

General Certificate of Education (A-level) January 2013

Mathematics

MD02

(Specification 6360)

Decision 2

Final

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
В	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

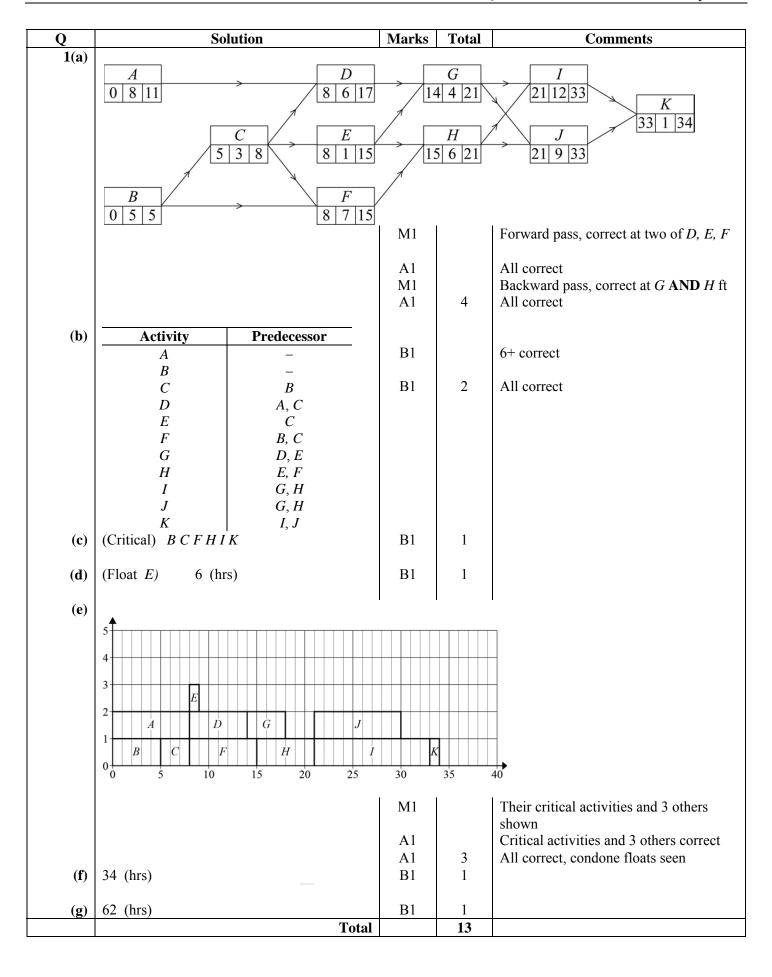
Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.



Q	Solution	Marks	Total	Comments
2(a)	$ \begin{pmatrix} 4 & -1 & 2 & 3 \\ 4 & 6 & 3 & 7 \\ 1 & 3 & -2 & 4 \end{pmatrix} -2 $ Max 4 6 3 7			
	Maximin (row) = 3	M1		Either correct, including correct values
	Minimax (col) = 3	A1 CSO		Both correct, written as equations PI by next line
	As Maximin (row) = Minimax (col) There is a stable solution	E1		Must have equation and statement and scored first 2 marks
	$ \begin{pmatrix} \text{Play safe} & \text{(H)} & B \\ \text{(Play safe} & \text{(W)} & F \end{pmatrix} $	B1	4	Both correct
(b)	Saddle point (B,F)	B1	1	
	Total		5	

MD02								
Q			Soluti	on		Marks	Total	Comments
3(a)	(8	5	0	9	6)			
	5	6	5	9	7	D.1		
	11	10	12	12	11	B1	1	
	9	5	8	12	9			
		3	O	12)			
(b)	Add at	n extra r	row ≥ 12	2		B1		
	(8	5	0	9	6 (0)	D 1		
	5	6	5	9	7 (5)			
	11	10	12	12	11 (10)			
					` ′			
	9	5	8	12	9 (5)			
	(12	12	12	12	12)(12)			
	8	5	0 9	9 6				
	0	1		4 2		M1		3 rows correct from row reduction
	1	0	2 2	2 1				
	4	0		7 4				
	0	0	0 (0 0		A1		All correct
	(8	5	0	9	6)			Alternatives
			\ \					(86-)
	¥	ļ	Y .	4	2			$\begin{bmatrix} 0 & 1 & 0 & 4 & 2 \end{bmatrix}$
		0	ť	2	1			
		0	3	7	4			
	 (• 		•	0	-0)			4 0 3 7 4
	(Zeros	correct	ly covere	d by 4 lir	nes)	B1F		(0 0 0 0)
	Cover	ed in 4 l	ines, not	optimal		E1		*
	(reduc	e by 1)						
	9.	5	n	8	٩			7 (8) 5 (6) D (0) 8 (9) 5 (6)
		ĺ	0	3	1			$\phi(0)$ 2 (2) 1 (0) 4 (4) 2 (2)
	1	6	2	1	Ō			$\phi(0)$ $\phi(0)$ $\phi(0)$ $\phi(0)$ $\phi(0)$
	4	þ	2 3	6	0 3 0			3 (3) 0 (0) 3 (2) 6 (6) 3 (3)
	1	1	1	0	0			0 (0) $1 (1)$ $1 (0)$ $0 (0)$ $0 (0)$
						B1		
	5 lines	needed	l, optimal			(E1)		*or earned here
	2 111103		, optimar					or carried note
			XA, YE,	, <i>ZB</i> , (-D)	B1		
	Value	= 151				B1	8	
					Total		9	

Q	Solution	Marks	Total	Comments
(4)(a)(i)	Max Flow = 50 (Min cut = 50)	E1		Either statement
(ii)	35 ≤ max flow ≤ 50 (or min cut)	E1, E1		E1 for strict inequalities
(iii)	Error or contradiction	E1	4	oe
(b)	At F,			
	flow in ≥ 8 flow out ≤ 7	M1 A1	2	Stating F and one of the 'flows'
	Total		6	

$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	Q Q	Solution	Marks	Total	Comments
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$			212001110		- Comments
(ii) $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		1 -1 2 -3 0 0 0 0	B2,1,0	2	All correct, 3 rows correct
(ii) $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	(b)(i)	z-col: 16 17 19	M1		
(ii) $1 $				2	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		Min, R_3 as pivot	Al	2	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	(ii)	$1 \frac{1}{2} -1 0 0 1\frac{1}{2} 0 \frac{51}{2}$	M1		Row operations
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		2 2	A 1		
Co(i) O 1 1 0 0 -1 1 2 A1 3 All correct		$0 \frac{1}{1} -1 1 0 \frac{1}{1} 0 \frac{17}{1}$	Al		One row (other than R_3) correct
Alternative 2 1 -2 0 0 3 0 51 0 1 4 0 2 -1 0 15 0 1 4 0 2 -1 0 15 0 1 -2 2 0 1 0 0 -1 1 2 0 0 0 -1 1 2 0 0 1 1 2 0 0 1 1 2 0 0 1 1 2 0 0 1 1 2 0 0 1 1 2 2 2 0 0 1 1 2 2 2 0 0 1 1 0 0 -1 1 2 2 0 0 1 1 0 0 -1 1 2 0 0 1 1 2 0 0 1 1 2 0 0 1 1 2 0 0 1 1 2 0 0 1 1 2 0 0 1 1 2 0 0 1 1 2 0 0 1 1 2 0 0 1 1 2 0 0 1 1 2 0 0 1 1 2 0 0 1 1 2 0 0 1 1 2 0 0 1 1 2 0 0 0 1 2 0 0 0 0 0 0 0 0 0		$\begin{array}{cccccccccccccccccccccccccccccccccccc$	A1	3	All correct
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$					
(c)(i) $y \cot \frac{15}{4}, \left(-\frac{17}{2}\right), \frac{2}{1}$ R_4 as pivot B1 Fully correct description $ \begin{array}{cccccccccccccccccccccccccccccccccc$		2 1 -2 0 0 3 0 51			
(c)(i) $y \text{ col } \frac{15}{4} \cdot \left(-\frac{17}{2}\right), \frac{2}{1}$ R_4 as pivot B1 $ \begin{array}{cccccccccccccccccccccccccccccccccc$		0 1 4 0 2 -1 0 15			
(c)(i) $y \cot \frac{15}{4}, \left(-\frac{17}{2}\right), \frac{2}{1}$ R_4 as pivot B1 Fully correct description $ \begin{array}{cccccccccccccccccccccccccccccccccc$		0 1 -2 2 0 1 0 17			
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		0 1 1 0 0 -1 1 2	(111)		
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	(c)(i)	$y \text{ col } \frac{15}{4}, \left(-\frac{17}{2}\right), \frac{2}{1}$ R_4 as pivot	B1		Fully correct description
Color Col		$0 -1\frac{1}{2} 0 0 1 1\frac{1}{2} -2 \frac{7}{2}$	M1		Row operations
Alternative 2 3 0 0 0 1 2 55 0 -3 0 0 2 3 -4 7 (M1) 0 3 0 2 0 -1 2 21 0 1 1 0 0 -1 1 2 (c)(ii) Optimal $P = \frac{55}{2}$ Puth statement and valve readed. OF			Λ 1	2	All correct
$ \begin{vmatrix} 2 & 3 & 0 & 0 & 0 & 1 & 2 & 55 \\ 0 & -3 & 0 & 0 & 2 & 3 & -4 & 7 \\ 0 & 3 & 0 & 2 & 0 & -1 & 2 & 21 \\ 0 & 1 & 1 & 0 & 0 & -1 & 1 & 2 \\ \end{vmatrix} $			AI	3	All collect
(c)(ii) Optimal $P = \frac{55}{2}$					
(c)(ii) Optimal $P = \frac{55}{2}$ Puth statement and value recorded. OF		0 -3 0 0 2 3 -4 7	(M1)		
(c)(ii) Optimal $P = \frac{55}{2}$ Puth statement and value recorded. OF		0 3 0 2 0 -1 2 21	(A1)		
$P = \frac{55}{2}$ Puth statement and valve needed. OF		0 1 1 0 0 -1 1 2	()		
	(c)(ii)				
$x = 0, y = 2, z = \frac{21}{2}$ $s = t = 0, r = \frac{7}{2}$ B1 B1 B1		$F = \frac{1}{2}$	B1		Both statement and value needed. OE
$s = t = 0, r = \frac{7}{2}$ B1 3		$x = 0, y = 2, z = \frac{21}{2}$	B1		
		$s = t = 0, \ r = \frac{7}{2}$	B1	3	
Total 13					

Q	Solution	Marks	Total	Comments
6(a)	$R_C > R_B$	E1	1	oe
(b)	$ \begin{array}{ccc} A \begin{pmatrix} -2 & 0 & 3 \\ 4 & 1 & -1 \end{pmatrix} \\ K \text{ plays } A \text{ prob } p \\ C \text{ prob } 1-p \end{array} $			
	P plays D, K wins $-2p + 4(1-p)$ $(=4-6p)$	M1		Allow 2 expressions in unsimplified form
	E, K wins $1-p$ F, K wins $3p-1(1-p)$ $(=-1+4p)$	A1		All 3 correct
	$ \begin{array}{c c} 4 \\ 1 \\ -1 \end{array} $	M1		Must have 3 lines
	l I	A1		With values shown
	Max at $1 - p = -1 + 4p$	M1		Identifying correct maximum from their graph
	$p = \frac{2}{5}$			
	(K plays) A prob $\frac{2}{5}$, C prob $\frac{3}{5}$	A1		Both stated, coming from equating correct two equations and M2 scored
	Value of game = $\frac{3}{5}$	B1	7	

Q	Solution	Marks	Total	Comments
6(c)	P plays D prob p			
	E " q			
	F " 1-p-q			
	W. wlasse A. D. lanes	N // 1		
	K plays A, P loses -2p + 3(1-p-q) = 3 - 5p - 3q	M1		Either (unsimplified) expression correct
	-2p + 3(1-p-q)=3-3p-3q			
	K plays C, P loses			
	4p+q-1(1-p-q)=-1+5p+2q			
	,			
	$3 - 5p - 3q = \frac{3}{5}$			
	$-1 + 5p + 2q = \frac{3}{5}$	m1		Equating BOTH of their expressions to value of their game
	$\frac{5}{2} \qquad -q = \frac{6}{5}$			value of their game
	$2 -q = \frac{1}{5}$			
	$q = \frac{4}{5}$	A1 CSO		Or for finding <i>p</i>
	3	CSO		
	$5p + \frac{8}{5} - 1 = \frac{3}{5}$			
	p = 0			
	p = 0			
	P plays D prob 0			
	E, prob $\frac{4}{5}$			
		E1	4	All three needed, must have scored
	F , prob $\frac{1}{5}$			previous A mark
	Alternative method			
	Probability of <i>D</i> is 0	(E1)		OE, might be earned in final line
		(M1)		
	$3(1-p) = \frac{3}{5}$ or $p-1(1-p) = \frac{3}{5}$, ,		Or equating the expressions
	$p = \frac{4}{5}$	(m1)		
		(A1)		
	$E \text{ prob } \frac{4}{5}$ $F \text{ prob } \frac{1}{5}$	(A1) CSO		
	Total		12	

Q			Solutio	on	Marks	Total	Comments
7(a)							
	Stage	State	From	Value			
	1	G	I	15			
		Н	I	12			
	2	E	G	15+15 = 30 ←			
			Н	12+16=28			
					B1		Stage 2 values correct
		F	G	15+13=28			
			Н	12+17= 29 ←			
	3	В	Ε	30+16 = 46			
					M1		Calculating 4 values at stage 3
		C	E	30+14 = 44 ←	m1		Using max values at E and F
			F	29+12 = 41	1111		osing man varies at 2 and 1
		D	F	29+15 = 44	A 1		All 4 values correct
	4	A	В	46+12 = 58			
				44.20.54			
			C	44+20= 64 ←			
				44.10.60	m1		Using max at C
			D	44+18 = 62	A 1		All correct
					B1	7	Identifying 64 as maximum value
	.	. ~ - ~	_		D.1		
(b)	Route A	A C E G	I		B1	1	
				Т-4-1		0	
				Total		8	

Q Q	Solution	Marks	Total	Comments
(8)(a)	ABEH 8			
	ACFH 5	D.1	1	
	A D G H 11	B1	1	
(b)(i)	ACEH 2	M1		One correct route and flow
	ACGH 4	A1		At least one other correct
	Either ADFH 1 and ABFH 2	A1		All correct
	Or ADFH 3			
	B			
	\wedge \wedge 0	M1		Forward and back flows on diagram
	810 82 24 8			Ç
	$\frac{1}{2^0}$ $\frac{1}{2^0}$ $\frac{1}{2^0}$ $\frac{1}{2^0}$			
	$A \xrightarrow{\begin{array}{c} 20 \\ 20 \\ 20 \\ 20 \\ 20 \\ 20 \\ 20 \\ 20 $			
	781 \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \			
	32 40 4			
	\mathcal{X}_{12} \mathcal{X}_{0} \mathcal{X}_{0} \mathcal{X}_{15} \mathcal{X}_{15}			
	$D \xrightarrow{\downarrow D} G$	A1	5	All correct
(ii)	Max flow 33	B1		
	B 8 E			
	[9] (8) 2[1] (0)			
	10			
	2/			
	$A \xrightarrow{11} C \xrightarrow{5} F \xrightarrow{8} H$	B1	2	OE
	4			
	[13] (14) [2] (3) 15			
	[13] (14)			
	D 11 G			
(c)	Cut through BE, CE, FH, CG, DG	B1	1	
		<i>D</i> 1		
	Total		9	
	TOTAL		75	