| Centre Number | | | | | | Candidate Number | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Surname | | | | | | | | | | |
| Other Names | | | | | | | | | | |
| Candidate Signature | | | | | | | | | | |

| For Examiner's Use | |
|---|---|
| Examiner's Initials | |
| Question | Mark |
| 1 | |
| 2 | |
| 3 | |
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| 8 | |
| 9 | |
| TOTAL | |

General Certificate of Education
Advanced Subsidiary Examination
June 2012

# Mathematics MD01

## Unit Decision 1

## Thursday 24 May 2012 9.00 am to 10.30 am

**For this paper you must have:**
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

**Time allowed**
- 1 hour 30 minutes

**Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.

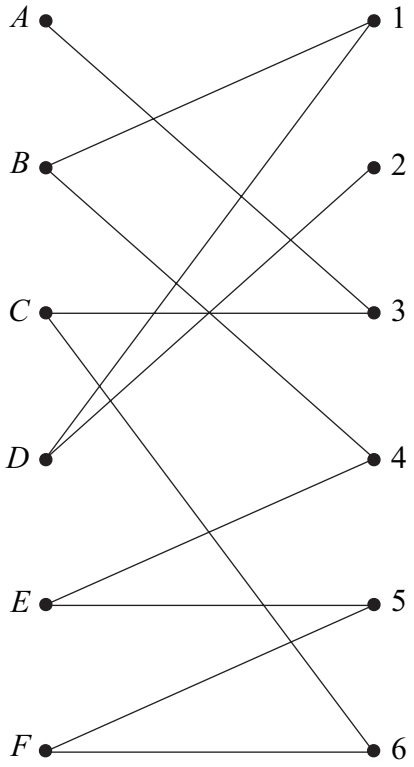
**Information**
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

**Advice**
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

**1** Six people, $A$, $B$, $C$, $D$, $E$ and $F$, are to be allocated to six tasks, 1, 2, 3, 4, 5 and 6. The following bipartite graph shows the tasks that each of the people is able to undertake.




**(a)** Represent this information in an adjacency matrix. *(2 marks)*

**(b)** Initially, $B$ is assigned to task 4, $C$ to task 3, $D$ to task 1, $E$ to task 5 and $F$ to task 6.

By using an algorithm from this initial matching, find a complete matching. *(3 marks)*

QUESTION PART REFERENCE

**Answer space for question 1**

QUESTION PART REFERENCE

## Answer space for question 1

**2** A student is using a shuttle sort algorithm to rearrange a set of numbers into ascending order.

Her correct solution for the first three passes is as follows.

| | | | | | | |
|---|---|---|---|---|---|---|
| Initial list | 10 | 7 | 4 | 22 | 23 | 26 |
| After 1st pass | 7 | 10 | 4 | 22 | 23 | 26 |
| After 2nd pass | 4 | 7 | 10 | 22 | 23 | 26 |
| After 3rd pass | 4 | 7 | 10 | 22 | 23 | 26 |

**(a)** Write down the number of comparisons on each of the three passes. *(2 marks)*

**(b)** Write down the number of swaps on each of the three passes. *(2 marks)*

**(c)** Explain whether or not the student has completed the algorithm. *(1 mark)*

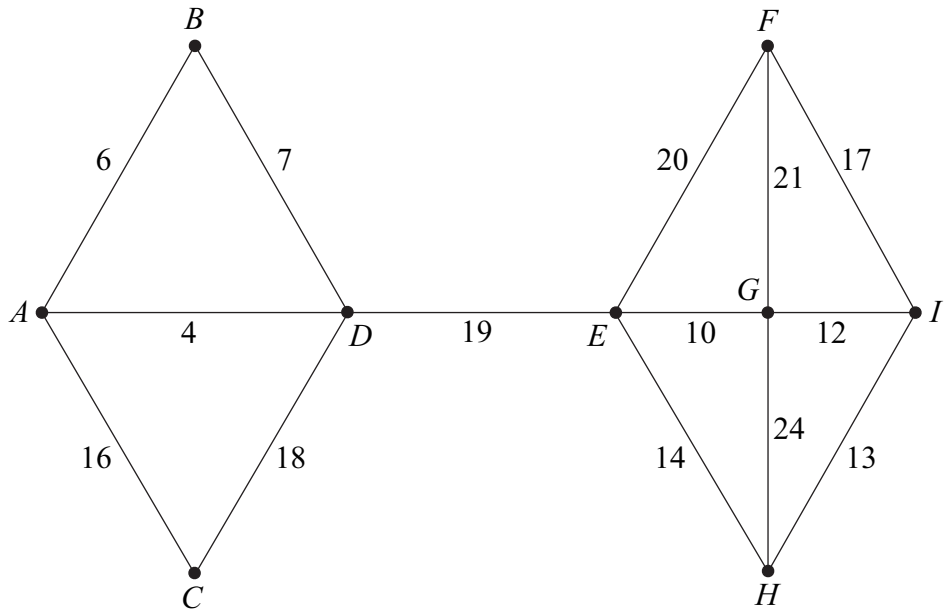
QUESTION PART REFERENCE

**Answer space for question 2**

QUESTION PART REFERENCE

**Answer space for question 2**

**3** The following network shows the lengths, in miles, of roads connecting nine villages, $A$, $B$, …, $I$.




**(a) (i)** Use Prim's algorithm starting from $A$, showing the order in which you select the edges, to find a minimum spanning tree for the network. *(4 marks)*

**(ii)** State the length of your minimum spanning tree. *(1 mark)*

**(iii)** Draw your minimum spanning tree. *(2 marks)*

**(b)** Prim's algorithm from different starting points produces the same minimum spanning tree for this network. State the final edge that would complete the minimum spanning tree using Prim's algorithm:

**(i)** starting from $D$; *(1 mark)*

**(ii)** starting from $H$. *(1 mark)*

QUESTION PART REFERENCE

**Answer space for question 3**

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QUESTION PART REFERENCE

**Answer space for question 3**

**4** The edges on the network below represent some major roads in a city. The number on each edge is the minimum time taken, in minutes, to drive along that road.

**(a) (i)** Use Dijkstra's algorithm on the network to find the shortest possible driving time from $A$ to $J$. *(5 marks)*

**(ii)** Write down the corresponding route. *(1 mark)*

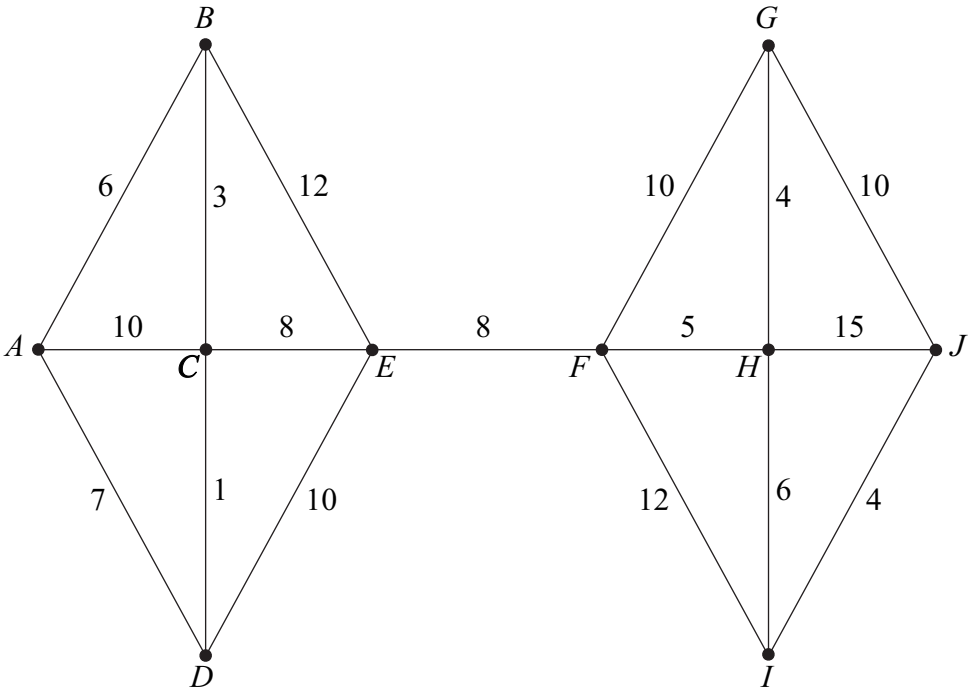
**(b)** A new ring road is to be constructed connecting $A$ to $J$ directly.

Find the maximum length of this new road from $A$ to $J$ if the time taken to drive along it, travelling at an average speed of 90 km/h, is to be no more than the time found in part **(a)(i)**. *(2 marks)*

QUESTION PART REFERENCE

**(a)(i)**

**Answer space for question 4**

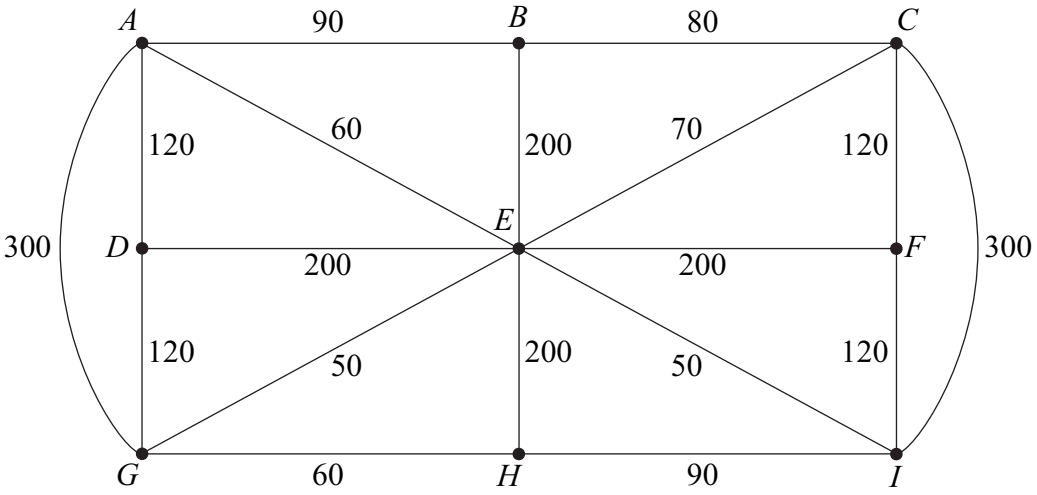


QUESTION PART REFERENCE

**Answer space for question 4**

**5** The network below shows some streets in a town. The number on each edge shows the length of that street, in metres.

Leaflets are to be distributed by a restaurant owner, Tony, from his restaurant located at vertex $B$. Tony must start from his restaurant, walk along all the streets at least once, before returning to his restaurant.




The total length of the streets is 2430 metres.

**(a)** Find the length of an optimal Chinese postman route for Tony. *(5 marks)*

**(b)** Colin also wishes to distribute some leaflets. He starts from his house at $H$, walks along all the streets at least once, before finishing at the restaurant at $B$.

Colin wishes to walk the minimum distance. Find the length of an optimal route for Colin. *(1 mark)*

**(c)** David also walks along all the streets at least once. He can start at any vertex and finish at any vertex. David also wishes to walk the minimum distance.

**(i)** Find the length of an optimal route for David. *(1 mark)*

**(ii)** State the vertices from which David could start in order to achieve this optimal route. *(1 mark)*

QUESTION PART REFERENCE

**Answer space for question 5**

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QUESTION PART REFERENCE

**Answer space for question 5**

**6** The complete graph $K_n$ $(n > 1)$ has every one of its $n$ vertices connected to each of the other vertices by a single edge.

**(a)** Draw the complete graph $K_4$. *(1 mark)*

**(b) (i)** Find the total number of edges for the graph $K_8$.

**(ii)** Give a reason why $K_8$ is not Eulerian. *(2 marks)*

**(c)** For the graph $K_n$, state in terms of $n$:

**(i)** the total number of edges;

**(ii)** the number of edges in a minimum spanning tree;

**(iii)** the condition for $K_n$ to be Eulerian;

**(iv)** the condition for the number of edges of a Hamiltonian cycle to be equal to the number of edges of an Eulerian cycle. *(4 marks)*

QUESTION PART REFERENCE

**Answer space for question 6**

QUESTION PART REFERENCE

**Answer space for question 6**

**7** Rupta, a sales representative, has to visit six shops, $A$, $B$, $C$, $D$, $E$ and $F$. Rupta starts at shop $A$ and travels to each of the other shops once, before returning to shop $A$. Rupta wishes to keep her travelling time to a minimum.

The table shows the travelling times, in minutes, between the shops.

| | ***A*** | ***B*** | ***C*** | ***D*** | ***E*** | ***F*** |
|---|---|---|---|---|---|---|
| ***A*** | – | 16 | 10 | 25 | 26 | 40 |
| ***B*** | 16 | – | 20 | 19 | 18 | 50 |
| ***C*** | 10 | 20 | – | 14 | 22 | 31 |
| ***D*** | 25 | 19 | 14 | – | 11 | 32 |
| ***E*** | 26 | 18 | 22 | 11 | – | 42 |
| ***F*** | 40 | 50 | 31 | 32 | 42 | – |

**(a)** Find the travelling time of the tour $ACFDEBA$. *(1 mark)*

**(b)** Use the nearest neighbour algorithm, starting at $A$, to find an upper bound for the travelling time for Rupta's tour. *(4 marks)*

**(c)** By deleting $A$, find a lower bound for the travelling time for Rupta's tour. *(4 marks)*

**(d)** Sketch a network showing the edges that give you the lower bound in part **(c)** and comment on its significance. *(2 marks)*

QUESTION PART REFERENCE

**Answer space for question 7**

QUESTION PART REFERENCE

**Answer space for question 7**

**8** The following algorithm finds an estimate of the value of the number represented by the symbol e:

Line 10 Let $A = 1$, $B = 1$, $C = 1$

Line 20 Let $D = A$

Line 30 Let $C = C \times B$

Line 40 Let $D = D + (1/C)$

Line 50 If $B = 4$ then go to Line 80

Line 60 Let $B = B + 1$

Line 70 Go to Line 30

Line 80 Print 'An estimate of e is', $D$

Line 90 End

**(a)** Trace the algorithm. *(6 marks)*

**(b)** A student miscopied Line 70.

His line was

Line 70 Go to Line 10

Explain what would happen if his algorithm were traced. *(2 marks)*

QUESTION PART REFERENCE

**Answer space for question 8**

QUESTION PART REFERENCE

**Answer space for question 8**

**9** Ollyin is buying new pillows for his hotel. He buys three types of pillow: soft, medium and firm.

He must buy at least 100 soft pillows and at least 200 medium pillows.

He must buy at least 400 pillows in total.

Soft pillows cost £4 each. Medium pillows cost £3 each. Firm pillows cost £4 each.

He wishes to spend no more than £1800 on new pillows.

At least 40% of the new pillows must be medium pillows.

Ollyin buys $x$ soft pillows, $y$ medium pillows and $z$ firm pillows.

**(a)** In addition to $x \geqslant 0$, $y \geqslant 0$ and $z \geqslant 0$, find five inequalities in $x$, $y$ and $z$ that model the above constraints. *(3 marks)*

**(b)** Ollyin decides to buy twice as many soft pillows as firm pillows.

**(i)** Show that three of your answers in part **(a)** become

$$3x + 2y \geqslant 800$$
$$2x + y \leqslant 600$$
$$y \geqslant x$$

*(3 marks)*

**(ii)** On the grid opposite, draw a suitable diagram to represent Ollyin's situation, indicating the feasible region. *(5 marks)*

**(iii)** Use your diagram to find the maximum total number of pillows that Ollyin can buy. *(2 marks)*

**(iv)** Find the number of each type of pillow that Ollyin can buy that corresponds to your answer to part **(b)(iii)**. *(1 mark)*

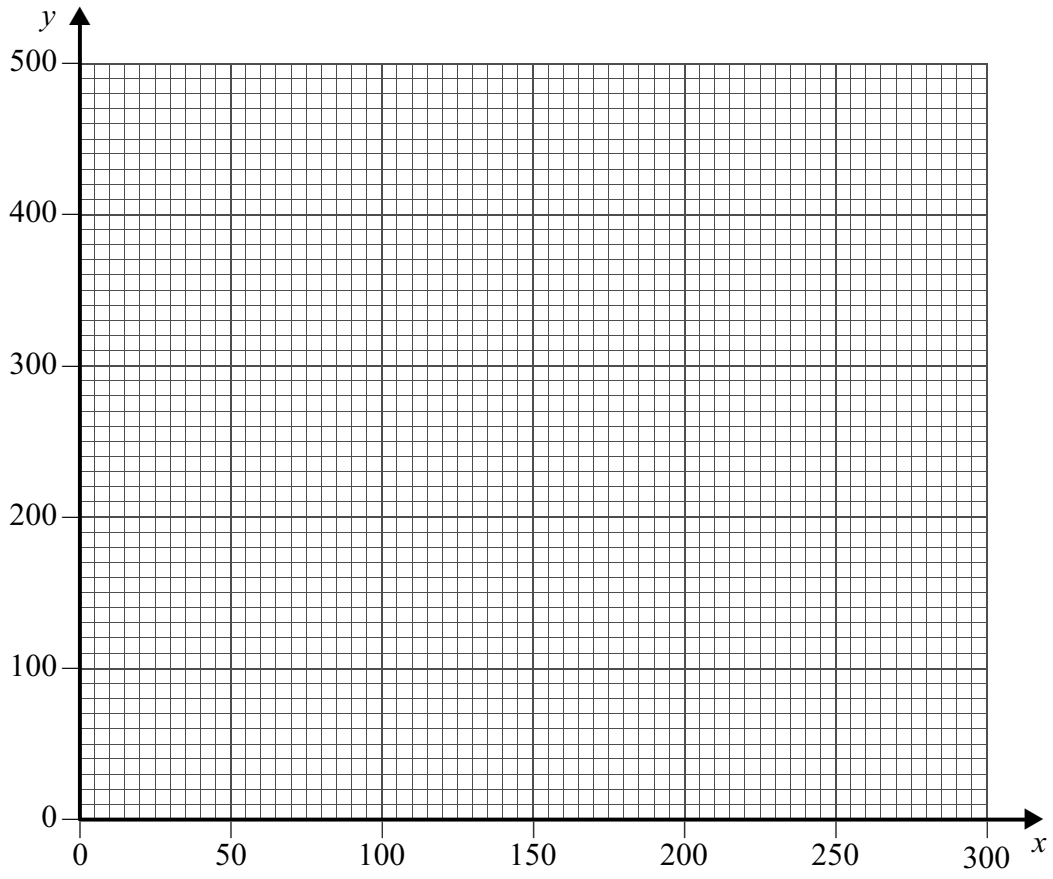
QUESTION PART REFERENCE

**Answer space for question 9**

QUESTION PART REFERENCE

**(b)(ii)**

**Answer space for question 9**



QUESTION PART REFERENCE

## Answer space for question 9

**END OF QUESTIONS**