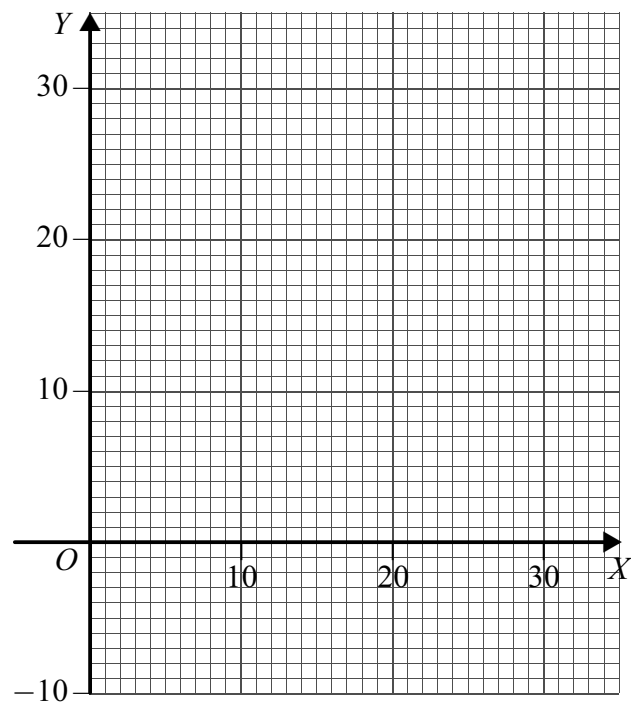
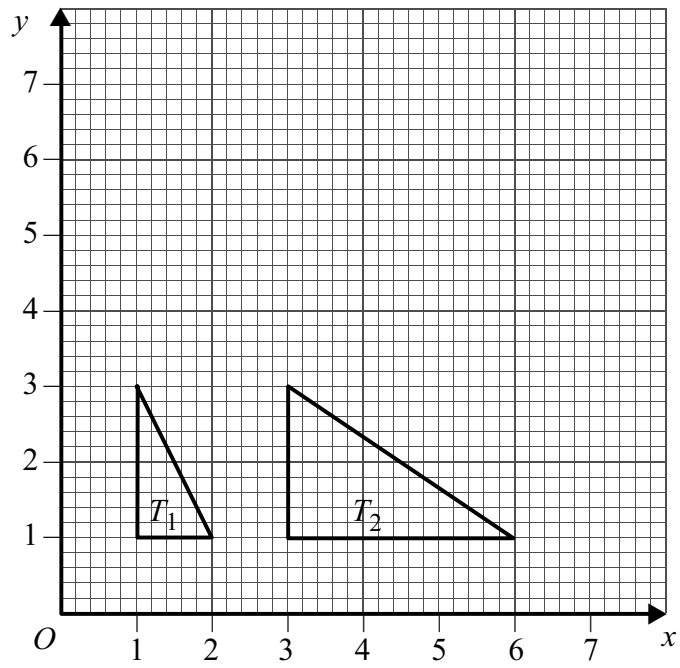
Figure 2 (for use in Question 4)

Figure 3 (for use in Question 8)

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