

General Certificate of Education
June 2007
Advanced Subsidiary Examination



MATHEMATICS
Unit Further Pure 1

MFP1

Wednesday 20 June 2007 1.30 pm to 3.00 pm

For this paper you must have:

- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables
- an insert for use in Questions 5 and 9 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- Fill in the boxes at the top of the insert.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 3 & 8 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

The matrix $\mathbf{M} = \mathbf{A} - 2\mathbf{B}$.

(a) Show that $\mathbf{M} = n \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$, where n is a positive integer. (2 marks)

(b) The matrix **M** represents a combination of an enlargement of scale factor p and a reflection in a line L . State the value of p and write down the equation of L . (2 marks)

(c) Show that

$$\mathbf{M}^2 = q\mathbf{I}$$

where q is an integer and **I** is the 2×2 identity matrix. (2 marks)

2 (a) Show that the equation

$$x^3 + x - 7 = 0$$

has a root between 1.6 and 1.8. (3 marks)

(b) Use interval bisection **twice**, starting with the interval in part (a), to give this root to one decimal place. (4 marks)

3 It is given that $z = x + iy$, where x and y are real numbers.

(a) Find, in terms of x and y , the real and imaginary parts of

$$z - 3iz^*$$

where z^* is the complex conjugate of z . (3 marks)

(b) Find the complex number z such that

$$z - 3iz^* = 16$$
 (3 marks)

4 The quadratic equation

$$2x^2 - x + 4 = 0$$

has roots α and β .

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. (2 marks)

(b) Show that $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{1}{4}$. (2 marks)

(c) Find a quadratic equation with integer coefficients such that the roots of the equation are

$$\frac{4}{\alpha} \text{ and } \frac{4}{\beta} \quad (3 \text{ marks})$$

5 [Figure 1 and Figure 2, printed on the insert, are provided for use in this question.]

The variables x and y are known to be related by an equation of the form

$$y = ab^x$$

where a and b are constants.

The following approximate values of x and y have been found.

x	1	2	3	4
y	3.84	6.14	9.82	15.7

(a) Complete the table in **Figure 1**, showing values of x and Y , where $Y = \log_{10} y$. Give each value of Y to three decimal places. (2 marks)

(b) Show that, if $y = ab^x$, then x and Y must satisfy an equation of the form

$$Y = mx + c \quad (3 \text{ marks})$$

(c) Draw on **Figure 2** a linear graph relating x and Y . (2 marks)

(d) Hence find estimates for the values of a and b . (4 marks)

- 6 Find the general solution of the equation

$$\sin\left(2x - \frac{\pi}{2}\right) = \frac{\sqrt{3}}{2}$$

giving your answer in terms of π .

(6 marks)

- 7 A curve has equation

$$y = \frac{3x - 1}{x + 2}$$

- (a) Write down the equations of the two asymptotes to the curve. (2 marks)

- (b) Sketch the curve, indicating the coordinates of the points where the curve intersects the coordinate axes. (5 marks)

- (c) Hence, or otherwise, solve the inequality

$$0 < \frac{3x - 1}{x + 2} < 3 \quad (2 \text{ marks})$$

- 8 For each of the following improper integrals, find the value of the integral **or** explain briefly why it does not have a value:

(a) $\int_0^1 (x^{\frac{1}{3}} + x^{-\frac{1}{3}}) \, dx;$ (4 marks)

(b) $\int_0^1 \frac{x^{\frac{1}{3}} + x^{-\frac{1}{3}}}{x} \, dx.$ (4 marks)

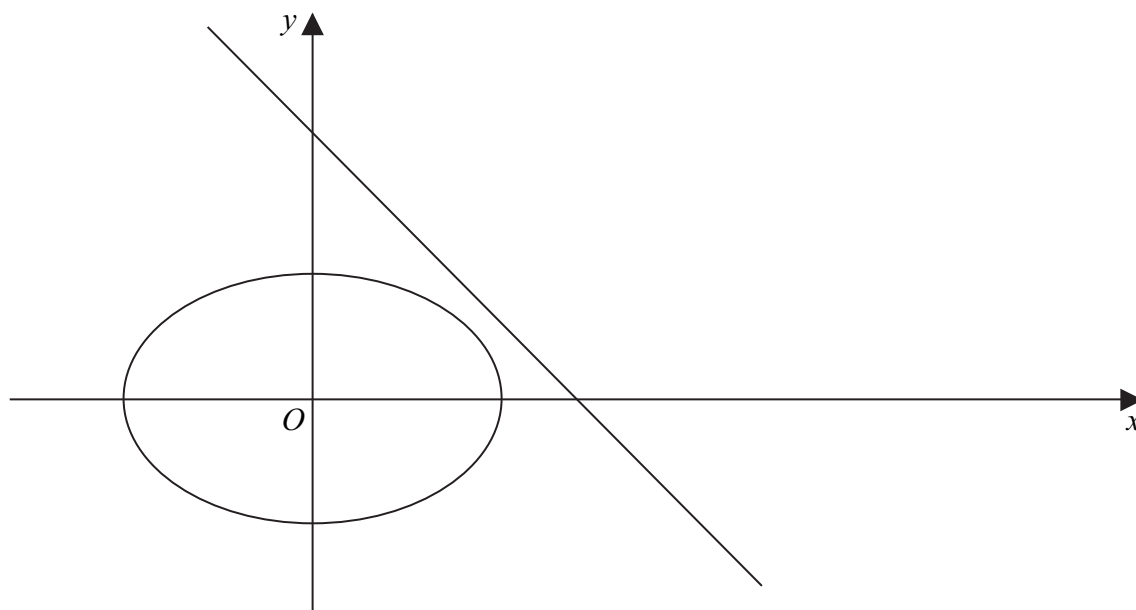
9 [Figure 3, printed on the insert, is provided for use in this question.]

The diagram shows the curve with equation

$$\frac{x^2}{2} + y^2 = 1$$

and the straight line with equation

$$x + y = 2$$



- (a) Write down the exact coordinates of the points where the curve with equation $\frac{x^2}{2} + y^2 = 1$ intersects the coordinate axes. (2 marks)
- (b) The curve is translated k units in the positive x direction, where k is a constant. Write down, in terms of k , the equation of the curve after this translation. (2 marks)
- (c) Show that, if the line $x + y = 2$ intersects the **translated** curve, the x -coordinates of the points of intersection must satisfy the equation
- $$3x^2 - 2(k + 4)x + (k^2 + 6) = 0 \quad (4 \text{ marks})$$
- (d) Hence find the two values of k for which the line $x + y = 2$ is a tangent to the translated curve. Give your answer in the form $p \pm \sqrt{q}$, where p and q are integers. (4 marks)
- (e) On **Figure 3**, show the translated curves corresponding to these two values of k . (3 marks)

END OF QUESTIONS

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Surname						Other Names					
Centre Number						Candidate Number					
Candidate Signature											

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Insert

Insert for use in **Questions 5 and 9**.

Fill in the boxes at the top of this page.

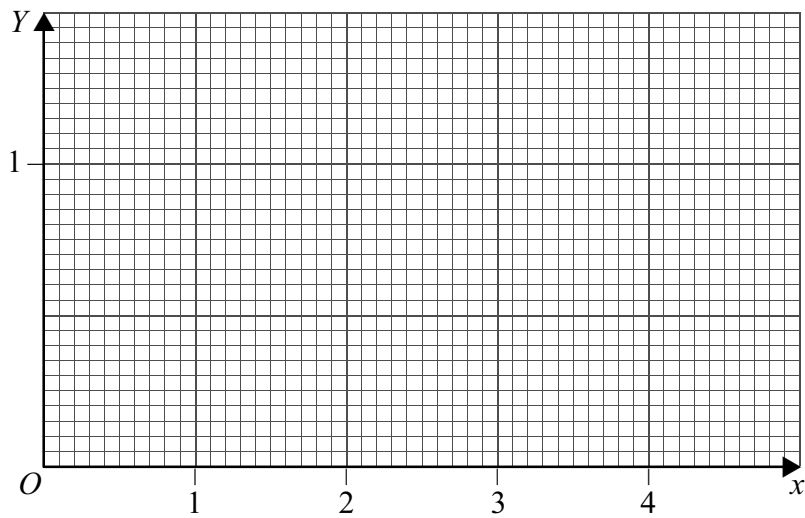
Fasten this insert securely to your answer book.

Turn over for Figure 1

Turn over ►

Figure 1 (for use in Question 5)

x	1	2	3	4
Y	0.584			

Figure 2 (for use in Question 5)**Figure 3 (for use in Question 9)**