



# General Certificate of Education

## Mathematics 6360

### *MD02 Decision 2*

## Mark Scheme

### *2006 examination – June series*

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

## Key To Mark Scheme And Abbreviations Used In Marking

|              |  |     |                            |
|--------------|--|-----|----------------------------|
| M            | mark is for method   |     |                            |
| m or dM      | mark is dependent on one or more M marks and is for method         |     |                            |
| A            | mark is dependent on M or m marks and is for accuracy              |     |                            |
| B            | mark is independent of M or m marks and is for method and accuracy |     |                            |
| E            | mark is for explanation  |     |                            |
| ✓ or ft or F | follow through from previous incorrect result                      | MC  | mis-copy                   |
| CAO          | correct answer only  | MR  | mis-read                   |
| CSO          | correct solution only  | RA  | required accuracy          |
| AWFW         | anything which falls within  | FW  | further work               |
| AWRT         | anything which rounds to   | ISW | ignore subsequent work     |
| ACF          | any correct form   | FIW | from incorrect work        |
| AG           | answer given   | BOD | given benefit of doubt     |
| SC           | special case   | WR  | work replaced by candidate |
| OE           | or equivalent  | FB  | formulae book              |
| A2,1         | 2 or 1 (or 0) accuracy marks                                       | NOS | not on scheme              |
| -x EE        | deduct x marks for each error                                      | G   | graph                      |
| NMS          | no method shown  | c   | candidate                  |
| PI           | possibly implied   | sf  | significant figure(s)      |
| SCA          | substantially correct approach                                     | dp  | decimal place(s)           |

### No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

## MD02

| Q            | Solution   | Marks  | Total                 | Comments   |   |   |       |   |   |   |   |          |   |  |
|--------------|--|--|-----------------------|--|---|---|-------|---|---|---|---|----------|---|--|
| 1(a)         |  |  |                       |  |   |   |       |   |   |   |   |          |   |  |
|              |  | M1<br>A1<br>A1   | 3                     | SCA<br>(almost correct 2 slips)<br>Correct   |   |   |       |   |   |   |   |          |   |  |
| (b)          | Forward pass for earliest start times  | M1<br>A1   | 2                     | All correct  |   |   |       |   |   |   |   |          |   |  |
| (c)          | Backward pass for latest finish times  | M1<br>A1   | 2                     | All correct  |   |   |       |   |   |   |   |          |   |  |
| (d)          | Critical path <i>A B E H I</i>   | B1   | 1                     |  |   |   |       |   |   |   |   |          |   |  |
| (e)          | <table border="1"> <tr> <td>Non critical</td><td>C</td><td>D</td><td>F</td><td>G</td></tr> <tr> <td>Float</td><td>4</td><td>2</td><td>3</td><td>3</td></tr> </table> | Non critical   | C                     | D  | F | G | Float | 4 | 2 | 3 | 3 | M1<br>A1 | 2 | At least one float time correct<br>All correct |
| Non critical | C  | D  | F                     | G  |   |   |       |   |   |   |   |          |   |  |
| Float        | 4  | 2  | 3                     | 3  |   |   |       |   |   |   |   |          |   |  |
| (f)          |  | 'their' critical path on chart<br>C from 6 to 14 (with space 2-6)<br>D from 9 to 17 (with slack 7-9)<br>F & G from 10 to 21 with appropriate slack | B1✓<br>M1<br>A1<br>A1 | 4<br>One other activity (condone no slack or earliest start)<br>2 other non critical activities<br>All correct |   |   |       |   |   |   |   |          |   |  |
|              | <b>Total</b>   |  | <b>14</b>             |  |   |   |       |   |   |   |   |          |   |  |

**MD02 (cont)**

| Q  | Solution  | Marks | Total   | Comments  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |     |     |   |    |   |   |     |    |                   |   |   |   |   |   |   |   |   |   |
|--|---|-------|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|-----|-----|---|----|---|---|-----|----|-------------------|---|---|---|---|---|---|---|---|---|
| 2(a)   | Add extra row with all values equal   | B1    | 1   | Usually + 25 and below rest<br>18    15    19    20    17<br>23    24    22    25    23<br>20    16    18    22    19<br>21    17    18    23    20<br>25    25    25    25    25 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |     |     |   |    |   |   |     |    |                   |   |   |   |   |   |   |   |   |   |
| (b)  | Reduce columns first  | M1    |   | Do not award if full row of zeros added   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |     |     |   |    |   |   |     |    |                   |   |   |   |   |   |   |   |   |   |
|  | <table><tr><td></td><td>P</td><td>Q</td><td>R</td><td>S</td><td>T</td></tr><tr><td>A</td><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td></tr><tr><td>B</td><td>5</td><td>9</td><td>4</td><td>5</td><td>6</td></tr><tr><td>C</td><td>2</td><td>1</td><td>0</td><td>2</td><td>2</td></tr><tr><td>D</td><td>3</td><td>2</td><td>0</td><td>3</td><td>3</td></tr><tr><td>(E)</td><td>7</td><td>10</td><td>7</td><td>5</td><td>8</td></tr></table> |       | P   | Q   | R | S | T | A | 0 | 0 | 1 | 0 | 0 | B | 5 | 9 | 4 | 5 | 6 | C | 2 | 1 | 0 | 2 | 2 | D | 3 | 2 | 0 | 3 | 3   | (E) | 7   | 10 | 7 | 5 | 8   | A1 |                   |   |   |   |   |   |   |   |   |   |
|  |   | P     | Q   | R   | S | T |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |     |     |   |    |   |   |     |    |                   |   |   |   |   |   |   |   |   |   |
|  | A   | 0     | 0   | 1   | 0 | 0 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |     |     |   |    |   |   |     |    |                   |   |   |   |   |   |   |   |   |   |
|  | B   | 5     | 9   | 4   | 5 | 6 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |     |     |   |    |   |   |     |    |                   |   |   |   |   |   |   |   |   |   |
| C  | 2   | 1     | 0   | 2   | 2 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |     |     |   |    |   |   |     |    |                   |   |   |   |   |   |   |   |   |   |
| D  | 3   | 2     | 0   | 3   | 3 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |     |     |   |    |   |   |     |    |                   |   |   |   |   |   |   |   |   |   |
| (E)  | 7   | 10    | 7   | 5   | 8 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |     |     |   |    |   |   |     |    |                   |   |   |   |   |   |   |   |   |   |
| Reduce rows next   | M1  |       | These 2 marks available for those who reduce row first  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |     |     |   |    |   |   |     |    |                   |   |   |   |   |   |   |   |   |   |
| <table><tr><td></td><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td></tr><tr><td></td><td>1</td><td>5</td><td>0</td><td>1</td><td>2</td></tr><tr><td></td><td>2</td><td>1</td><td>0</td><td>2</td><td>2</td></tr><tr><td></td><td>3</td><td>2</td><td>0</td><td>3</td><td>3</td></tr><tr><td></td><td>2</td><td>5</td><td>2</td><td>0</td><td>3</td></tr></table>  |   | 0     | 0   | 1   | 0 | 0 |   | 1 | 5 | 0 | 1 | 2 |   | 2 | 1 | 0 | 2 | 2 |   | 3 | 2 | 0 | 3 | 3 |   | 2 | 5 | 2 | 0 | 3 | A1✓ |     | <table><tr><td></td><td></td><td></td></tr><tr><td>.</td><td>.</td><td>.</td></tr><tr><td>.</td><td>.</td><td>.</td></tr><tr><td>.</td><td>.</td><td>.</td></tr><tr><td>.</td><td>.</td><td>.</td></tr></table><br>One error only |    |   |   | .   | .  | .                 | . | . | . | . | . | . | . | . | . |
|  | 0   | 0     | 1   | 0   | 0 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |     |     |   |    |   |   |     |    |                   |   |   |   |   |   |   |   |   |   |
|  | 1   | 5     | 0   | 1   | 2 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |     |     |   |    |   |   |     |    |                   |   |   |   |   |   |   |   |   |   |
|  | 2   | 1     | 0   | 2   | 2 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |     |     |   |    |   |   |     |    |                   |   |   |   |   |   |   |   |   |   |
|  | 3   | 2     | 0   | 3   | 3 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |     |     |   |    |   |   |     |    |                   |   |   |   |   |   |   |   |   |   |
|  | 2   | 5     | 2   | 0   | 3 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |     |     |   |    |   |   |     |    |                   |   |   |   |   |   |   |   |   |   |
|  |   |       |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |     |     |   |    |   |   |     |    |                   |   |   |   |   |   |   |   |   |   |
| .  | .   | .     |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |     |     |   |    |   |   |     |    |                   |   |   |   |   |   |   |   |   |   |
| .  | .   | .     |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |     |     |   |    |   |   |     |    |                   |   |   |   |   |   |   |   |   |   |
| .  | .   | .     |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |     |     |   |    |   |   |     |    |                   |   |   |   |   |   |   |   |   |   |
| .  | .   | .     |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |     |     |   |    |   |   |     |    |                   |   |   |   |   |   |   |   |   |   |
| Covering zeros requires 3 lines so adjust with least entry remaining being 1   | M1  |       | SC if full row of zeros, award M1 for further stage of adjustment and A1 for final correct matrix |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |     |     |   |    |   |   |     |    |                   |   |   |   |   |   |   |   |   |   |
| <table><tr><td></td><td>P</td><td>Q</td><td>R</td><td>S</td><td>T</td></tr><tr><td>A</td><td>0</td><td>0</td><td>2</td><td>1</td><td>0</td></tr><tr><td>R</td><td>0</td><td>4</td><td>0</td><td>1</td><td>1</td></tr><tr><td>C</td><td>1</td><td>0</td><td>0</td><td>2</td><td>1</td></tr><tr><td>D</td><td>2</td><td>1</td><td>0</td><td>3</td><td>2</td></tr><tr><td>E</td><td>1</td><td>4</td><td>2</td><td>0</td><td>2</td></tr></table> |   | P     | Q   | R   | S | T | A | 0 | 0 | 2 | 1 | 0 | R | 0 | 4 | 0 | 1 | 1 | C | 1 | 0 | 0 | 2 | 1 | D | 2 | 1 | 0 | 3 | 2 | E   | 1   | 4   | 2  | 0 | 2 | A1✓ |    | ft one error only |   |   |   |   |   |   |   |   |   |
|  | P   | Q     | R   | S   | T |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |     |     |   |    |   |   |     |    |                   |   |   |   |   |   |   |   |   |   |
| A  | 0   | 0     | 2   | 1   | 0 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |     |     |   |    |   |   |     |    |                   |   |   |   |   |   |   |   |   |   |
| R  | 0   | 4     | 0   | 1   | 1 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |     |     |   |    |   |   |     |    |                   |   |   |   |   |   |   |   |   |   |
| C  | 1   | 0     | 0   | 2   | 1 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |     |     |   |    |   |   |     |    |                   |   |   |   |   |   |   |   |   |   |
| D  | 2   | 1     | 0   | 3   | 2 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |     |     |   |    |   |   |     |    |                   |   |   |   |   |   |   |   |   |   |
| E  | 1   | 4     | 2   | 0   | 2 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |     |     |   |    |   |   |     |    |                   |   |   |   |   |   |   |   |   |   |
| Match: A-Tim; B-Phil; C-Quin; D-Ros  | B1  |       |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |     |     |   |    |   |   |     |    |                   |   |   |   |   |   |   |   |   |   |
| Min <sup>m</sup> Time = 17 + 23 + 16 + 18 = 74 secs  | B1  |       | 8   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |     |     |   |    |   |   |     |    |                   |   |   |   |   |   |   |   |   |   |
|  | Total   |       | 9   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |     |     |   |    |   |   |     |    |                   |   |   |   |   |   |   |   |   |   |

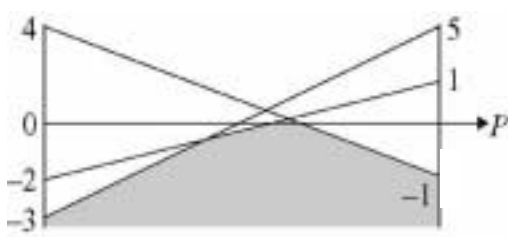
## MD02 (cont)

| Q            | Solution   | Marks                                | Total     | Comments  |
|--------------|--|--------------------------------------|-----------|---|
| 3(a)         | Working back from H<br>Starting from A (network)<br><br>$B \ 8^1$ $F \ 5^2 \ 4^3$<br>$C \ 7^1 \ 6^2$ $H \ 16^2 \ 14^4 \ 14^5$<br><br>$D \ 9^1 \ 6^2 \ 5^3$ $G \ 12^2 \ 8^4$<br>$E \ 8^1$ | B1<br>M1<br>M1<br>M1<br><br>A1<br>A1 | 6         | Alternatively, from A<br><br>First (stage) costs<br>second stage attempt<br>second stage indicated eg $15^2$ etc<br>Third stage attempt (two numbers crossed out)<br>Final value of 14 Dep on M2 earned<br>All “correct” with 2 clear routes to cost of 14<br>(or equivalent in tabular form) |
| (b)          | Min cost = 14<br><i>ABCFH</i><br>and <i>ABCDGH</i>   | B1<br>B1<br>B1                       | 3         |   |
| <b>Total</b> |  |                                      | <b>9</b>  |   |
| 4(a)         | D  | B1                                   | 1         |   |
| (b)          | $(17 + 25 + 35 + 13 + 12 + 13 = 115)$  | B1                                   | 1         |   |
| (c)          | $ABD_{\max} = 25$ ; $GED_{\max} = 12$  | B1B1                                 | 2         |   |
| (d)(i)       | <br>Route <i>ABD</i> <i>GED</i> <i>GFD</i> <i>GD</i> <i>AD</i> <i>AFD</i> <i>GED</i><br>Flow    25    12    16    13    17    15    7  | M1<br>M1<br>M1<br>A1<br>A1<br><br>A1 | 6         | Forward and backward flows<br>Adjusting flows on diagram<br>Routes and flows in chart<br>One correct other than ABD, GED<br>Another correct   |
| (ii)         | Total = 105<br>Max flow<br>  | B1<br><br>B1                         | 2         |   |
| (iii)        | Cut through <i>AF</i> , <i>AD</i> , <i>BD</i> , <i>DE</i> , <i>DG</i> , and <i>GF</i>  | M1<br>A1                             | 2         | Through 3 saturated arcs ( <i>fairly generous</i> )<br>Correct  |
| (e)          | Reduce max flow by their <i>EG</i><br>changing 19 to 15<br>$\Rightarrow$ New max = 101   | M1<br><br>A1                         | 2         | Reduce by 4 since everywhere else saturated<br>Correct answer $\Rightarrow$ 2 marks   |
| <b>Total</b> |  |                                      | <b>16</b> |   |

## MD02 (cont)

| Q      | Solution  | Marks   | Total   | Comments                                 |                |     |                |               |    |   |    |   |   |                 |                |   |   |   |   |    |                |                |   |   |   |   |   |                |               |   |    |               |   |    |   |   |    |   |               |
|--------|---|---|---|--|----------------|-----|----------------|---------------|----|---|----|---|---|-----------------|----------------|---|---|---|---|----|----------------|----------------|---|---|---|---|---|----------------|---------------|---|----|---------------|---|----|---|---|----|---|---------------|
| 5(a)   | $3x+7y \leq 33$   | M1  | 2   | One correct inequality, or all using $<$ |                |     |                |               |    |   |    |   |   |                 |                |   |   |   |   |    |                |                |   |   |   |   |   |                |               |   |    |               |   |    |   |   |    |   |               |
|        | $x+2y \leq 10$  | A1  |   | All correct                              |                |     |                |               |    |   |    |   |   |                 |                |   |   |   |   |    |                |                |   |   |   |   |   |                |               |   |    |               |   |    |   |   |    |   |               |
| (b)(i) | $2x+7y \leq 26$   |   | 2   |  |                |     |                |               |    |   |    |   |   |                 |                |   |   |   |   |    |                |                |   |   |   |   |   |                |               |   |    |               |   |    |   |   |    |   |               |
|        | Compare $\frac{33}{3}, \frac{10}{1}, \frac{26}{2}$  | E1  |   |  |                |     |                |               |    |   |    |   |   |                 |                |   |   |   |   |    |                |                |   |   |   |   |   |                |               |   |    |               |   |    |   |   |    |   |               |
|        | Choose smallest positive value $\Rightarrow$<br>pivot = 1   | E1  |   |  |                |     |                |               |    |   |    |   |   |                 |                |   |   |   |   |    |                |                |   |   |   |   |   |                |               |   |    |               |   |    |   |   |    |   |               |
| (ii)   | <table><tr><td><math>P</math></td><td><math>x</math></td><td><math>y</math></td><td><math>r</math></td><td><math>s</math></td><td><math>t</math></td><td>Value</td></tr><tr><td>1</td><td>0</td><td>-1</td><td>0</td><td>4</td><td>0</td><td>40</td></tr><tr><td>0</td><td>0</td><td>1</td><td>1</td><td>-3</td><td>0</td><td>3</td></tr><tr><td>0</td><td>1</td><td>2</td><td>0</td><td>1</td><td>0</td><td>10</td></tr><tr><td>0</td><td>0</td><td>3</td><td>0</td><td>-2</td><td>1</td><td>6</td></tr></table> | $P$   | $x$   | $y$                                      | $r$            | $s$ | $t$            | Value         | 1  | 0 | -1 | 0 | 4 | 0               | 40             | 0 | 0 | 1 | 1 | -3 | 0              | 3              | 0 | 1 | 2 | 0 | 1 | 0              | 10            | 0 | 0  | 3             | 0 | -2 | 1 | 6 | M1 | 7 | Row operation |
|        | $P$   | $x$   | $y$   | $r$                                      | $s$            | $t$ | Value          |               |    |   |    |   |   |                 |                |   |   |   |   |    |                |                |   |   |   |   |   |                |               |   |    |               |   |    |   |   |    |   |               |
|        | 1   | 0   | -1  | 0  | 4              | 0   | 40             |               |    |   |    |   |   |                 |                |   |   |   |   |    |                |                |   |   |   |   |   |                |               |   |    |               |   |    |   |   |    |   |               |
|        | 0   | 0   | 1   | 1  | -3             | 0   | 3              |               |    |   |    |   |   |                 |                |   |   |   |   |    |                |                |   |   |   |   |   |                |               |   |    |               |   |    |   |   |    |   |               |
|        | 0   | 1   | 2   | 0  | 1              | 0   | 10             |               |    |   |    |   |   |                 |                |   |   |   |   |    |                |                |   |   |   |   |   |                |               |   |    |               |   |    |   |   |    |   |               |
|        | 0   | 0   | 3   | 0  | -2             | 1   | 6              |               |    |   |    |   |   |                 |                |   |   |   |   |    |                |                |   |   |   |   |   |                |               |   |    |               |   |    |   |   |    |   |               |
|        |   | A1  | Correct one row ( <i>other than pivot row</i> ) |  |                |     |                |               |    |   |    |   |   |                 |                |   |   |   |   |    |                |                |   |   |   |   |   |                |               |   |    |               |   |    |   |   |    |   |               |
|        |   | A1  | All correct                                     |  |                |     |                |               |    |   |    |   |   |                 |                |   |   |   |   |    |                |                |   |   |   |   |   |                |               |   |    |               |   |    |   |   |    |   |               |
|        | next $y$ pivot on 3   |   | M1  |  |                |     |                |               |    |   |    |   |   |                 |                |   |   |   |   |    |                |                |   |   |   |   |   |                |               |   |    |               |   |    |   |   |    |   |               |
|        |   | <table><tr><td>1</td><td>0</td><td>0</td><td>0</td><td><math>3\frac{1}{3}</math></td><td><math>\frac{1}{3}</math></td><td>42</td></tr><tr><td>0</td><td>0</td><td>0</td><td>1</td><td><math>-2\frac{1}{3}</math></td><td><math>-\frac{1}{3}</math></td><td>1</td></tr><tr><td>0</td><td>1</td><td>0</td><td>0</td><td><math>2\frac{1}{3}</math></td><td><math>-\frac{2}{3}</math></td><td>6</td></tr><tr><td>0</td><td>0</td><td>1</td><td>0</td><td><math>-\frac{2}{3}</math></td><td><math>\frac{1}{3}</math></td><td>2</td></tr></table> | 1   | 0  | 0              | 0   | $3\frac{1}{3}$ | $\frac{1}{3}$ | 42 | 0 | 0  | 0 | 1 | $-2\frac{1}{3}$ | $-\frac{1}{3}$ | 1 | 0 | 1 | 0 | 0  | $2\frac{1}{3}$ | $-\frac{2}{3}$ | 6 | 0 | 0 | 1 | 0 | $-\frac{2}{3}$ | $\frac{1}{3}$ | 2 | m1 | Row operation |   |    |   |   |    |   |               |
| 1      | 0   | 0   | 0   | $3\frac{1}{3}$                           | $\frac{1}{3}$  | 42  |                |               |    |   |    |   |   |                 |                |   |   |   |   |    |                |                |   |   |   |   |   |                |               |   |    |               |   |    |   |   |    |   |               |
| 0      | 0   | 0   | 1   | $-2\frac{1}{3}$                          | $-\frac{1}{3}$ | 1   |                |               |    |   |    |   |   |                 |                |   |   |   |   |    |                |                |   |   |   |   |   |                |               |   |    |               |   |    |   |   |    |   |               |
| 0      | 1   | 0   | 0   | $2\frac{1}{3}$                           | $-\frac{2}{3}$ | 6   |                |               |    |   |    |   |   |                 |                |   |   |   |   |    |                |                |   |   |   |   |   |                |               |   |    |               |   |    |   |   |    |   |               |
| 0      | 0   | 1   | 0   | $-\frac{2}{3}$                           | $\frac{1}{3}$  | 2   |                |               |    |   |    |   |   |                 |                |   |   |   |   |    |                |                |   |   |   |   |   |                |               |   |    |               |   |    |   |   |    |   |               |
|        | A1  | Correct one row (other than pivot row)  |   |  |                |     |                |               |    |   |    |   |   |                 |                |   |   |   |   |    |                |                |   |   |   |   |   |                |               |   |    |               |   |    |   |   |    |   |               |
|        | A1  | All correct (condone multiples of given rows) (maximum 6 if $y$ -pivot used first)  |   |  |                |     |                |               |    |   |    |   |   |                 |                |   |   |   |   |    |                |                |   |   |   |   |   |                |               |   |    |               |   |    |   |   |    |   |               |
|        |   |   |   |  |                |     |                |               |    |   |    |   |   |                 |                |   |   |   |   |    |                |                |   |   |   |   |   |                |               |   |    |               |   |    |   |   |    |   |               |
|        |   |   |   |  |                |     |                |               |    |   |    |   |   |                 |                |   |   |   |   |    |                |                |   |   |   |   |   |                |               |   |    |               |   |    |   |   |    |   |               |
|        |   |   |   |  |                |     |                |               |    |   |    |   |   |                 |                |   |   |   |   |    |                |                |   |   |   |   |   |                |               |   |    |               |   |    |   |   |    |   |               |
| (iii)  | No negative number in top row   | E1  | 3   | ft if M3 scored and optimum reached      |                |     |                |               |    |   |    |   |   |                 |                |   |   |   |   |    |                |                |   |   |   |   |   |                |               |   |    |               |   |    |   |   |    |   |               |
|        | $P_{\max} = 42$   | B1✓   |   |  |                |     |                |               |    |   |    |   |   |                 |                |   |   |   |   |    |                |                |   |   |   |   |   |                |               |   |    |               |   |    |   |   |    |   |               |
|        | $x = 6 \ y = 2$   | B1✓   |   |  |                |     |                |               |    |   |    |   |   |                 |                |   |   |   |   |    |                |                |   |   |   |   |   |                |               |   |    |               |   |    |   |   |    |   |               |
|        | Total   |   | 14  |  |                |     |                |               |    |   |    |   |   |                 |                |   |   |   |   |    |                |                |   |   |   |   |   |                |               |   |    |               |   |    |   |   |    |   |               |

## MD02 (cont)

| Q      | Solution   | Marks  | Total     | Comments  |
|--------|--|--|-----------|---|
| 6(a)   | Gain for Rowan + gain for Colleen in each strategy = 0   | E1   | 1         | Gain for one = loss of other  |
| (b)    | $  \begin{array}{ccc c}  & & & \underline{\text{min}} \\  -3 & -4 & 1 & -4 \\  1 & 5 & -1 & \boxed{-1} \\  -2 & -3 & 4 & -3 \\  \hline  \text{Max} & \boxed{1} & 5 & 4  \end{array}  $   | M1<br><br>A1   |           | $\left\{ \begin{array}{l} \text{minimum of rows \& max of columns} \\ \text{or} \\ \text{maximum of minima or minimax} \end{array} \right.$ |
|        | $1 \neq -1 \Rightarrow$ no stable solution   | E1   | 3         |   |
| (c)    | $R_3$ dominates $R_1$<br>$(-3, -4, 1) < (-2, -3, 4)$ so never play $R_1$   | E1   | 1         |   |
| (d)(i) | $R$ chooses $R_2$ with prob $p$<br>$\Rightarrow$ choose $R_3$ with prob $1 - p$<br>$\Rightarrow$ expected gain when $C$ plays<br>$C_1: p - 2(1 - p) = 3p - 2$<br>$C_2: 5p - 3(1 - p) = 8p - 3$<br>$C_3: -p + 4(1 - p) = 4 - 5p$<br>Plot expected gains for $0 \leq p \leq 1$ | M1<br><br><br><br><br><br><br><br><br><br>A1<br><br>M1 |           | Attempt at one expression   |
|        |   | A1   |           | Condone mirror image  |
|        | Choosing their "highest" point<br>$C_1$ & $C_3$ intersect $\Rightarrow 3p - 2 = 4 - 5p$<br>$\Rightarrow p = \frac{3}{4}$   | M1<br><br>A1   |           | Any 2 lines   |
|        | $\Rightarrow$ play $R_2$ with prob $\frac{3}{4}$<br>and $R_3$ with prob $\frac{1}{4}$  | E1✓  | 7         | Statement of strategy   |
| (ii)   | Value of game is $3 \times \frac{3}{4} - 2 = \frac{1}{4}$  | B1   | 1         | CSO or equivalent, eg 0.25  |
|        | <b>Total</b>   |  | <b>13</b> |   |
|        | <b>TOTAL</b>   |  | <b>75</b> |   |