Answer **all** questions.

1 Five trainers, Ali, Bo, Chas, Dee and Eve, held an initial training session with each of four teams over an assault course. The completion times in minutes are recorded below.

	Ali	**Bo**	**Chas**	**Dee**	**Eve**
Team 1	16	19	18	25	24
Team 2	22	21	20	26	25
Team 3	21	22	23	21	24
Team 4	20	21	21	23	20

Each of the four teams is to be allocated a trainer and the overall time for the four teams is to be minimised. No trainer can train more than one team.

(a) Modify the table of values by adding an extra row of values so that the Hungarian algorithm can be applied. *(1 mark)*

(b) Use the Hungarian algorithm, reducing **columns first** then rows, to decide which four trainers should be allocated to which team. State the minimum total training time for the four teams using this matching. *(8 marks)*

2 A manufacturing company is planning to build three new machines, A, B and C, at the rate of one per month. The order in which they are built is a matter of choice, but the profits will vary according to the number of workers available and the suppliers' costs. The expected profits in thousands of pounds are given in the table.

Month	**Already built**	**Profit (in units of £1000)** A	B	C
1	—	52	47	48
2	A	—	58	54
	B	70	—	54
	C	68	63	—
3	A **and** B	—	—	64
	A **and** C	—	67	—
	B **and** C	69	—	—

(a) Draw a labelled network such that the most profitable order of manufacture corresponds to the longest path within that network. *(2 marks)*

(b) Use dynamic programming to determine the order of manufacture that **maximises** the total profit, and state this maximum profit. *(7 marks)*

3 [**Figures 1 and 2**, printed on the insert, are provided for use in this question.]

A building project is to be undertaken. The table shows the activities involved.

Activity	**Immediate Predecessors**	**Duration (days)**	**Number of Workers Required**
A	—	2	3
B	*A*	4	2
C	*A*	6	1
D	*B*, *C*	8	3
E	*C*	3	2
F	*D*	2	2
G	*D*, *E*	4	2
H	*D*, *E*	6	1
I	*F*, *G*, *H*	2	3

(a) Complete the activity network for the project on **Figure 1**. *(3 marks)*

(b) Find the earliest start time for each activity. *(2 marks)*

(c) Find the latest finish time for each activity. *(2 marks)*

(d) Find the critical path and state the minimum time for completion. *(2 marks)*

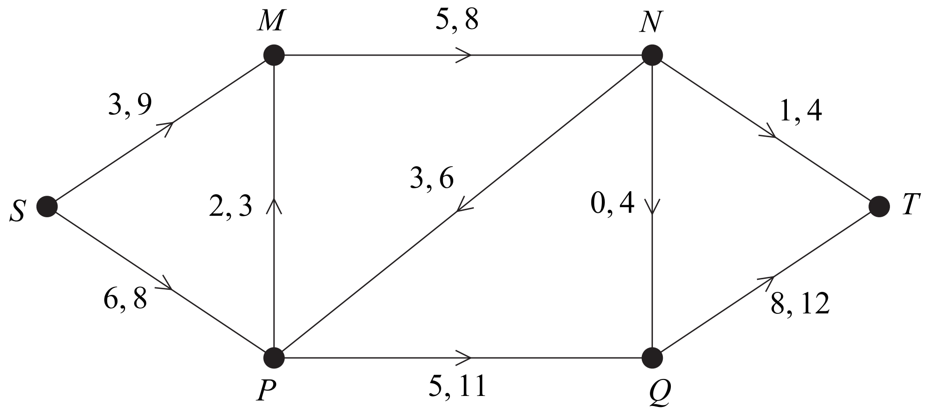
(e) State the float time for each non-critical activity. *(2 marks)*

(f) Given that each activity starts as early as possible, draw a resource histogram for the project on **Figure 2**. *(4 marks)*

(g) There are only 3 workers available at any time. Use resource levelling to explain why the project will overrun and state the minimum extra time required. *(3 marks)*

4 [**Figures 3**, **4 and 5**, printed on the insert, are provided for use in this question.]

The network shows a system of pipes, with the lower and upper capacities for each pipe in litres per second.



(a) **Figure 3**, on the insert, shows a partially completed diagram for a feasible flow of 10 litres per second from S to T. Indicate, on **Figure 3**, the flows along the edges MN, PQ, NP and NT. *(4 marks)*

(b) (i) Taking your answer from part (a) as an initial flow, use flow augmentation on **Figure 4** to find the maximum flow from S to T. *(6 marks)*

(ii) State the value of the maximum flow and illustrate this flow on **Figure 5**. *(2 marks)*

(c) Find a cut with capacity equal to that of the maximum flow. *(2 marks)*

5 (a) Display the following linear programming problem in a Simplex tableau.

Maximise $P = 3x + 2y + 4z$

subject to
$$x + 4y + 2z \leqslant 8$$
$$2x + 7y + 3z \leqslant 21$$
$$x \geqslant 0, y \geqslant 0, z \geqslant 0$$
(3 marks)

(b) Use the Simplex method to perform **one** iteration of your tableau for part (a), choosing a value in the z-column as pivot. *(3 marks)*

(c) (i) Perform one further iteration. *(5 marks)*

(ii) State whether or not this is the optimal solution, and give a reason for your answer. *(2 marks)*

6 Sam is playing a computer game in which he is trying to drive a car in different road conditions. He chooses a car and the computer decides the road conditions. The points scored by Sam are shown in the table.

Road Conditions

		C_1	C_2	C_3
Sam's Car	S_1	-2	2	4
	S_2	2	4	5
	S_3	5	1	2

Sam is trying to maximise his total points and the computer is trying to stop him.

(a) Explain why Sam should never choose S_1 and why the computer should not choose C_3. *(2 marks)*

(b) Find the play-safe strategies for the reduced 2 by 2 game for Sam and the computer, and hence show that this game does not have a stable solution. *(4 marks)*

(c) Sam uses random numbers to choose S_2 with probability p and S_3 with probability $1 - p$.

(i) Find expressions for the expected gain for Sam when the computer chooses each of its two remaining strategies. *(3 marks)*

(ii) Calculate the value of p for Sam to maximise his total points. *(2 marks)*

(iii) Hence find the expected points gain for Sam. *(1 mark)*

END OF QUESTIONS

For use in Question 3

Figure 1 (for use in part (a))

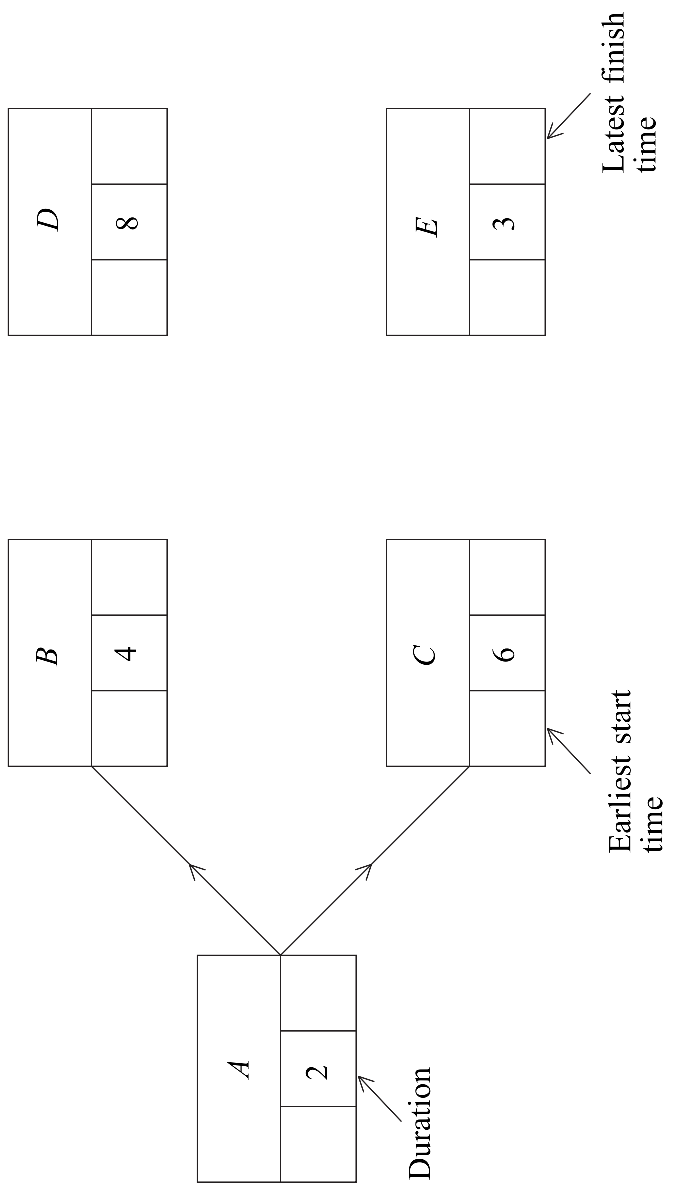
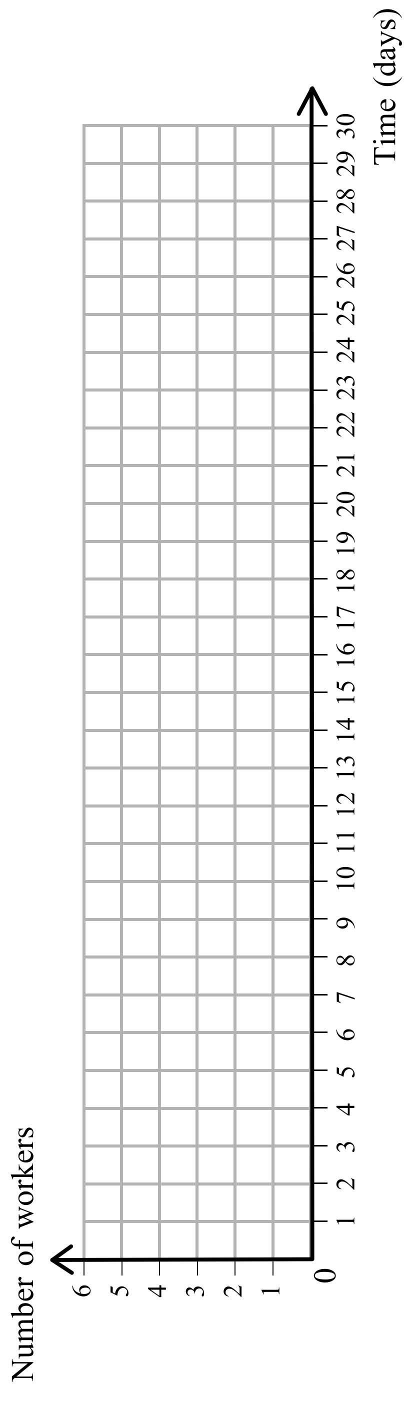


Figure 2 (for use in part (f))



For use in Question 4

Figure 3 (for use in part (a))

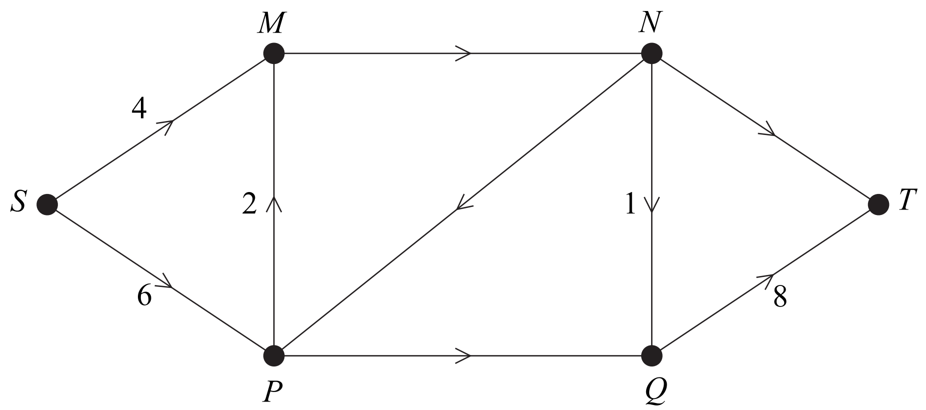
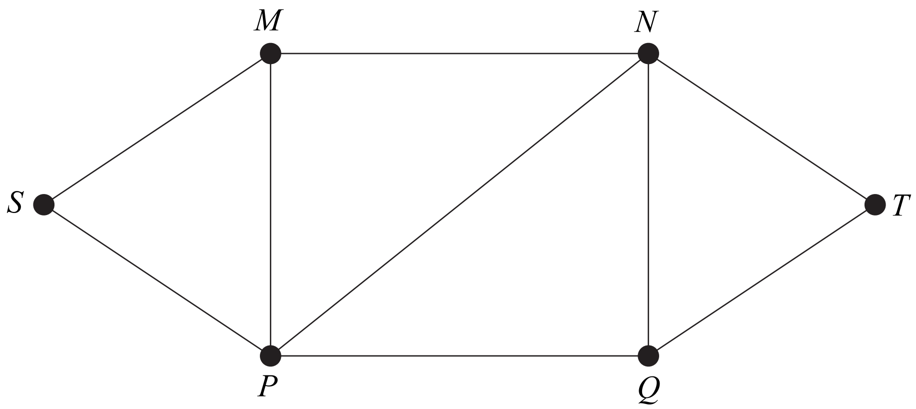
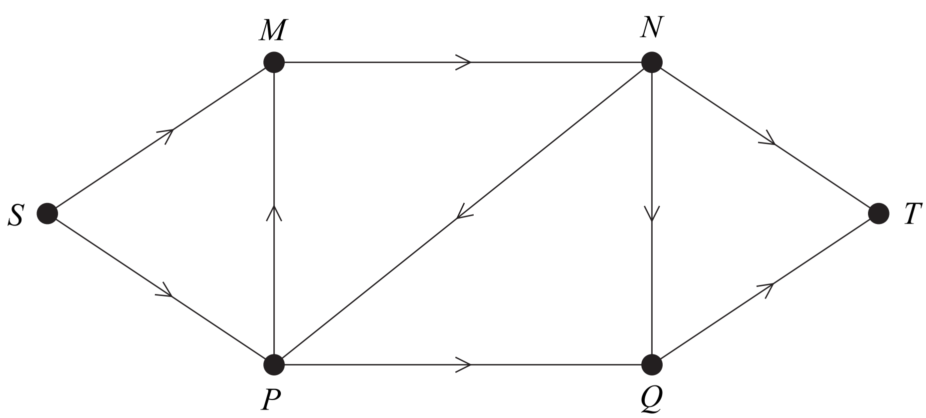


Figure 4 (for use in part (b)(i))



Route	Extra flow

Figure 5 (for use in part (b)(ii))



Practice 2

1. Explain what is meant, in a network, by

(a) a walk
(2)

(b) a tour
(2)

(Total 4 marks)

---

2. Jameson cars are made in two factories A and B. Sales have been made at the two main showrooms in London and Edinburgh. Cars are to be transported from the factories to the showrooms. The table below shows the cost, in pounds, of transporting one car from each factory to each showroom. It also shows the number of cars available at each factory and the number required at each showroom.

	London (L)	Edinburgh (E)	Supply
A	80	70	55
B	60	50	45
Demand	35	60	

It is decided to use the transportation algorithm to obtain a minimal cost solution.

(a) Explain why it is necessary to add a dummy demand point.
(2)

(b) Complete table 1 in the answer booklet.
(2)

(c) Use the north-west corner rule to obtain a possible pattern of distribution.
(1)

(d) Taking the most negative improvement index to indicate the entering square, use the stepping-stone method to obtain an optimal solution. You must make your shadow costs and improvement indices clear and demonstrate that your solution is optimal.
(7)

(e) State the cost of your optimal solution.
(1)

(Total 13 marks)

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3. (a) Explain the difference between a maximin route and a minimax route in dynamic programming.

(2)

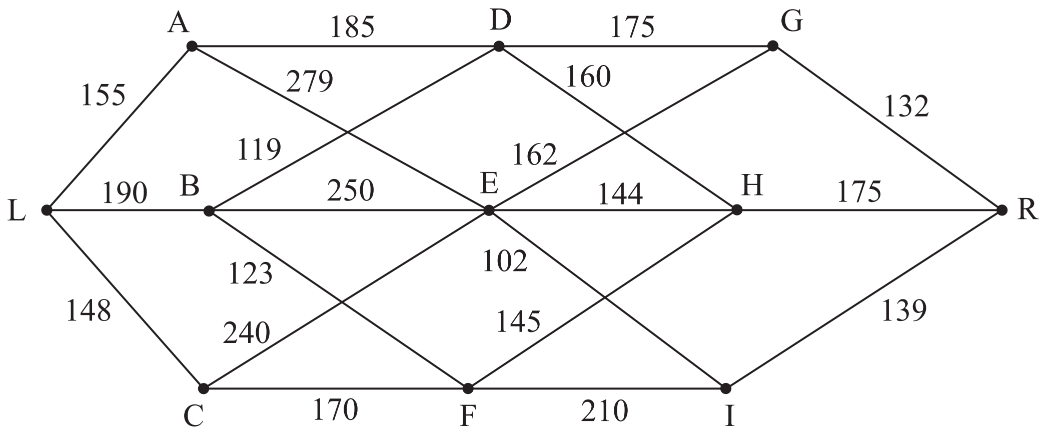

Figure 1

A Maximin route from L to R is to be found through the staged network shown in Figure 1.

(b) Use dynamic programming to complete the table in the answer book and hence find a maximin route.

(10)

(Total 12 marks)

4. (a) In game theory, explain the circumstances under which column (x) dominates column (y) in a two-person zero-sum game.

(2)

Liz and Mark play a zero-sum game. This game is represented by the following pay-off matrix for Liz.

	Mark plays 1	*Mark plays 2*	*Mark plays 3*
Liz plays 1	5	3	2
Liz plays 2	4	5	6
Liz plays 3	6	4	3

(b) Verify that there is no stable solution to this game.

(3)

(c) Find the best strategy for Liz and the value of the game to her.

(9)

The game now changes so that when Liz plays 1 and Mark plays 3 the pay-off to Liz changes from 2 to 4. All other pay-offs for this zero-sum game remain the same.

(d) Explain why a graphical approach is no longer possible and briefly describe the method Liz should use to determine her best strategy.

(2)

(Total 16 marks)

5. Four salespersons, Joe, Min-Seong, Olivia and Robert, are to attend four business fairs, A, B, C and D. Each salesperson must attend just one fair and each fair must be attended by just one salesperson. The expected sales, in thousands of pounds, that each salesperson would make at each fair is shown in the table below.

	A	*B*	*C*	*D*
Joe	48	49	42	42
Min-Seong	53	49	51	50
Olivia	51	53	48	48
Robert	47	50	46	43

(a) Use the Hungarian algorithm, reducing rows first, to obtain an allocation that maximises the total expected sales from the four salespersons. You must make your method clear and show the table after each stage.

(10)

(b) State all possible optimal allocations and the optimal total value.

(4)

(Total 14 marks)

6.

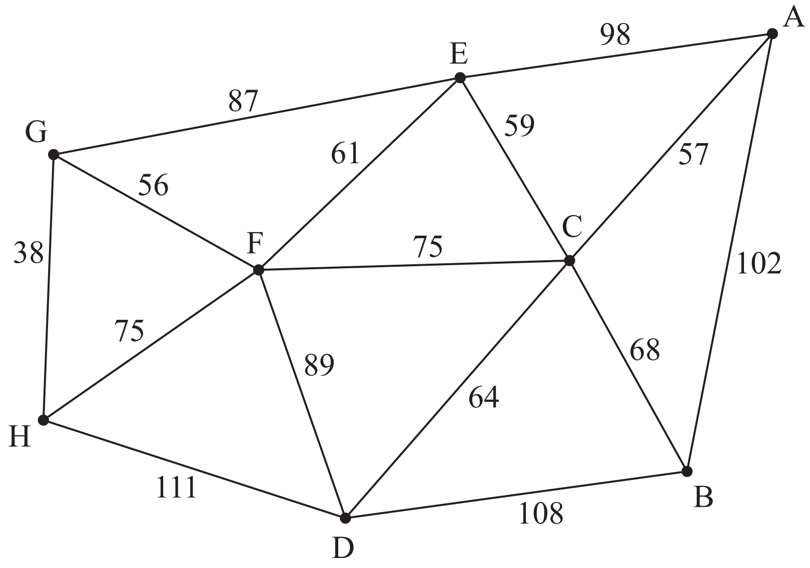

Figure 2

The network in figure 2 shows the distances, in km, between eight weather data collection points. Starting and finishing at A, Alice needs to visit each collection point at least once, in a minimum distance.

(a) Obtain a minimum spanning tree for the network using Kruskal's algorithm, stating the order in which you select the arcs.

(2)

(b) Use your answer to part (a) to determine an initial upper bound for the length of the route.

(1)

(c) Starting from your initial upper bound use short cuts to find an upper bound, which is below 630km. State the corresponding route.

(4)

(d) Use the nearest neighbour algorithm starting at B to find a second upper bound for the length of the route.

(3)

(e) By deleting C, and all of its arcs, find a lower bound for the length of the route.

(4)

(f) Use your results to write down the smallest interval which you are confident contains the optimal length of the route.

(2)

(Total 16 marks)

TOTAL FOR PAPER: 75 MARKS

END

Answer book for Practice 2

Leave blank

1.

(Total 4 marks)

Q1

Leave blank

2. (a)

(b) Table 1

	L	E	Dummy	Supply
A	80	70		55
B	60	50		45
Demand	35	60		100

(c)

	L	E	D
A			
B			

(d) *You may not need to use all of these tables*

	L	E	D
A			
B			

	L	E	D
A			
B			

Leave blank

	L	E	D
A			
B			

	L	E	D
A			
B			

	L	E	D
A			
B			

	L	E	D
A			
B			

Leave blank

	L	E	D
A			
B			

	L	E	D
A			
B			

Q2

(Total 13 marks)

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Leave blank

3. (a)

(b)

Stage	State	Action	Destination	Value

Leave blank

Stage	State	Action	Destination	Value

Q3

(Total 12 marks)

Leave blank

4.

Leave blank

(Total 16 marks)

Q4

Leave blank

5.

	A	*B*	*C*	*D*
J				
M				
O				
R				

	A	*B*	*C*	*D*
J				
M				
O				
R				

	A	*B*	*C*	*D*
J				
M				
O				
R				

	A	*B*	*C*	*D*
J				
M				
O				
R				

	A	*B*	*C*	*D*
J				
M				
O				
R				

Leave blank

	A	B	C	D
J				
M				
O				
R				

	A	B	C	D
J				
M				
O				
R				

	A	B	C	D
J				
M				
O				
R				

(Total 14 marks)

Q5

Leave blank

6.

Leave blank

Leave blank

Q6

(Total 16 marks)

TOTAL FOR PAPER: 75 MARKS

END

Answer **all** questions.

1 [**Figure 1**, printed on the insert, is provided for use in this question.]

A building project is to be undertaken. The table shows the activities involved.

Activity	**Immediate Predecessors**	**Duration (weeks)**
A	–	2
B	–	1
C	*A*	3
D	*A*, *B*	2
E	*B*	4
F	*C*	1
G	*C*, *D*, *E*	3
H	*E*	5
I	*F*, *G*	2
J	*H*, *I*	3

(a) Complete an activity network for the project on **Figure 1**. *(3 marks)*

(b) Find the earliest start time for each activity. *(2 marks)*

(c) Find the latest finish time for each activity. *(2 marks)*

(d) State the minimum completion time for the building project and identify the critical paths. *(4 marks)*

2 Five successful applicants received the following scores when matched against suitability criteria for five jobs in a company.

	Job 1	**Job 2**	**Job 3**	**Job 4**	**Job 5**
Alex	13	11	9	10	13
Bill	15	12	12	11	12
Cath	12	10	8	14	14
Don	11	12	13	14	10
Ed	12	14	14	13	14

It is intended to allocate each applicant to a different job so as to maximise the total score of the five applicants.

(a) Explain why the Hungarian algorithm may be used if each number, x, in the table is replaced by $15 - x$. *(2 marks)*

(b) Form a new table by subtracting each number in the table from 15. Use the Hungarian algorithm to allocate the jobs to the applicants so that the total score is maximised. *(8 marks)*

(c) It is later discovered that Bill has already been allocated to Job 4. Decide how to alter the allocation of the other jobs so as to maximise the score now possible. *(3 marks)*

3 (a) Display the following linear programming problem in a Simplex tableau.

Maximise $P = 5x + 8y + 7z$

subject to
$$3x + 2y + z \leqslant 12$$
$$2x + 4y + 5z \leqslant 16$$
$$x \geqslant 0,\ y \geqslant 0,\ z \geqslant 0$$
(3 marks)

(b) The Simplex method is to be used by initially choosing a value in the y-column as a pivot.

(i) Explain why the initial pivot is 4. *(1 mark)*

(ii) Perform **two** iterations of your tableau from part (a) using the Simplex method. *(6 marks)*

(iii) State the values of P, x, y and z after your second iteration. *(2 marks)*

(iv) State, giving a reason, whether the maximum value of P has been achieved. *(1 mark)*

4 (a) Two people, Ros and Col, play a zero-sum game. The game is represented by the following pay-off matrix for Ros.

		Col		
	Strategy	**X**	**Y**	**Z**
Ros	**I**	−4	−3	0
	II	5	−2	2
	III	1	−1	3

(i) Show that this game has a stable solution. *(3 marks)*

(ii) Find the play-safe strategy for each player and state the value of the game. *(2 marks)*

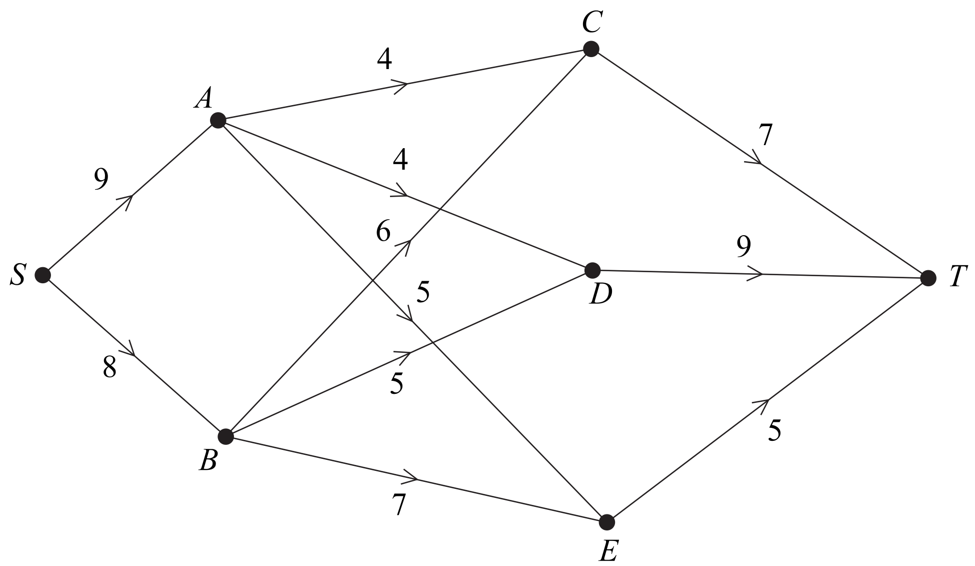
(b) Ros and Col play a different zero-sum game for which there is no stable solution. The game is represented by the following pay-off matrix for Ros.

		Col		
	Strategy	$\mathbf{C_1}$	$\mathbf{C_2}$	$\mathbf{C_3}$
Ros	$\mathbf{R_1}$	3	2	1
	$\mathbf{R_2}$	−2	−1	2

(i) Find the optimal mixed strategy for Ros. *(7 marks)*

(ii) Calculate the value of the game. *(1 mark)*

5 A three-day journey is to be made from S to T, with overnight stops at the end of the first day at either A or B and at the end of the second day at one of the locations C, D or E. The network shows the number of hours of sunshine forecast for each day of the journey.



The optimal route, known as the maximin route, is that for which the least number of hours of sunshine during a day's journey is as large as possible.

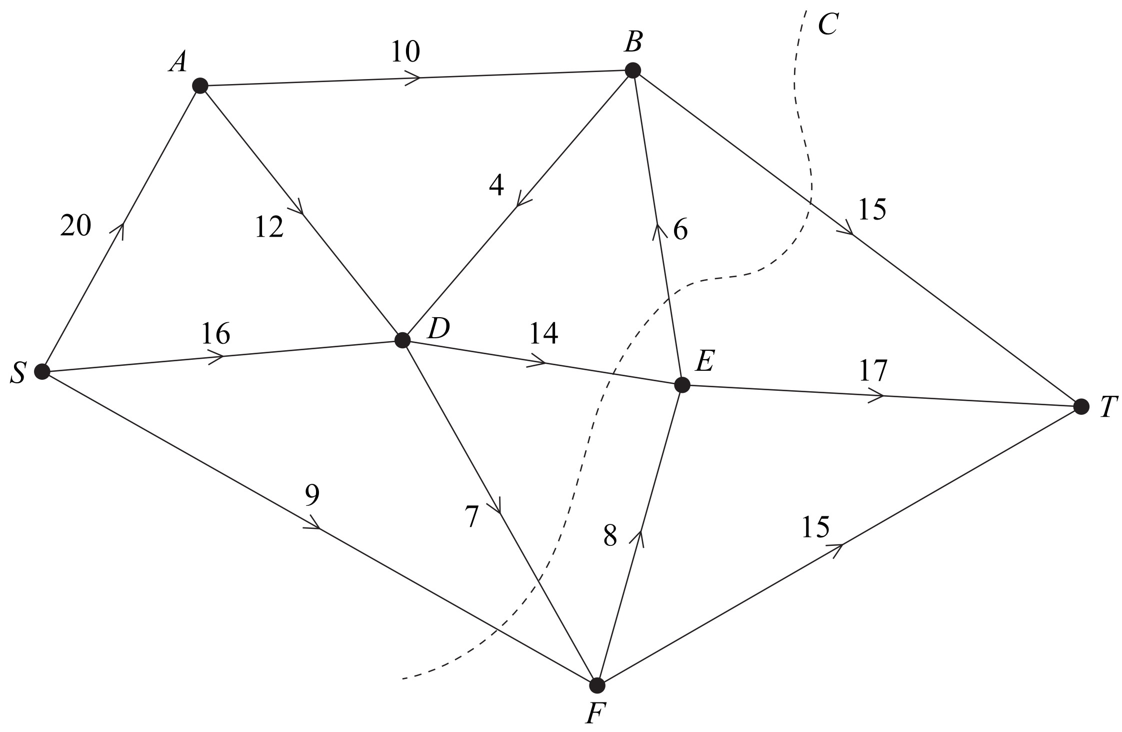
(a) Explain why the three-day route $SAET$ is better than $SACT$. *(2 marks)*

(b) Use dynamic programming to find the optimal (maximin) three-day route from S to T. *(8 marks)*

Turn over for the next question

6 [**Figures 2** and **3**, printed on the insert, are provided for use in this question.]

The diagram shows a network of pipelines through which oil can travel. The oil field is at S, the refinery is at T and the other vertices are intermediate stations. The weights on the edges show the capacities in millions of barrels per hour that can flow through each pipeline.



(a) (i) Find the value of the cut marked C on the diagram. *(1 mark)*

(ii) Hence make a deduction about the maximum flow of oil through the network. *(2 marks)*

(b) State the maximum possible flows along the routes $SABT$, $SDET$ and SFT. *(2 marks)*

(c) (i) Taking your answer to part (b) as the initial flow, use a labelling procedure on **Figure 2** to find the maximum flow from S to T. Record your routes and flows in the table provided and show the augmented flows on the network diagram. *(6 marks)*

(ii) State the value of the maximum flow, and, on **Figure 3**, illustrate a possible flow along each edge corresponding to this maximum flow. *(2 marks)*

(iii) Prove that your flow in part (c)(ii) is a maximum. *(2 marks)*

END OF QUESTIONS

Figure 1 (for use in Question 1)

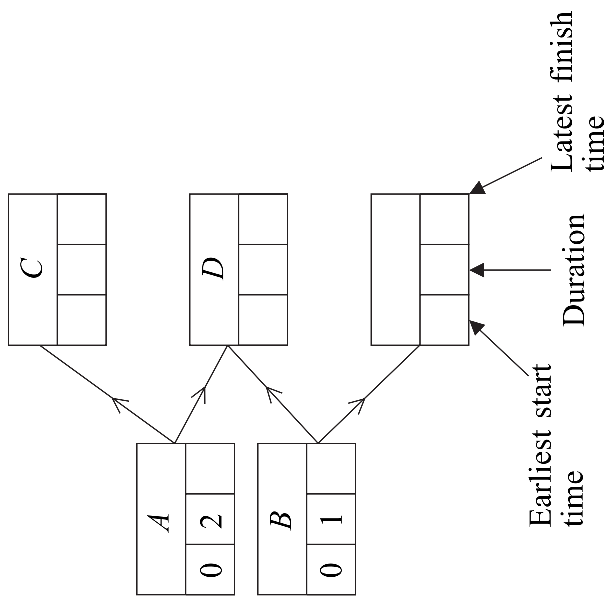
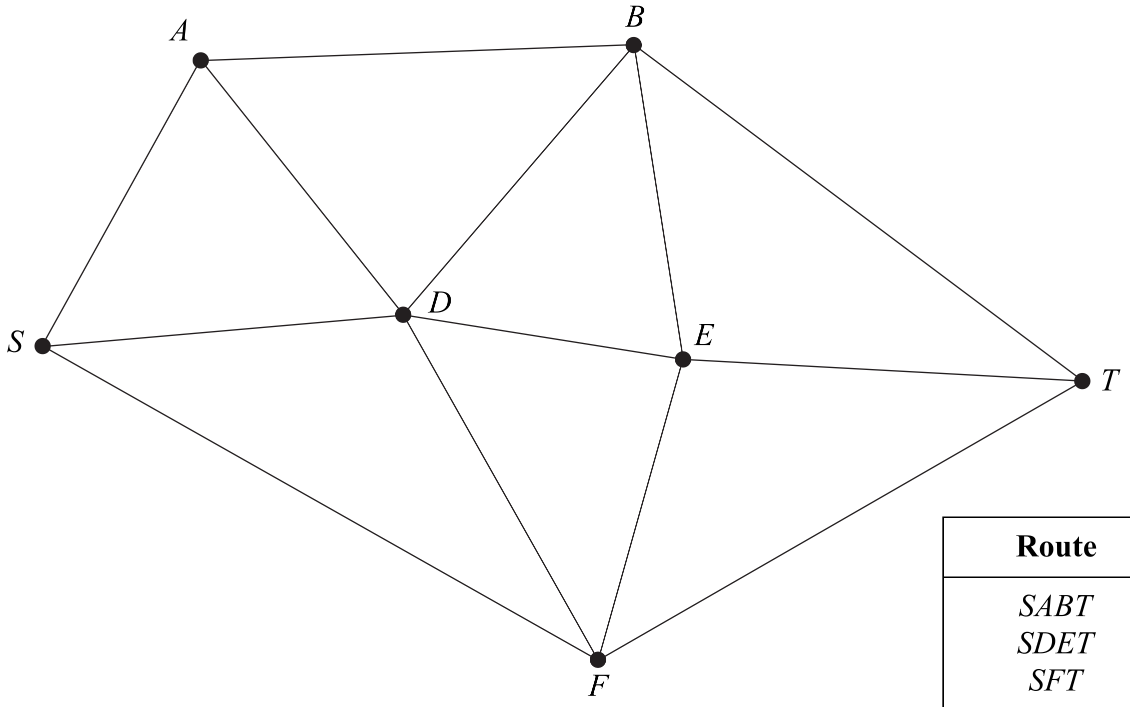
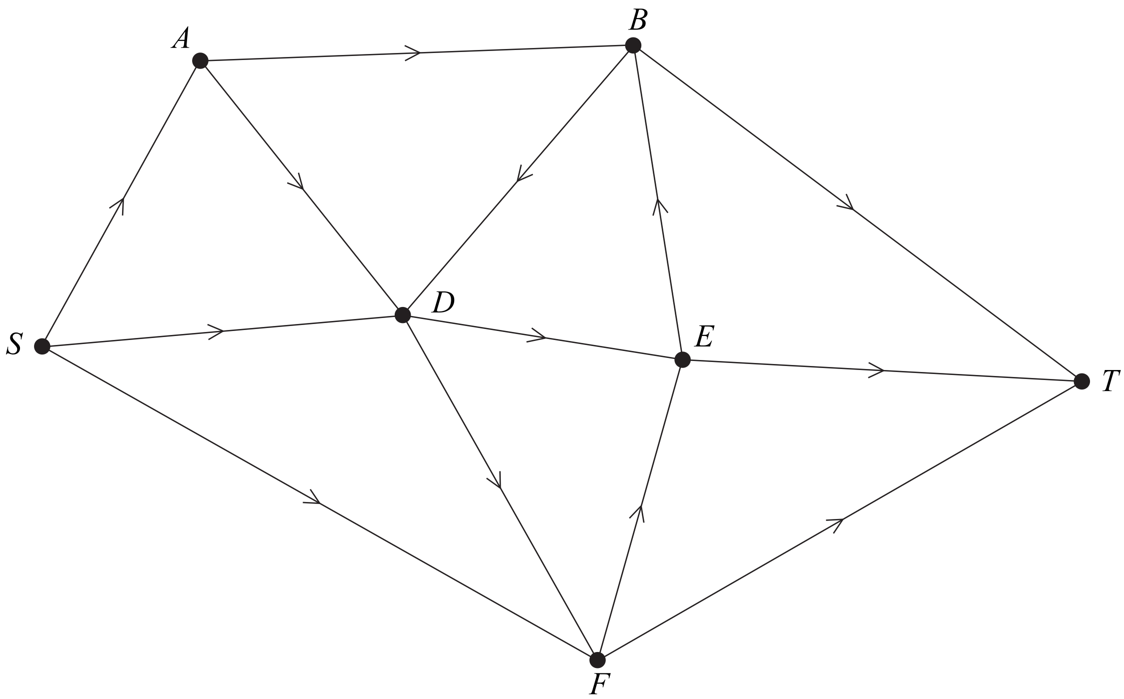


Figure 2 (for use in Question 6(c)(i))



Route	Flow
SABT	
SDET	
SFT	

Figure 3 (for use in Question 6(c)(ii))



Practice 4

1. A company, Kleenitquick, has developed a new stain remover. To promote sales, three salespersons, Jess, Matt and Rachel, will be assigned to three of four department stores 1, 2, 3 and 4, to demonstrate the stain remover. Each salesperson can only be assigned to one department store.

The table below shows the cost, in pounds, of assigning each salesperson to each department store.

	1	**2**	**3**	**4**
Jess	15	11	14	12
Matt	13	8	17	13
Rachel	14	9	13	15

(a) Explain why a dummy row needs to be added to the table.
(1)

(b) Complete Table 1 in the answer book.
(1)

(c) Reducing rows first, use the Hungarian algorithm to obtain an allocation that minimises the cost of assigning salespersons to department stores. You must make your method clear and show the table after each iteration.
(6)

(d) Find the minimum cost.
(1)

(Total 9 marks)

2. (a) Explain the difference between the classical and the practical travelling salesperson problems.

(2)

The table below shows the distances, in km, between six data collection points, A, B, C, D, E, and F.

	A	**B**	**C**	**D**	**E**	**F**
A	-	77	34	56	67	21
B	77	-	58	58	36	74
C	34	58	-	73	70	42
D	56	58	73	-	68	38
E	67	36	70	68	-	71
F	21	74	42	38	71	-

Rachel must visit each collection point. She will start and finish at A and wishes to minimise the total distance travelled.

(b) Starting at A, use the nearest neighbour algorithm to obtain an upper bound. Make your method clear.

(3)

Starting at B, a second upper bound of 293 km was found.

(c) State the better upper bound of these two, giving a reason for your answer.

(1)

By deleting A, a lower bound was found to be 245 km.

(d) By deleting B, find a second lower bound. Make your method clear.

(4)

(e) State the better lower bound of these two, giving a reason for your answer.

(1)

(f) Taking your answers to (c) and (e), use inequalities to write down an interval that must contain the length of Rachel's optimal route.

(1)

(Total 12 marks)

3. A two-person zero-sum game is represented by the following pay-off matrix for player A.

	B plays 1	B plays 2	B plays 3
A plays 1	–5	6	–3
A plays 2	1	–4	13
A plays 3	–2	3	–1

(a) Verify that there is no stable solution to this game.
(3)

(b) Reduce the game so that player B has a choice of only two actions.
(1)

(c) Write down the reduced pay-off matrix **for player B**.
(2)

(d) Find the best strategy for player B and the value of the game to player B.
(7)

(Total 13 marks)

4.

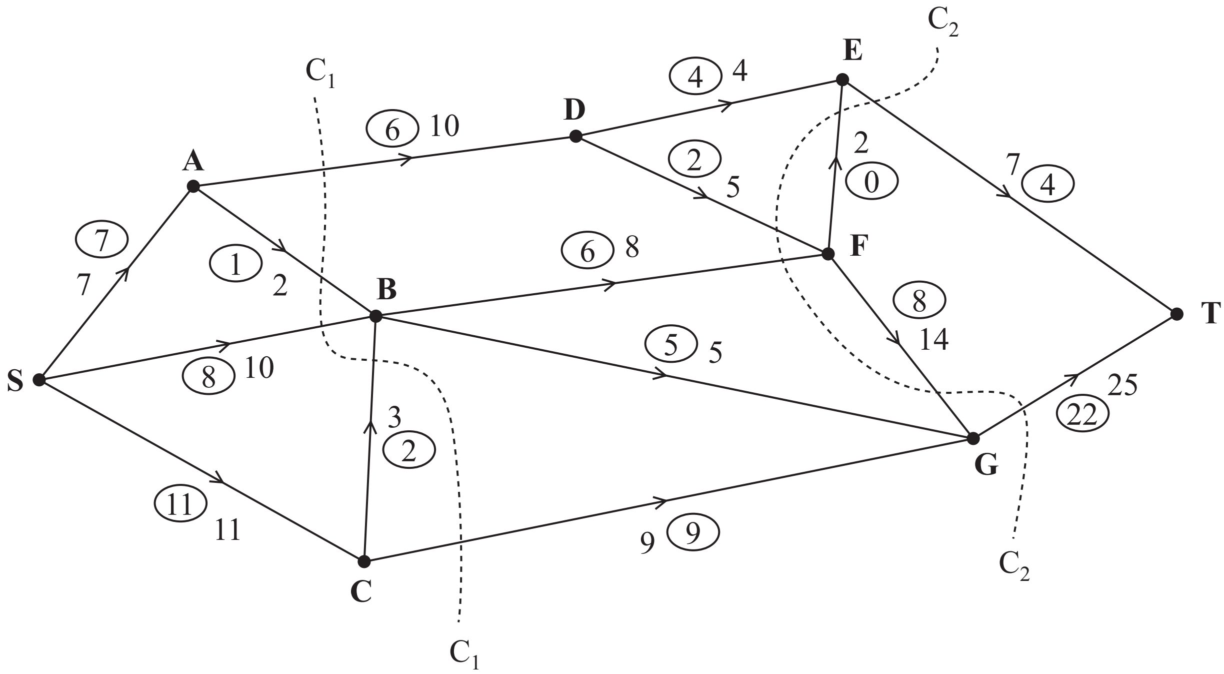


Figure 1

Figure 1 shows a capacitated network. The capacity of each arc is shown on the arc. The numbers in circles represent an initial flow from S to T.

Two cuts C_1 and C_2 are shown in Figure 1.

(a) Find the capacity of each of the two cuts.

(2)

(b) Find the maximum flow through the network. You must list each flow-augmenting route you use together with its flow.

(3)

(Total 5 marks)

5. While solving a maximising linear programming problem, the following tableau was obtained.

Basic Variable	x	y	z	r	s	t	value
z	$\frac{1}{4}$	$-\frac{1}{4}$	1	$\frac{1}{4}$	0	0	2
s	$\frac{5}{4}$	$\frac{7}{4}$	0	$-\frac{3}{4}$	1	0	4
t	3	$\frac{5}{2}$	0	$-\frac{1}{2}$	0	1	2
P	−2	−4	0	$\frac{5}{4}$	0	0	10

(a) Write down the values of x, y and z as indicated by this tableau.

(2)

(b) Write down the profit equation from the tableau.

(2)

(Total 4 marks)

6. The table below shows the cost, in pounds, of transporting one unit of stock from each of three supply points, X, Y and Z to three demand points, A, B and C. It also shows the stock held at each supply point and the stock required at each demand point.

	A	**B**	**C**	Supply
X	17	8	7	22
Y	16	12	15	17
Z	6	10	9	15
Demand	16	15	23	

(a) This is a **balanced problem**. Explain what this means.
(1)

(b) Use the north west corner method to obtain a possible solution.
(1)

(c) Taking ZA as the entering cell, use the stepping-stone method to find an improved solution. Make your route clear and state your exiting cell.
(3)

(d) Perform one more iteration of the stepping-stone method to find a further improved solution. You must make your shadow costs, improvement indices, entering cell, exiting cell and route clear.
(6)

(e) State the cost of the solution you found in part (d).
(1)

(Total 12 marks)

7. Minty has £250 000 to allocate to three investment schemes. She will allocate the money to these schemes in units of £50 000. The net income generated by each scheme, in £1000s, is given in the table below.

	£0	**£50 000**	**£100 000**	**£150 000**	**£200 000**	**£250 000**
Scheme 1	0	60	120	180	240	300
Scheme 2	0	65	125	190	235	280
Scheme 3	0	55	110	170	230	300

Minty wishes to maximise the net income. She decides to use dynamic programming to determine the optimal allocation, and starts the table shown in your answer book.

(a) Complete the table in the answer book to determine the amount Minty should allocate to each scheme in order to maximise the income. State the maximum income and the amount that should be allocated to each scheme.

(10)

(b) For this problem give the meaning of the table headings

(i) Stage,

(ii) State,

(iii) Action.

(3)

(Total 13 marks)

---

8. Laura (L) and Sam (S) play a two-person zero-sum game which is represented by the following pay-off matrix for Laura.

	S plays 1	S plays 2	S plays 3
L plays 1	–2	8	–1
L plays 2	7	4	–3
L plays 3	1	–5	4

Formulate the game as a linear programming problem for Laura, writing the constraints as inequalities. Define your variables clearly.

(Total 7 marks)

TOTAL FOR PAPER: 75 MARKS

END

Leave blank

1. (a)

(b)

	1	**2**	**3**	**4**
Jess	15	11	14	12
Matt	13	8	17	13
Rachel	14	9	13	15
Dummy				

Table 1

(c) *You may not need to use all of these tables*

	1	**2**	**3**	**4**
J				
M				
R				
D				

	1	**2**	**3**	**4**
J				
M				
R				
D				

	1	**2**	**3**	**4**
J				
M				
R				
D				

Leave blank

(Question 1 continued)

	1	2	3	4
J				
M				
R				
D				

	1	2	3	4
J				
M				
R				
D				

	1	2	3	4
J				
M				
R				
D				

	1	2	3	4
J				
M				
R				
D				

Allocation Jess does ________________

Matt does ________________

Rachel does ________________

(d) Minimum cost: ________________________________

Q1

(Total 9 marks)

Leave blank

2. (a)

	A	**B**	**C**	**D**	**E**	**F**
A	-	77	34	56	67	21
B	77	-	58	58	36	74
C	34	58	-	73	70	42
D	56	58	73	-	68	38
E	67	36	70	68	-	71
F	21	74	42	38	71	-

Leave blank

(Question 2 continued)

	A	B	C	D	E	F
A	-	77	34	56	67	21
B	77	-	58	58	36	74
C	34	58	-	73	70	42
D	56	58	73	-	68	38
E	67	36	70	68	-	71
F	21	74	42	38	71	-

Q2

(Total 12 marks)

Leave blank

3.

	B plays 1	B plays 2	B plays 3
A plays 1	–5	6	–3
A plays 2	1	–4	13
A plays 3	–2	3	–1

Leave blank

(Question 3 continued)

Q3

(Total 13 marks)

Leave blank

4. (a)

Value of cut C_1:

Value of cut C_2:

(b)

Q4

(Total 5 marks)

Leave blank

5. (a) x = ____________________

y = ____________________

z = ____________________

(b) ____

____

____

Q5

(Total 4 marks)

Leave blank

6. (a)

(b)

	A	B	C
X			
Y			
Z			

You may not need to use all of these tables

	A	B	C
X			
Y			
Z			

	A	B	C
X			
Y			
Z			

	A	B	C
X			
Y			
Z			

	A	B	C
X			
Y			
Z			

Leave blank

(Question 6 continued)

	A	B	C
X			
Y			
Z			

	A	B	C
X			
Y			
Z			

	A	B	C
X			
Y			
Z			

	A	B	C
X			
Y			
Z			

	A	B	C
X			
Y			
Z			

	A	B	C
X			
Y			
Z			

Q6

(Total 12 marks)

Leave blank

7. (a)

Stage	State (in £1000s)	Action (in £1000s)	Destination (in £1000s)	Value (in £1000s)
1	250	250	0	300 *
	200	200	0	240 *
	150	150	0	180 *
	100	100	0	120 *
	50	50	0	60 *
	0	0	0	0 *
2	250	250	0	280 + 0 = 280
		200	50	235 + 60 = 295
		150	100	
		100	150	
		50	200	
		0	250	
	200	200	0	
		150	50	
		100		
		50		
		0		
	150	150		
		100		
		50		
		0		
	100			

Leave blank

(Question 7 continued)

Stage	State (in £1000s)	Action (in £1000s)	Destination (in £1000s)	Value (in £1000s)

Maximum income: ____________________

Scheme	1	2	3
Amount to be invested (in £1000s)			

(b) ____________________

Q7

(Total 13 marks)

Leave blank

8.

Leave blank

(Question 8 continued)

Q8

(Total 7 marks)

TOTAL FOR PAPER: 75 MARKS

END

Answer **all** questions.

1 [**Figures 1 and 2**, printed on the insert, are provided for use in this question.]

A group of workers is involved in a building project. The table shows the activities involved. Each worker can perform any of the given activities.

Activity	**Immediate predecessors**	**Duration (days)**	**Number of workers required**
A	–	3	5
B	*A*	8	2
C	*A*	7	3
D	*B*, *C*	8	4
E	*C*	10	2
F	*C*	3	3
G	*D*, *E*	3	4
H	*F*	6	1
I	*G*, *H*	2	3

(a) Complete the activity network for the project on **Figure 1**. *(2 marks)*

(b) Find the earliest start time and the latest finish time for each activity, inserting their values on **Figure 1**. *(4 marks)*

(c) Find the critical path and state the minimum time for completion. *(2 marks)*

(d) The number of workers required for each activity is given in the table above. Given that each activity starts as early as possible and assuming there is no limit to the number of workers available, draw a resource histogram for the project on **Figure 2**, indicating clearly which activities take place at any given time. *(4 marks)*

(e) It is later discovered that there are only 7 workers available at any time. Use resource levelling to explain why the project will overrun and indicate which activities need to be delayed so that the project can be completed with the minimum extra time. State the minimum extra time required. *(3 marks)*

2 The following table shows the times taken, in minutes, by five people, Ash, Bob, Col, Dan and Emma, to carry out the tasks 1, 2, 3 and 4. Dan cannot do task 3.

	Ash	**Bob**	**Col**	**Dan**	**Emma**
Task 1	14	10	12	12	14
Task 2	11	13	10	12	12
Task 3	13	11	12	**	12
Task 4	13	10	12	13	15

Each of the four tasks is to be given to a different one of the five people so that the overall time for the four tasks is minimised.

(a) Modify the table of values by adding an extra row of **non-zero** values so that the Hungarian algorithm can be applied. *(1 mark)*

(b) Use the Hungarian algorithm, reducing **columns first** then rows, to decide which four people should be allocated to which task. State the minimum total time for the four tasks using this matching. *(8 marks)*

(c) After special training, Dan is able to complete task 3 in 12 minutes. Determine, giving a reason, whether the minimum total time found in part (b) could be improved. *(2 marks)*

3 Two people, Rob and Con, play a zero-sum game.

The game is represented by the following pay-off matrix for Rob.

Con

	Strategy	$\mathbf{C_1}$	$\mathbf{C_2}$	$\mathbf{C_3}$
	$\mathbf{R_1}$	-2	5	3
Rob	$\mathbf{R_2}$	3	-3	-1
	$\mathbf{R_3}$	-3	3	2

(a) Explain what is meant by the term 'zero-sum game'. *(1 mark)*

(b) Show that this game has no stable solution. *(3 marks)*

(c) Explain why Rob should never play strategy R_3. *(1 mark)*

(d) (i) Find the optimal mixed strategy for Rob. *(7 marks)*

(ii) Find the value of the game. *(1 mark)*

4 A linear programming problem involving the variables x, y and z is to be solved. The objective function to be maximised is $P = 2x + 3y + 5z$. The initial Simplex tableau is given below.

P	x	y	z	s	t	u	*value*
1	−2	−3	−5	0	0	0	0
0	1	0	1	1	0	0	9
0	2	1	4	0	1	0	40
0	4	2	3	0	0	1	33

(a) In addition to $x \geqslant 0$, $y \geqslant 0$, $z \geqslant 0$, write down **three** inequalities involving x, y and z for this problem. *(2 marks)*

(b) (i) By choosing the first pivot from the z-column, perform **one** iteration of the Simplex method. *(4 marks)*

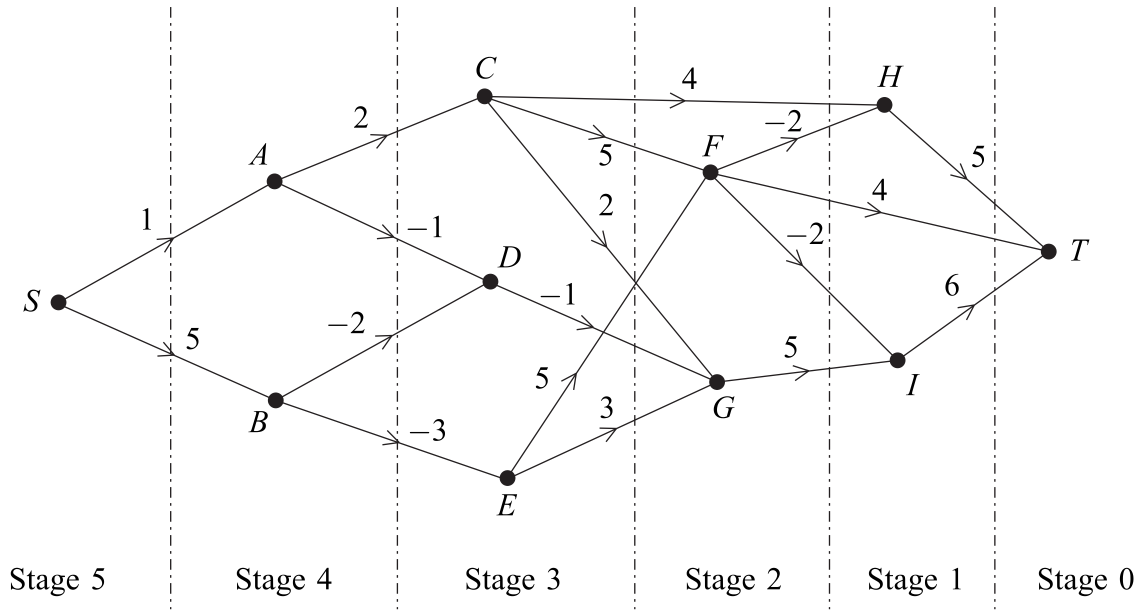
(ii) Explain how you know that the optimal value has not been reached. *(1 mark)*

(c) (i) Perform one further iteration. *(4 marks)*

(ii) Interpret the final tableau and state the values of the slack variables. *(3 marks)*

5 [**Figure 3**, printed on the insert, is provided for use in this question.]

The following network shows 11 vertices. The number on each edge is the cost of travelling between the corresponding vertices. A negative number indicates a reduction by the amount shown.



(a) **Working backwards from *T***, use dynamic programming to find the minimum cost of travelling from *S* to *T*. You may wish to complete the table on **Figure 3** as your solution. *(6 marks)*

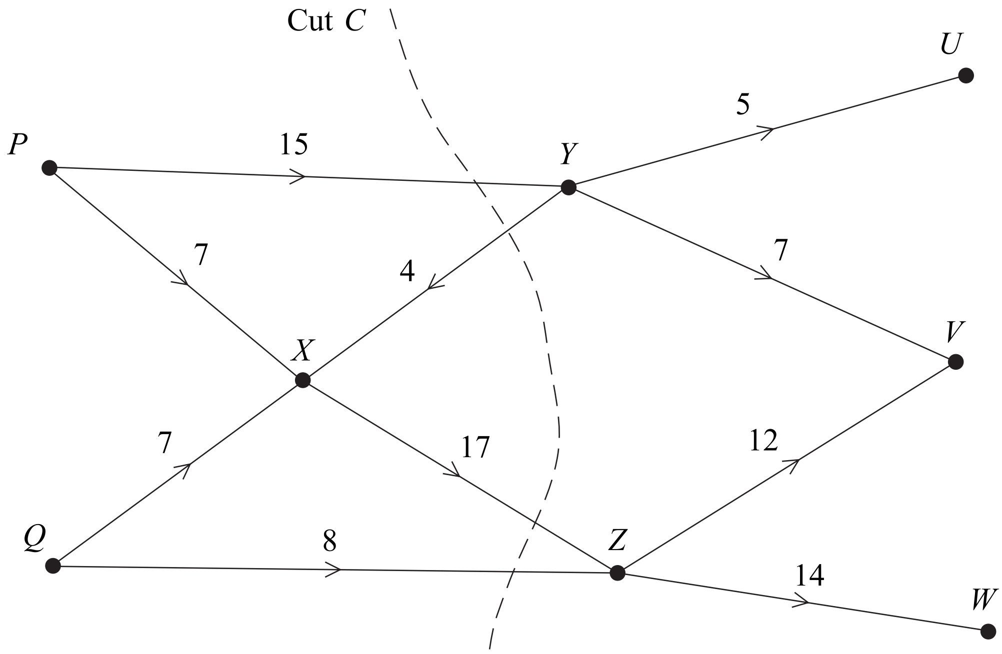
(b) State the minimum cost and the routes corresponding to this minimum cost. *(3 marks)*

Turn over for the next question

6 [**Figures 4, 5 and 6**, printed on the insert, are provided for use in this question.]

Water has to be transferred from two mountain lakes P and Q to three urban reservoirs U, V and W. There are pumping stations at X, Y and Z.

The possible routes with the capacities along each edge, in millions of litres per hour, are shown in the following diagram.



(a) On **Figure 4**, add a super-source, S, and a super-sink, T, and appropriate edges so as to produce a directed network with a single source and a single sink. Indicate the capacity of each of the edges you have added. *(2 marks)*

(b) (i) Find the value of the cut C. *(1 mark)*

(ii) State what can be deduced about the maximum flow from S to T. *(1 mark)*

(c) On **Figure 5**, write down the maximum flows along the routes $SQZWT$ and $SPYXZVT$. *(2 marks)*

(d) (i) On **Figure 6**, add the vertices S and T and the edges connecting S and T to the network. Using the maximum flows along the routes $SQZWT$ and $SPYXZVT$ found in part (c) as the initial flow, indicate the potential increases and decreases of flow on each edge. *(2 marks)*

(ii) Use flow augmentation to find the maximum flow from S to T. You should indicate any flow augmenting paths on **Figure 5** and modify the potential increases and decreases of the flow on **Figure 6**. *(4 marks)*

(e) State the value of the flow from Y to X in millions of litres per hour when the maximum flow is achieved. *(1 mark)*

END OF QUESTIONS

Figure 1 (for use in Question 1)

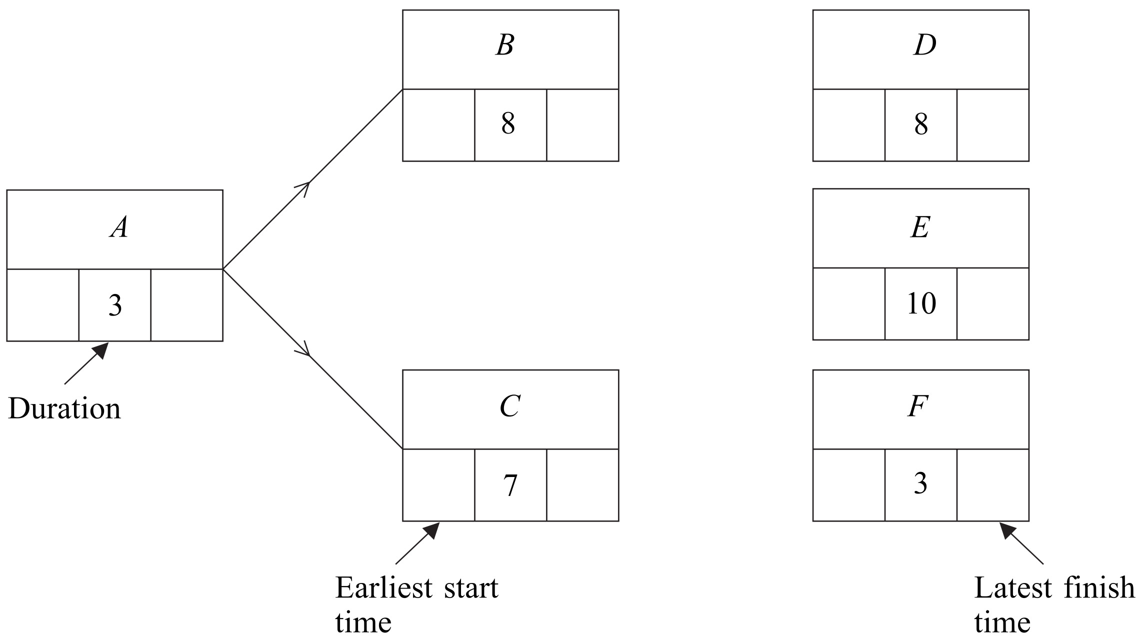


Figure 2 (for use in Question 1)

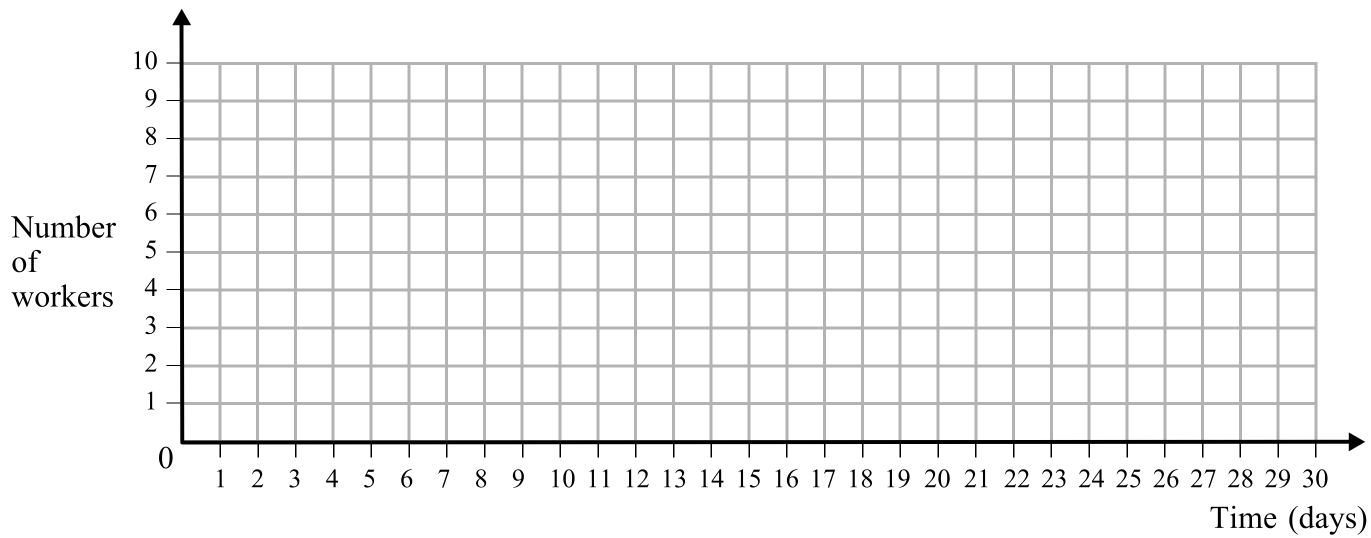


Figure 3 (for use in Question 5)

Stage	State	From	Value	
1	H	T	5	
	I	T	6	
2	F	H	$-2 + 5 = 3$	
		T	4	
		I		
	G	I		
3	C	H		
		F		
		G		

Turn over ▶

Figure 4 (for use in Question 6)

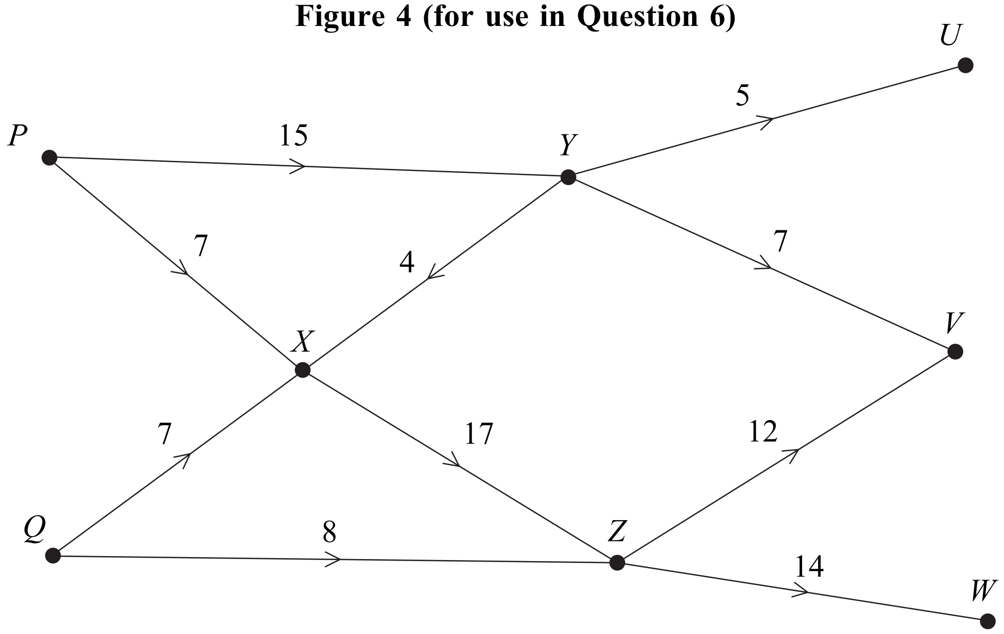


Figure 5 (for use in Question 6)

Route	Flow
SQZWT	
SPYXZVT	

Figure 6 (for use in Question 6)