

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										



General Certificate of Education  
Advanced Subsidiary Examination  
June 2012

# Mathematics

**MFP1**

**Unit Further Pure 1**

**Friday 18 May 2012 9.00 am to 10.30 am**

**For this paper you must have:**

- the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

**Time allowed**

- 1 hour 30 minutes

**Instructions**

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

**Information**

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

**Advice**

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
<b>TOTAL</b>	



J U N 1 2 M F P 1 0 1

Answer **all** questions.

Answer each question in the space provided for that question.

**1** The quadratic equation

$$5x^2 - 7x + 1 = 0$$

has roots  $\alpha$  and  $\beta$ .

**(a)** Write down the values of  $\alpha + \beta$  and  $\alpha\beta$ . (2 marks)

**(b)** Show that  $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{39}{5}$ . (3 marks)

**(c)** Find a quadratic equation, with integer coefficients, which has roots

$$\alpha + \frac{1}{\alpha} \quad \text{and} \quad \beta + \frac{1}{\beta} \quad \text{(5 marks)}$$

QUESTION  
PART  
REFERENCE

**Answer space for question 1**



QUESTION  
PART  
REFERENCE

Answer space for question 1

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**2** A curve has equation  $y = x^4 + x$ .

- (a)** Find the gradient of the line passing through the point  $(-2, 14)$  and the point on the curve for which  $x = -2 + h$ . Give your answer in the form

$$p + qh + rh^2 + h^3$$

where  $p$ ,  $q$  and  $r$  are integers.

(5 marks)

- (b)** Show how the answer to part **(a)** can be used to find the gradient of the curve at the point  $(-2, 14)$ . State the value of this gradient. (2 marks)

QUESTION  
PART  
REFERENCE

### Answer space for question 2



QUESTION  
PART  
REFERENCE

Answer space for question 2

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**3** It is given that  $z = x + iy$ , where  $x$  and  $y$  are real numbers.

**(a)** Find, in terms of  $x$  and  $y$ , the real and imaginary parts of

$$i(z + 7) + 3(z^* - i) \quad (3 \text{ marks})$$

**(b)** Hence find the complex number  $z$  such that

$$i(z + 7) + 3(z^* - i) = 0 \quad (3 \text{ marks})$$

QUESTION  
PART  
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**Answer space for question 3**



**Answer space for question 3**

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4

Find the general solution, in degrees, of the equation

$$\sin\left(70^\circ - \frac{2}{3}x\right) = \cos 20^\circ$$

(6 marks)

QUESTION  
PART  
REFERENCE**Answer space for question 4**



QUESTION  
PART  
REFERENCE

Answer space for question 4

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**5** The curve  $C$  has equation  $y = \frac{x}{(x+1)(x-2)}$ .

The line  $L$  has equation  $y = -\frac{1}{2}$ .

**(a)** Write down the equations of the asymptotes of  $C$ . (3 marks)

**(b)** The line  $L$  intersects the curve  $C$  at two points. Find the  $x$ -coordinates of these two points. (2 marks)

**(c)** Sketch  $C$  and  $L$  on the same axes.

(You are given that the curve  $C$  has no stationary points.) (3 marks)

**(d)** Solve the inequality

$$\frac{x}{(x+1)(x-2)} \leq -\frac{1}{2} \quad (3 \text{ marks})$$

QUESTION  
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**Answer space for question 5**



**Answer space for question 5**

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**6 (a)** Using surd forms, find the matrix of a rotation about the origin through  $135^\circ$  anticlockwise. (2 marks)

**(b)** The matrix  $\mathbf{M}$  is defined by  $\mathbf{M} = \begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix}$ .

**(i)** Given that  $\mathbf{M}$  represents an enlargement followed by a rotation, find the scale factor of the enlargement and the angle of the rotation. (3 marks)

**(ii)** The matrix  $\mathbf{M}^2$  also represents an enlargement followed by a rotation. State the scale factor of the enlargement and the angle of the rotation. (2 marks)

**(iii)** Show that  $\mathbf{M}^4 = k\mathbf{I}$ , where  $k$  is an integer and  $\mathbf{I}$  is the  $2 \times 2$  identity matrix. (2 marks)

**(iv)** Deduce that  $\mathbf{M}^{2012} = -2^n\mathbf{I}$  for some positive integer  $n$ . (2 marks)

QUESTION  
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### Answer space for question 6



QUESTION  
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**Answer space for question 6**

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**Answer space for question 6**

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7 The equation

$$24x^3 + 36x^2 + 18x - 5 = 0$$

has one real root,  $\alpha$ .

- (a) Show that  $\alpha$  lies in the interval  $0.1 < x < 0.2$ . (2 marks)
- (b) Starting from the interval  $0.1 < x < 0.2$ , use interval bisection **twice** to obtain an interval of width 0.025 within which  $\alpha$  must lie. (3 marks)
- (c) Taking  $x_1 = 0.2$  as a first approximation to  $\alpha$ , use the Newton–Raphson method to find a second approximation,  $x_2$ , to  $\alpha$ . Give your answer to four decimal places. (4 marks)

QUESTION  
PART  
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Answer space for question 7





**Answer space for question 7**

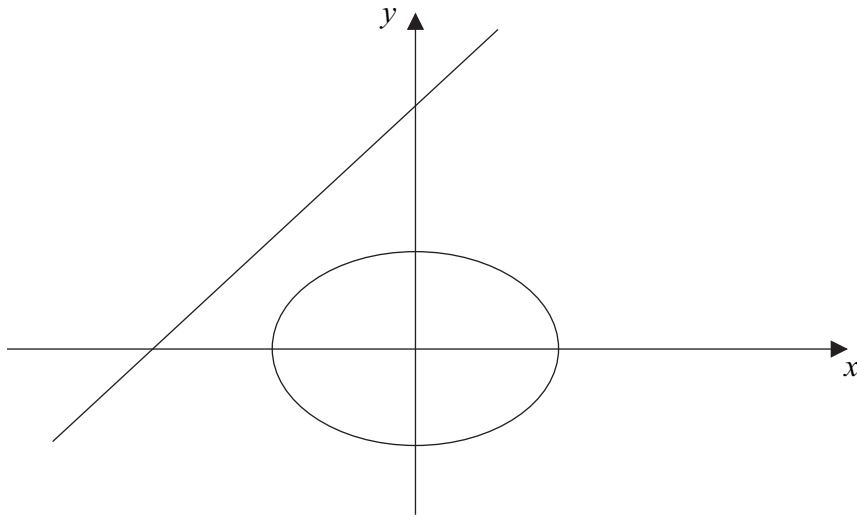
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- 8** The diagram shows the ellipse  $E$  with equation

$$\frac{x^2}{5} + \frac{y^2}{4} = 1$$

and the straight line  $L$  with equation

$$y = x + 4$$



- (a)** Write down the coordinates of the points where the ellipse  $E$  intersects the coordinate axes. (2 marks)

- (b)** The ellipse  $E$  is translated by the vector  $\begin{bmatrix} p \\ 0 \end{bmatrix}$ , where  $p$  is a constant. Write down the equation of the translated ellipse. (2 marks)

- (c)** Show that, if the translated ellipse intersects the line  $L$ , the  $x$ -coordinates of the points of intersection must satisfy the equation

$$9x^2 - (8p - 40)x + (4p^2 + 60) = 0 \quad (3 \text{ marks})$$

- (d)** Given that the line  $L$  is a tangent to the translated ellipse, find the coordinates of the two possible points of contact.

(No credit will be given for solutions based on differentiation.) (8 marks)



**Answer space for question 8**

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**Answer space for question 8**

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Answer space for question 8

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END OF QUESTIONS



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