

General Certificate of Education
June 2006
Advanced Subsidiary Examination



MATHEMATICS
Unit Further Pure 1

MFP1

Monday 12 June 2006 1.30 pm to 3.00 pm

For this paper you must have:

- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The quadratic equation

$$3x^2 - 6x + 2 = 0$$

has roots α and β .

(a) Write down the numerical values of $\alpha + \beta$ and $\alpha\beta$. (2 marks)

(b) (i) Expand $(\alpha + \beta)^3$. (1 mark)

(ii) Show that $\alpha^3 + \beta^3 = 4$. (3 marks)

(c) Find a quadratic equation with roots α^3 and β^3 , giving your answer in the form $px^2 + qx + r = 0$, where p , q and r are integers. (3 marks)

2 A curve satisfies the differential equation

$$\frac{dy}{dx} = \log_{10} x$$

Starting at the point (2, 3) on the curve, use a step-by-step method with a step length of 0.2 to estimate the value of y at $x = 2.4$. Give your answer to three decimal places. (6 marks)

3 Show that

$$\sum_{r=1}^n (r^2 - r) = kn(n+1)(n-1)$$

where k is a rational number. (4 marks)

4 Find, in **radians**, the general solution of the equation

$$\cos 3x = \frac{\sqrt{3}}{2}$$

giving your answers in terms of π . (5 marks)

5 The matrix $\mathbf{M}$ is defined by

$$\mathbf{M} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

(a) Find the matrix:

(i) $\mathbf{M}^2$; (3 marks)

(ii) $\mathbf{M}^4$. (1 mark)

(b) Describe fully the geometrical transformation represented by $\mathbf{M}$. (2 marks)

(c) Find the matrix $\mathbf{M}^{2006}$. (3 marks)

6 It is given that $z = x + iy$, where x and y are real numbers.

(a) Write down, in terms of x and y , an expression for

$$(z + i)^*$$

where $(z + i)^*$ denotes the complex conjugate of $(z + i)$. (2 marks)

(b) Solve the equation

$$(z + i)^* = 2iz + 1$$

giving your answer in the form $a + bi$. (5 marks)

Turn over for the next question

Turn over ►

- 7 (a) Describe a geometrical transformation by which the hyperbola

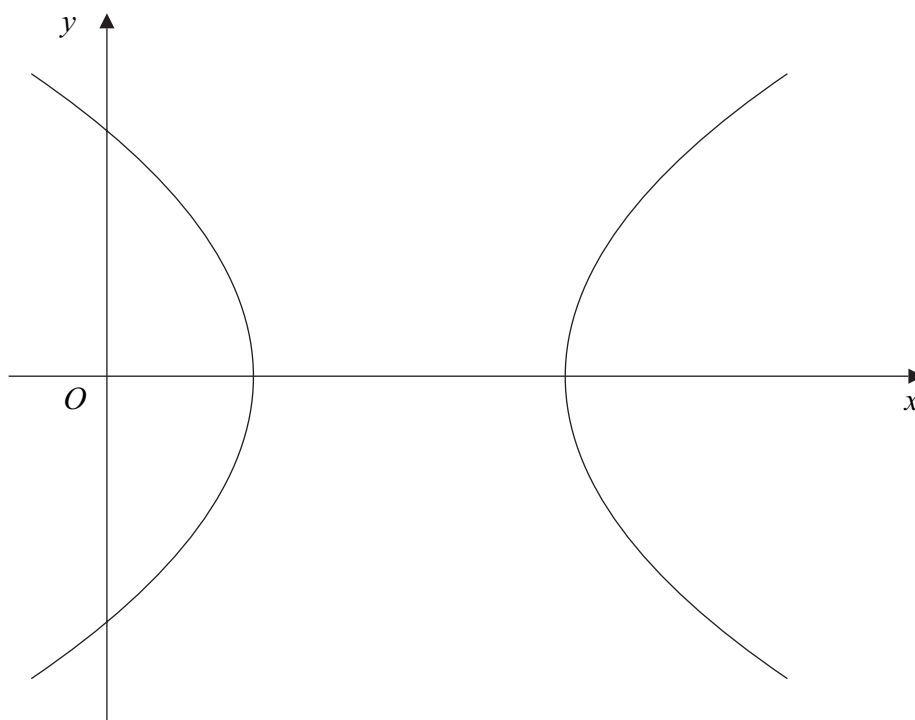
$$x^2 - 4y^2 = 1$$

can be obtained from the hyperbola $x^2 - y^2 = 1$.

(2 marks)

- (b) The diagram shows the hyperbola H with equation

$$x^2 - y^2 - 4x + 3 = 0$$



By completing the square, describe a geometrical transformation by which the hyperbola H can be obtained from the hyperbola $x^2 - y^2 = 1$.

(4 marks)

- 8 (a) The function f is defined for all real values of x by

$$f(x) = x^3 + x^2 - 1$$

- (i) Express $f(1+h) - f(1)$ in the form

$$ph + qh^2 + rh^3$$

where p , q and r are integers.

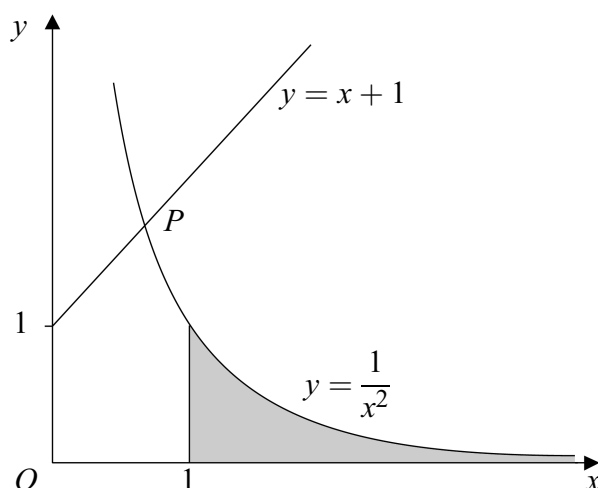
(4 marks)

- (ii) Use your answer to part (a)(i) to find the value of $f'(1)$.

(2 marks)

- (b) The diagram shows the graphs of

$$y = \frac{1}{x^2} \quad \text{and} \quad y = x + 1 \quad \text{for} \quad x > 0$$



The graphs intersect at the point P .

- (i) Show that the x -coordinate of P satisfies the equation $f(x) = 0$, where f is the function defined in part (a). (1 mark)
- (ii) Taking $x_1 = 1$ as a first approximation to the root of the equation $f(x) = 0$, use the Newton–Raphson method to find a second approximation x_2 to the root. (3 marks)
- (c) The region enclosed by the curve $y = \frac{1}{x^2}$, the line $x = 1$ and the x -axis is shaded on the diagram. By evaluating an improper integral, find the area of this region. (3 marks)

Turn over ►

9 A curve C has equation

$$y = \frac{(x+1)(x-3)}{x(x-2)}$$

- (a) (i) Write down the coordinates of the points where C intersects the x -axis. (2 marks)
- (ii) Write down the equations of all the asymptotes of C . (3 marks)
- (b) (i) Show that, if the line $y = k$ intersects C , then

$$(k-1)(k-4) \geq 0 \quad (5 \text{ marks})$$

- (ii) Given that there is only one stationary point on C , find the coordinates of this stationary point.

(No credit will be given for solutions based on differentiation.) (3 marks)

- (c) Sketch the curve C . (3 marks)

END OF QUESTIONS

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