

General Certificate of Education
June 2007
Advanced Level Examination



MATHEMATICS
Unit Decision 2

MD02

Friday 22 June 2007 9.00 am to 10.30 am

For this paper you must have:

- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables
- an insert for use in Questions 1, 5 and 6 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil or coloured pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MD02.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Fill in the boxes at the top of the insert.

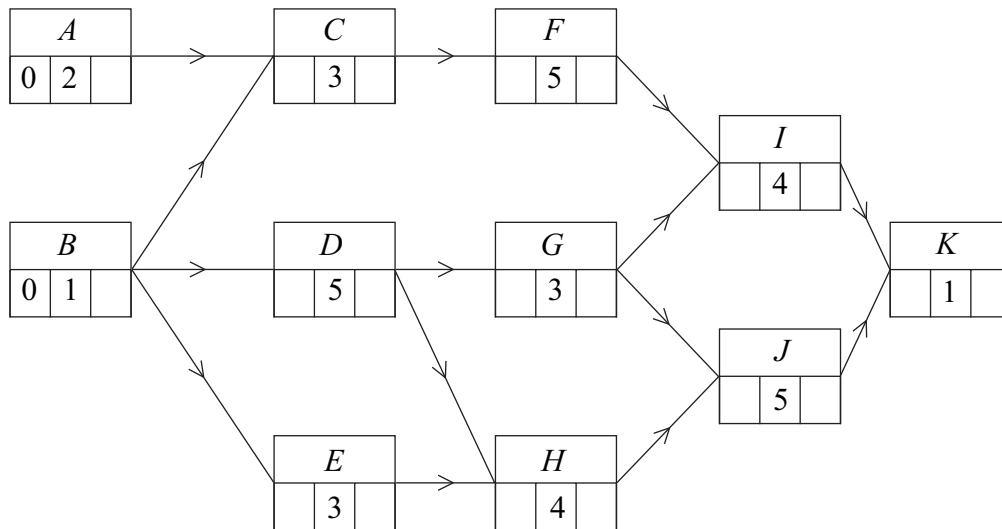
Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Answer **all** questions.

1 [Figures 1 and 2, printed on the insert, are provided for use in this question.]

The following diagram shows an activity diagram for a building project. The time needed for each activity is given in days.



- Complete the precedence table for the project on **Figure 1**. (2 marks)
- Find the earliest start times and latest finish times for each activity and insert their values on **Figure 2**. (4 marks)
- Find the critical path and state the minimum time for completion of the project. (2 marks)
- Find the activity with the greatest float time and state the value of its float time. (2 marks)

- 2 The daily costs, in pounds, for five managers A, B, C, D and E to travel to five different centres are recorded in the table below.

	A	B	C	D	E
Centre 1	10	11	8	12	5
Centre 2	11	5	11	6	7
Centre 3	12	8	7	11	4
Centre 4	10	9	14	10	6
Centre 5	9	9	7	8	9

Using the Hungarian algorithm, each of the five managers is to be allocated to a different centre so that the overall total travel cost is minimised.

- (a) By reducing the **rows first** and then the columns, show that the new table of values is

3	6	3	6	0
4	0	6	0	2
6	4	3	6	0
2	3	8	3	0
0	2	0	0	2

(3 marks)

- (b) Show that the zeros in the table in part (a) can be covered with three lines and use adjustments to produce a table where five lines are required to cover the zeros.

(5 marks)

- (c) Hence find the two possible ways of allocating the five managers to the five centres with the least possible total travel cost.

(3 marks)

- (d) Find the value of this minimum daily total travel cost.

(1 mark)

- 3 Two people, Rose and Callum, play a zero-sum game. The game is represented by the following pay-off matrix for Rose.

		Callum		
		C_1	C_2	C_3
Rose	R_1	5	2	-1
	R_2	-3	-1	5
	R_3	4	1	-2

- (a) (i) State the play-safe strategy for Rose and give a reason for your answer. (2 marks)
- (ii) Show that there is no stable solution for this game. (2 marks)
- (b) Explain why Rose should never play strategy R_3 . (1 mark)
- (c) Rose adopts a mixed strategy, choosing R_1 with probability p and R_2 with probability $1 - p$.
- (i) Find expressions for the expected gain for Rose when Callum chooses each of his three possible strategies. Simplify your expressions. (3 marks)
- (ii) Illustrate graphically these expected gains for $0 \leq p \leq 1$. (2 marks)
- (iii) Hence determine the optimal mixed strategy for Rose. (3 marks)
- (iv) Find the value of the game. (1 mark)

- 4 A linear programming problem involving variables x and y is to be solved. The objective function to be maximised is $P = 3x + 5y$. The initial Simplex tableau is given below.

P	x	y	s	t	u	<i>value</i>
1	-3	-5	0	0	0	0
0	1	2	1	0	0	36
0	1	1	0	1	0	20
0	4	1	0	0	1	39

- (a) In addition to $x \geq 0$, $y \geq 0$, write down **three** inequalities involving x and y for this problem. (2 marks)
- (b) (i) By choosing the first pivot from the **y -column**, perform **one** iteration of the Simplex method. (4 marks)
- (ii) Explain how you know that the optimal value has not been reached. (1 mark)
- (c) (i) Perform one further iteration. (4 marks)
- (ii) Interpret the final tableau and state the values of the slack variables. (3 marks)

Turn over for the next question

Turn over ►

5 [Figure 3, printed on the insert, is provided for use in this question.]

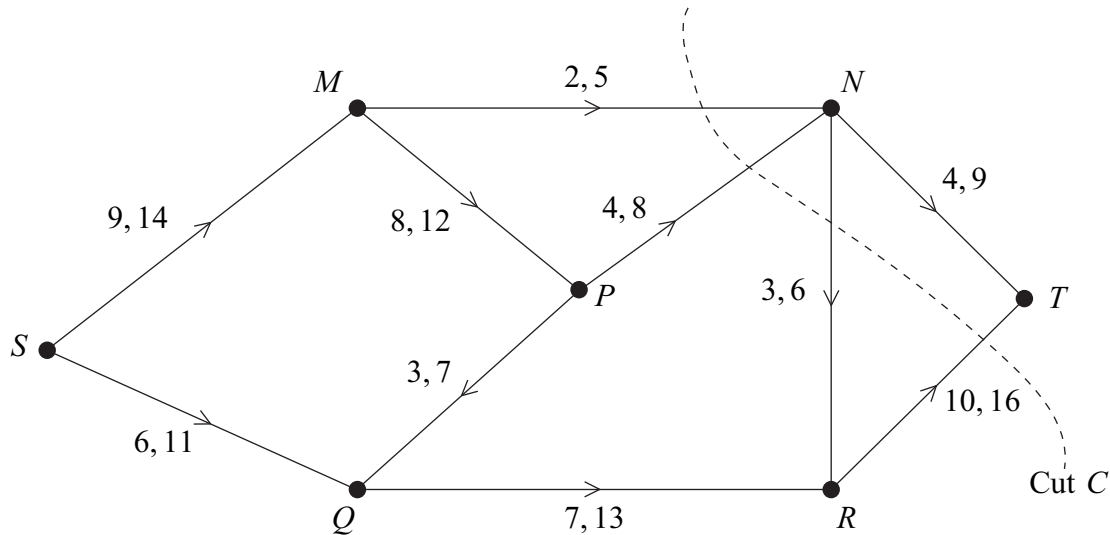
A maker of exclusive furniture is planning to build three cabinets *A*, *B* and *C* at the rate of one per month. The order in which they are built is a matter of choice, but the costs will vary because of the materials available and suppliers' costs. The expected costs, in pounds, are given in the table.

Month	Already built	Cost		
		<i>A</i>	<i>B</i>	<i>C</i>
1	–	500	440	475
2	<i>A</i>	–	440	490
	<i>B</i>	510	–	500
	<i>C</i>	520	490	–
3	<i>A and B</i>	–	–	520
	<i>A and C</i>	–	500	–
	<i>B and C</i>	510	–	–

- (a) Use dynamic programming, working **backwards** from month 3, to determine the order of manufacture that **minimises** the total cost. You may wish to use **Figure 3** for your working. (6 marks)
- (b) It is discovered that the figures given were actually the profits, not the costs, for each item. Modify your solution to find the order of manufacture that **maximises** the total profit. You may wish to use the final column of **Figure 3** for your working. (4 marks)

6 [Figures 4, 5 and 6, printed on the insert, are provided for use in this question.]

The network shows a system of pipes with the lower and upper capacities for each pipe in litres per second.



- (a) (i) Find the value of the cut C . (1 mark)
- (ii) State what can be deduced about the maximum flow from S to T . (1 mark)
- (b) **Figure 4**, printed on the insert, shows a partially completed diagram for a feasible flow of 20 litres per second from S to T . Indicate, on **Figure 4**, the flows along the edges MP , PN , QR and NR . (4 marks)
- (c) (i) Taking your answer from part (b) as an initial flow, indicate potential increases and decreases of the flow along each edge on **Figure 5**. (2 marks)
- (ii) Use flow augmentation on **Figure 5** to find the maximum flow from S to T . You should indicate any flow augmenting paths in the table and modify the potential increases and decreases of the flow on the network. (5 marks)
- (iii) Illustrate the maximum flow on **Figure 6**. (2 marks)

END OF QUESTIONS

There are no questions printed on this page

Surname						Other Names					
Centre Number						Candidate Number					
Candidate Signature											

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Insert

Insert for use in **Questions 1, 5 and 6**.

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

Turn over for Figure 1

Turn over ►

Figure 1 (for use in Question 1)

Activity	Immediate Predecessors
<i>A</i>	–
<i>B</i>	–
<i>C</i>	
<i>D</i>	
<i>E</i>	
<i>F</i>	
<i>G</i>	
<i>H</i>	
<i>I</i>	
<i>J</i>	
<i>K</i>	

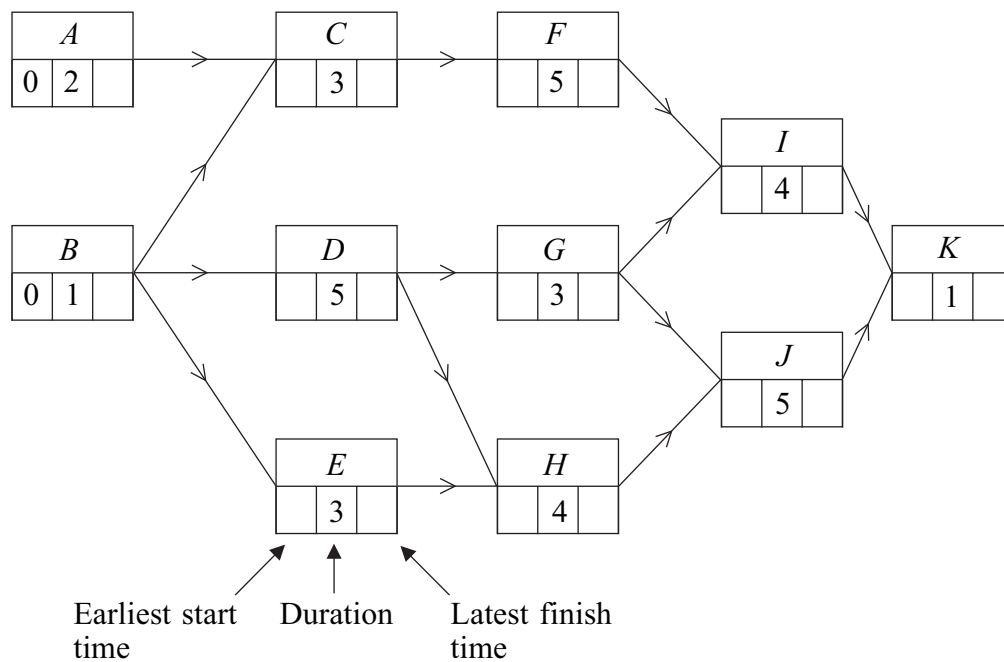
Figure 2 (for use in Question 1)

Figure 3 (for use in Question 5)

Month	Already built	Machine built	For use in part (a)	For use in part (b)
3	<i>A</i> and <i>B</i>	<i>C</i>		
	<i>A</i> and <i>C</i>	<i>B</i>		
	<i>B</i> and <i>C</i>	<i>A</i>		
2	<i>A</i>	<i>B</i>		
		<i>C</i>		
	<i>B</i>	<i>A</i>		
		<i>C</i>		
	<i>C</i>	<i>A</i>		
		<i>B</i>		

Figure 4 (for use in Question 6)

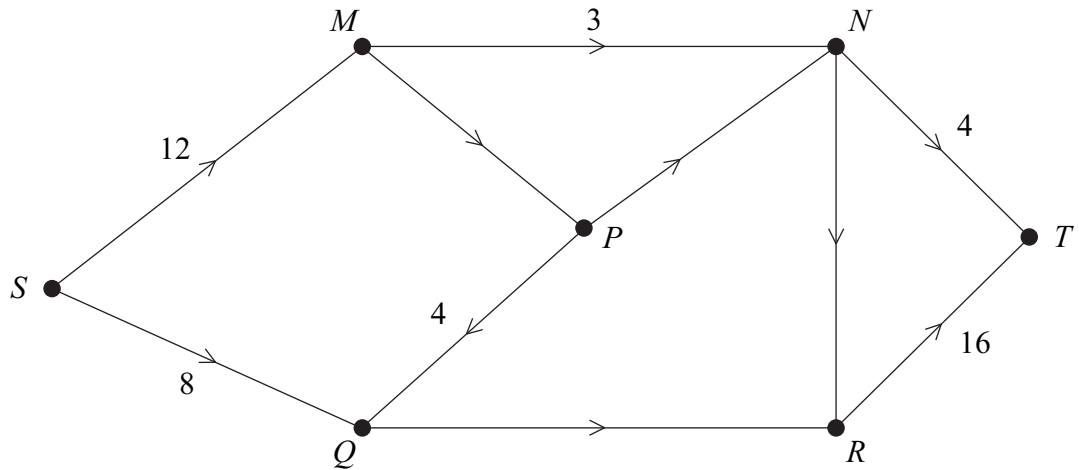
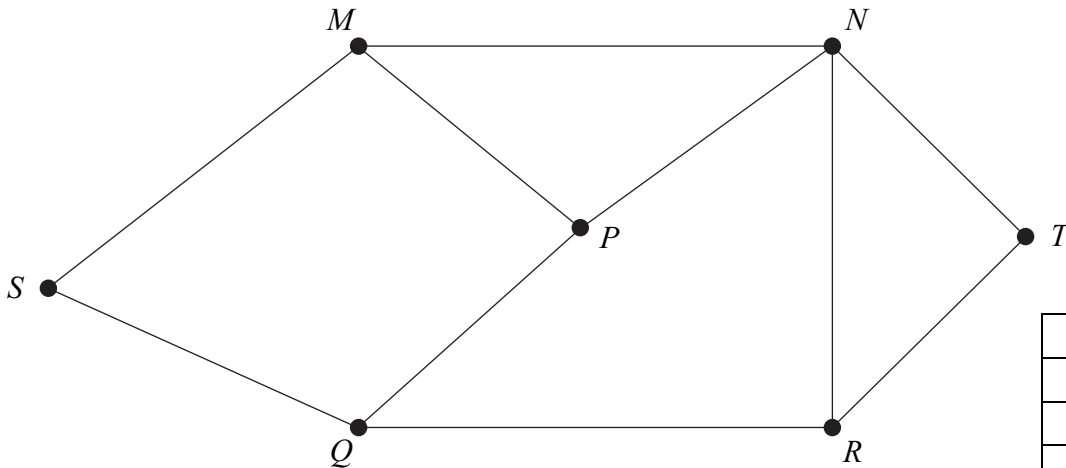


Figure 5 (for use in Question 6)



Path	Flow

Figure 6 (for use in Question 6)

