Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
TOTAL	

AQA

General Certificate of Education
Advanced Level Examination
June 2012

Mathematics MD02

Unit Decision 2

Thursday 21 June 2012 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 **Figure 1** below shows an activity diagram for a construction project. The time needed for each activity is given in days.

(a) Find the earliest start time and the latest finish time for each activity and insert their values on **Figure 1**. *(4 marks)*

(b) Find the critical paths and state the minimum time for completion of the project. *(3 marks)*

(c) On **Figure 2** opposite, draw a cascade diagram (Gantt chart) for the project, assuming that each activity starts as early as possible. *(5 marks)*

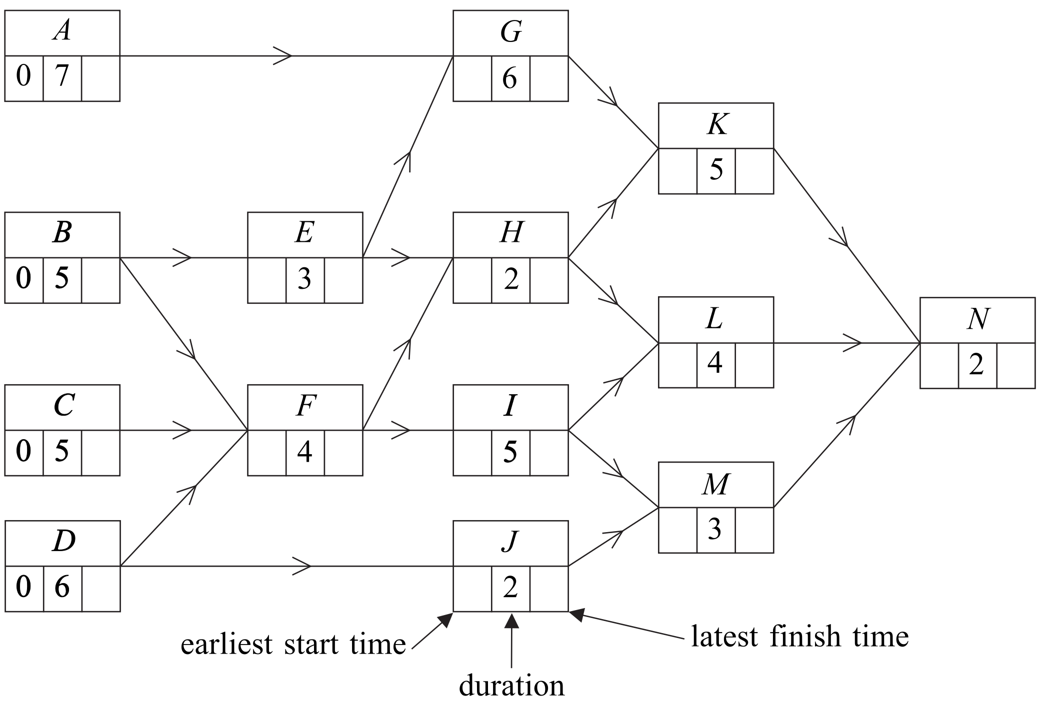
(d) Activity J takes longer than expected so that its duration is x days, where $x \geqslant 3$. Given that the minimum time for completion of the project is unchanged, find a further inequality relating to the maximum value of x. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 1

(a)

Figure 1



(b) Critical paths are ....

....

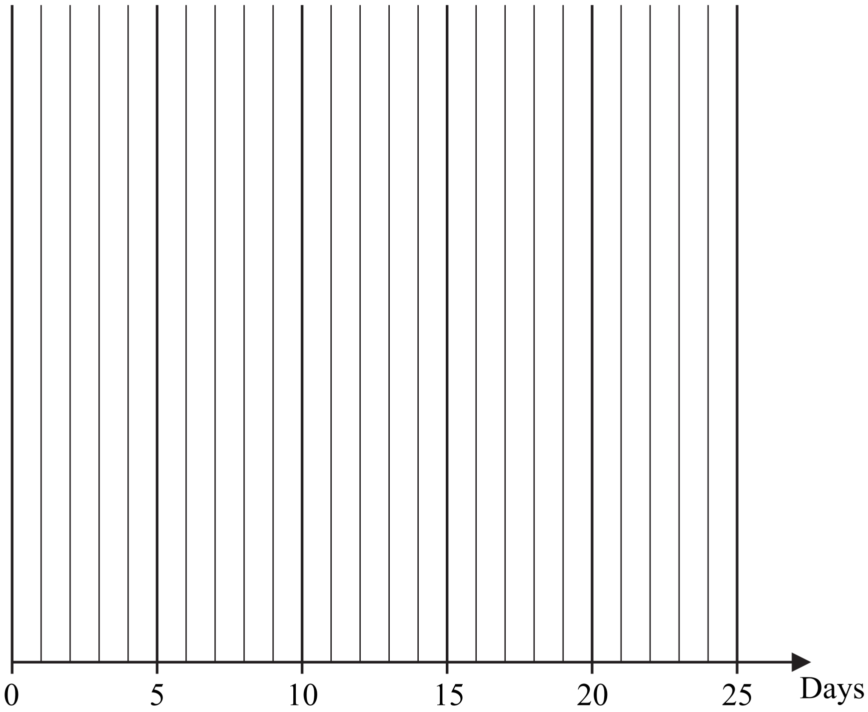
Minimum completion time is .... days.

QUESTION PART REFERENCE

Answer space for question 1

(c)

Figure 2



(d)

2 The times taken in minutes for five people, Ann, Baz, Cal, Di and Ez, to complete each of five different tasks are recorded in the table below. Neither Ann nor Di can do task 2, as indicated by the asterisks in the table.

	Ann	**Baz**	**Cal**	**Di**	**Ez**
Task 1	13	14	15	17	16
Task 2	***	21	21	***	18
Task 3	16	19	19	17	15
Task 4	16	16	18	16	16
Task 5	20	23	22	20	20

Using the Hungarian algorithm, each of the five people is to be allocated to a different task so that the total time for completing the five tasks is minimised.

(a) By reducing the **rows first** and then the columns, show that the zeros in the new table of values can be covered with four lines. *(3 marks)*

(b) Use adjustments to produce a table where five lines are required to cover the zeros. *(3 marks)*

(c) Hence find the possible ways of allocating the five people to the five tasks in the minimum total time. *(3 marks)*

(d) State the minimum total time for completing the five tasks. *(1 mark)*

QUESTION PART REFERENCE

Answer space for question 2

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QUESTION PART REFERENCE

Answer space for question 2

Turn over ▶

3 (a) Given that k is a constant, complete the Simplex tableau below for the following linear programming problem.

Maximise $P = kx + 6y + 5z$

subject to
$$2x + y + 4z \leqslant 11$$
$$x + 3y + 6z \leqslant 18$$
$$x \geqslant 0,\ y \geqslant 0,\ z \geqslant 0$$
(2 marks)

(b) Use the Simplex method to perform **one** iteration of your tableau for part **(a)**, choosing a value in the ***y*-column** as pivot. *(4 marks)*

(c) (i) In the case when $k = 1$, explain why the maximum value of P has now been reached and write down this maximum value of P. *(2 marks)*

(ii) In the case when $k = 3$, perform one further iteration and interpret your new tableau. *(6 marks)*

QUESTION PART REFERENCE

Answer space for question 3

(a)

P	x	y	z	s	t	*value*
1	$-k$	-6	-5	0	0	0
0						
0						

....

....

....

(b)

P	x	y	z	s	t	*value*

....

....

....

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QUESTION PART REFERENCE

Answer space for question 3

(c)(i)

(c)(ii)

P	x	y	z	s	t	*value*

4 (a) Two people, Adam and Bill, play a zero-sum game. The game is represented by the following pay-off matrix for Adam.

Bill

Adam

Strategy	**B_1**	**B_2**	**B_3**
A_1	-6	-1	-5
A_2	5	2	-3
A_3	-5	4	-4
A_4	2	1	-4

(i) Show that this game has a stable solution. *(3 marks)*

(ii) Find the play-safe strategy for each player. *(1 mark)*

(iii) State the value of the game for **Bill**. *(1 mark)*

QUESTION PART REFERENCE

Answer space for question 4(a)

QUESTION PART REFERENCE

Answer space for question 4(a)

Question 4 continues on the next page

Turn over ▶

4 (b) Roza plays a different zero-sum game against a computer. The game is represented by the following pay-off matrix for Roza.

Computer

	Strategy	**C_1**	**C_2**	**C_3**
Roza	**R_1**	3	4	-3
	R_2	-2	-1	5

(i) State which strategy the computer should never play, giving a reason for your answer. *(1 mark)*

(ii) Roza chooses strategy R_1 with probability p. Find expressions for the expected gains for Roza when the computer chooses each of its two remaining strategies. *(2 marks)*

(iii) Hence find the value of p for which Roza will maximise her expected gains. *(2 marks)*

(iv) Find the value of the game for Roza. *(1 mark)*

QUESTION PART REFERENCE

Answer space for question 4(b)

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QUESTION PART REFERENCE

Answer space for question 4(b)

Turn over ▶

5 Dave plans to renovate three houses, A, B and C, at the rate of one per year. The order in which they are renovated is a matter of choice, but some costs vary over the three years. The expected costs, in thousands of pounds, are given in the table below.

Year	Already renovated	Cost		
		A	B	C
1	–	60	70	65
2	A B C	– 55 65	75 – 80	70 60 –
3	A and B A and C B and C	– – 60	– 80 –	75 – –

For tax reasons, Dave needs to choose the order for renovation so that the least annual cost is as large as possible. Solving the maximin problem will produce this optimum order for renovation.

(a) (i) State the least annual cost when the order of renovation is BAC.

(ii) Determine, with a reason, whether the order ABC is better than the order BAC. *(3 marks)*

(b) By completing the table opposite, or otherwise, use dynamic programming, **working backwards from Year 3**, to find the optimum order for renovation. *(7 marks)*

QUESTION PART REFERENCE

Answer space for question 5

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QUESTION PART REFERENCE

(b)

Answer space for question 5

Year	**Already renovated**	**House renovated**	**Calculation**	**Value**
3	A and B	C		
	A and C	B		
	B and C	A		
2	A	B		
		C		
	B	A		
		C		
	C	A		
		B		
1				

Optimum order ....

....

....

....

....

....

....

Turn over ▶

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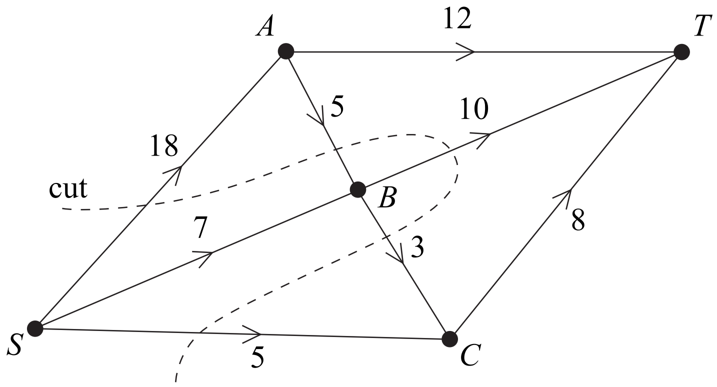
6 (a) The network shows a flow from S to T along a system of pipes, with the capacity in litres per second indicated on each edge.



(i) Show that the value of the cut shown on the diagram is 36. *(1 mark)*

....

....

(ii) The cut shown on the diagram can be represented as $\{S, B\}$, $\{A, C, T\}$.

Complete the table below to give the value of each of the 8 possible cuts. *(3 marks)*

Cut		**Value**
$\{S\}$	$\{A, B, C, T\}$	30
$\{S, A\}$	$\{B, C, T\}$	29
$\{S, B\}$	$\{A, C, T\}$	36
$\{S, C\}$	$\{A, B, T\}$	33
$\{S, A, B\}$	$\{C, T\}$	
$\{S, A, C\}$	$\{B, T\}$	
$\{S, B, C\}$	$\{A, T\}$	
$\{S, A, B, C\}$	$\{T\}$	30

(iii) State the value of the maximum flow through the network, giving a reason for your answer. *(2 marks)*

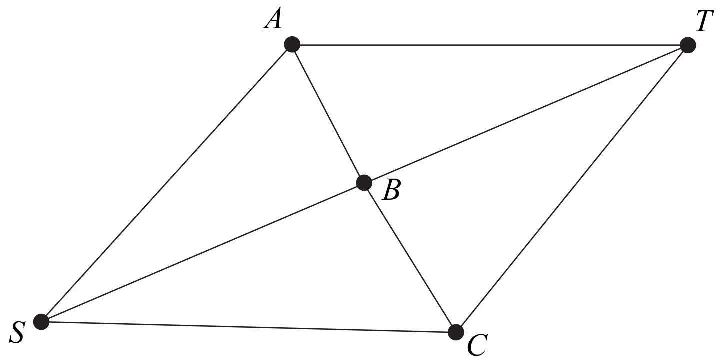
Maximum flow is ....

because ....

....

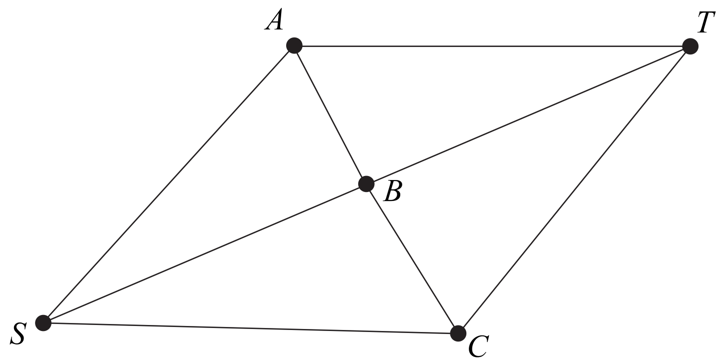
....

(iv) Indicate on the diagram below a possible flow along each edge corresponding to this maximum flow. *(1 mark)*



(b) The capacities along SC and along AT are each increased by 4 litres per second.

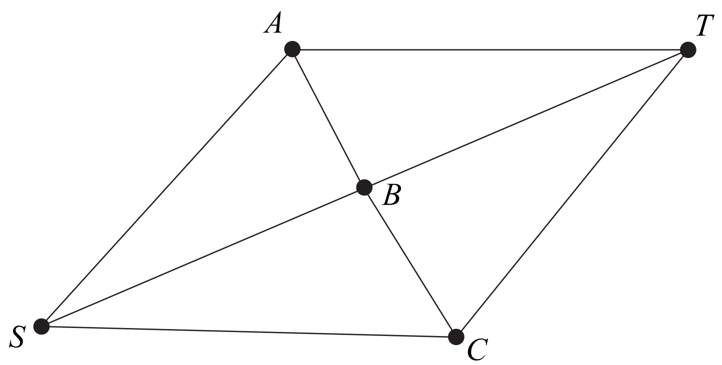
(i) Using your **values from part (a)(iv) as the initial flow**, indicate potential increases and decreases on the diagram below and use the labelling procedure to find the new maximum flow through the network. You should indicate any flow augmenting paths in the table and modify the potential increases and decreases of the flow on the diagram. *(6 marks)*



Path	**Additional Flow**

(ii) Use your results from part **(b)(i)** to illustrate the flow along each edge that gives this new maximum flow, and state the value of the new maximum flow. *(3 marks)*

New maximum flow is ....



END OF QUESTIONS

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