General Certificate of Education January 2007 Advanced Subsidiary Examination



MFP1

# MATHEMATICS Unit Further Pure 1

Further Pure 1

Friday 26 January 2007 1.30 pm to 3.00 pm

### For this paper you must have:

- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

#### **Instructions**

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP1.
- Answer all questions.
- Show all necessary working; otherwise marks for method may be lost.

#### **Information**

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

#### **Advice**

• Unless stated otherwise, you may quote formulae, without proof, from the booklet.

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## Answer all questions.

1 (a) Solve the following equations, giving each root in the form a + bi:

(i) 
$$x^2 + 16 = 0$$
; (2 marks)

(ii) 
$$x^2 - 2x + 17 = 0$$
. (2 marks)

(b) (i) Expand 
$$(1+x)^3$$
. (2 marks)

(ii) Express 
$$(1+i)^3$$
 in the form  $a+bi$ . (2 marks)

(iii) Hence, or otherwise, verify that x = 1 + i satisfies the equation

$$x^3 + 2x - 4i = 0 (2 marks)$$

2 The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$$

(a) Calculate:

(i) 
$$\mathbf{A} + \mathbf{B}$$
; (2 marks)

(b) Describe fully the geometrical transformation represented by each of the following matrices:

(ii) 
$$\mathbf{B}$$
; (2 marks)

3 The quadratic equation

$$2x^2 + 4x + 3 = 0$$

has roots  $\alpha$  and  $\beta$ .

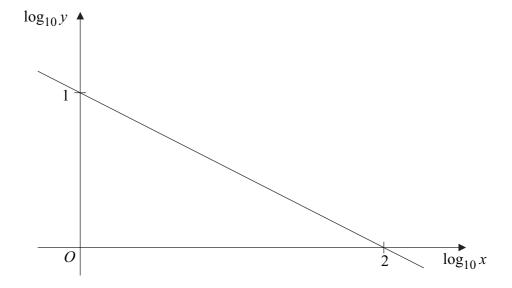
- (a) Write down the values of  $\alpha + \beta$  and  $\alpha\beta$ . (2 marks)
- (b) Show that  $\alpha^2 + \beta^2 = 1$ . (3 marks)
- (c) Find the value of  $\alpha^4 + \beta^4$ . (3 marks)

4 The variables x and y are related by an equation of the form

$$y = ax^b$$

where a and b are constants.

- (a) Using logarithms to base 10, reduce the relation  $y = ax^b$  to a linear law connecting  $\log_{10} x$  and  $\log_{10} y$ . (2 marks)
- (b) The diagram shows the linear graph that results from plotting  $\log_{10} y$  against  $\log_{10} x$ .



Find the values of a and b.

(4 marks)

5 A curve has equation

$$y = \frac{x}{x^2 - 1}$$

- (a) Write down the equations of the three asymptotes to the curve. (3 marks)
- (b) Sketch the curve.

(You are given that the curve has no stationary points.) (4 marks)

(c) Solve the inequality

$$\frac{x}{x^2 - 1} > 0 \tag{3 marks}$$

- **6** (a) (i) Expand  $(2r-1)^2$ . (1 mark)
  - (ii) Hence show that

$$\sum_{r=1}^{n} (2r-1)^2 = \frac{1}{3}n(4n^2 - 1)$$
 (5 marks)

(b) Hence find the sum of the squares of the odd numbers between 100 and 200. (4 marks)

7 The function f is defined for all real numbers by

$$f(x) = \sin\left(x + \frac{\pi}{6}\right)$$

- (a) Find the general solution of the equation f(x) = 0. (3 marks)
- (b) The quadratic function g is defined for all real numbers by

$$g(x) = \frac{1}{2} + \frac{\sqrt{3}}{2}x - \frac{1}{4}x^2$$

It can be shown that g(x) gives a good approximation to f(x) for small values of x.

- (i) Show that g(0.05) and f(0.05) are identical when rounded to four decimal places. (2 marks)
- (ii) A chord joins the points on the curve y = g(x) for which x = 0 and x = h. Find an expression in terms of h for the gradient of this chord. (2 marks)
- (iii) Using your answer to part (b)(ii), find the value of g'(0). (1 mark)

**8** A curve C has equation

$$\frac{x^2}{25} - \frac{y^2}{9} = 1$$

- (a) Find the y-coordinates of the points on C for which x = 10, giving each answer in the form  $k\sqrt{3}$ , where k is an integer. (3 marks)
- (b) Sketch the curve C, indicating the coordinates of any points where the curve intersects the coordinate axes. (3 marks)
- (c) Write down the equation of the tangent to C at the point where C intersects the positive x-axis. (1 mark)
- (d) (i) Show that, if the line y = x 4 intersects C, the x-coordinates of the points of intersection must satisfy the equation

$$16x^2 - 200x + 625 = 0 (3 marks)$$

(ii) Solve this equation and hence state the relationship between the line y = x - 4 and the curve C. (2 marks)

## END OF QUESTIONS

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