**NOTICE TO CUSTOMER:**

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Answer **all** questions.

**1** Three forces $2\mathbf{i}$, $3\mathbf{i} - 5\mathbf{j} + a\mathbf{k}$ and $b\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}$ act at the points with coordinates $(1, 1, 0)$, $(0, 0, 0)$ and $(-1, 2, 1)$ respectively, where $a$ and $b$ are constants.

Given that the three forces form a couple, find:

(a) the values of $a$ and $b$; *(2 marks)*

(b) the moment of the couple. *(4 marks)*

**2** Forces $\begin{bmatrix} 8 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 6 \\ 5 \end{bmatrix}$, $\begin{bmatrix} -2 \\ -2 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ -2 \end{bmatrix}$ act at the points with coordinates $(0, 0)$, $(0, 3)$, $(3, 4)$ and $(4, 0)$ respectively.

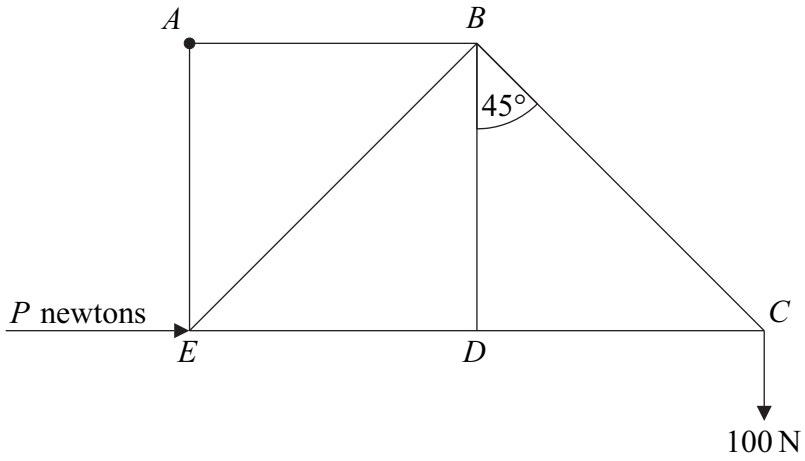
(a) (i) Find the magnitude of the resultant $\mathbf{F}$ of this system of forces. *(3 marks)*

(ii) Show that the line of action of $\mathbf{F}$ cuts the $y$-axis at the point $(0, 2)$. *(4 marks)*

(b) The system of forces is equivalent to a force acting at the origin together with a couple $C$. Write down the magnitude of $C$ and indicate its sense on a diagram. *(2 marks)*

**3** A framework is composed of seven light smoothly-jointed rods $AB$, $AE$, $BE$, $BD$, $ED$, $BC$ and $DC$, so that $ABDE$ is a square and $BDC$ is a right-angled triangle. The rod $AB$ has length $l$ and angle $CBD = 45°$.

The framework is in a vertical plane and is freely hinged at $A$ to a fixed support. A vertical force of 100 N acts at $C$. The rod $AE$ is kept vertical by a horizontal force of magnitude $P$ newtons applied at $E$, as shown in the diagram.




(a) Show that $P = 200$. *(2 marks)*

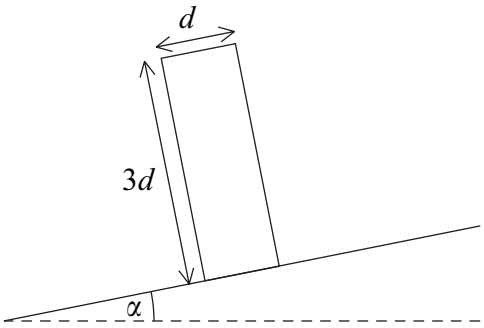
(b) (i) Find the magnitude of the reaction force on the framework at $A$. *(2 marks)*

(ii) Find the angle between this reaction force and the horizontal, giving your answer to the nearest degree. *(1 mark)*

(c) Find the magnitudes of the forces in each of the rods $AB$, $AE$ and $BE$, stating whether they are in tension or compression. *(5 marks)*

**Turn over for the next question**

**4** A uniform solid circular cylinder is in equilibrium with one plane face on a rough inclined plane. The plane is inclined to the horizontal at an angle $\alpha$ degrees, which can be varied. The cylinder has weight $W$, diameter $d$ and height $3d$.




(a) Draw a diagram showing the forces acting on the cylinder. *(2 marks)*

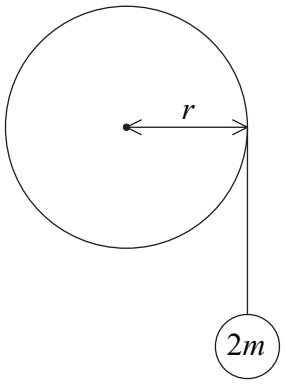
(b) If the plane is sufficiently rough to prevent sliding, find the maximum value of $\alpha$ for the cylinder to remain in equilibrium. *(3 marks)*

(c) The coefficient of friction between the cylinder and the plane is $\frac{2}{9}$. If the value of $\alpha$ is gradually increased from zero, show that the cylinder will slide before it topples. *(5 marks)*

**5** A light inextensible string is wrapped several times around a uniform cylinder and a particle of mass $2m$ is attached to the free end of the string.

The cylinder, of radius $r$, is free to rotate about a smooth fixed horizontal axis through its centre, perpendicular to its plane face. The moment of inertia of the cylinder about this axis is $4mr^2$.

The system is released from rest with the particle hanging freely. After time $t$, the cylinder has turned through an angle $\theta$ radians. Assume that during this subsequent motion no slipping of the string occurs.

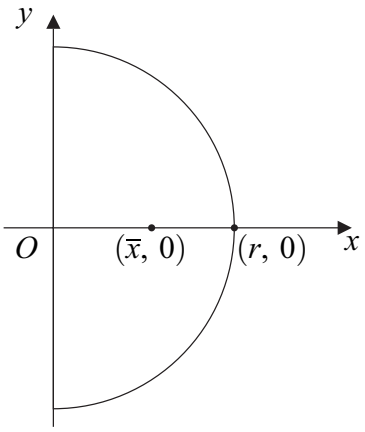



(a) Show that $\ddot{\theta} = \dfrac{g}{3r}$. *(6 marks)*

(b) Hence find an expression for the tension in the string in terms of $m$ and $g$. *(1 mark)*

**Turn over for the next question**

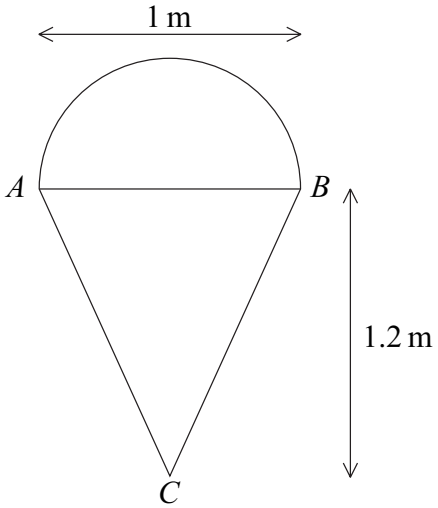
**6** (a) A uniform semicircular lamina of radius $r$ has its centre at the origin and its axis of symmetry along $Ox$. The position of its centre of mass has coordinates $(\overline{x}, 0)$.




(i) Show that $\frac{1}{2}\pi r^2\overline{x} = \int_0^r 2x\sqrt{r^2 - x^2}\,\mathrm{d}x$. *(4 marks)*

(ii) Hence prove that $\overline{x} = \dfrac{4r}{3\pi}$. *(3 marks)*

(b) The diagram below shows a uniform lamina, used as an advertising feature in a local supermarket to promote ice cream. It consists of a semicircle, of diameter $AB = 1$m, and an isosceles triangle $ABC$, where $C$ is at a distance 1.2 m from $AB$.




(i) State the distance of the centre of mass of the **triangle** from $C$. *(1 mark)*

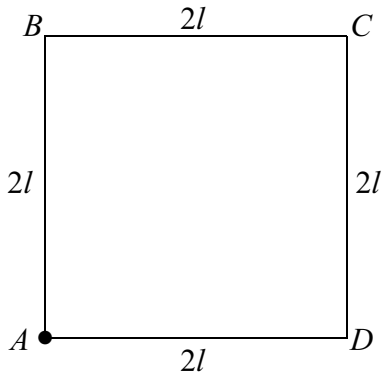
(ii) Show that the distance of the centre of mass of the **semicircle** from $C$ is approximately 1.41 m. *(1 mark)*

(iii) Find the distance of the centre of mass of the complete lamina from $C$. *(4 marks)*

(c) The lamina is freely suspended from $A$. Find the angle that $AB$ makes with the vertical through $A$, giving your answer to the nearest degree. *(3 marks)*

**7** A rigid square framework $ABCD$ is formed from four identical uniform rods. Each rod has length $2l$ and mass $m$.

The framework can rotate freely in a vertical plane about a horizontal axis through $A$ perpendicular to the plane of the square $ABCD$.




(a) Show that the moment of inertia of the **rod** ***BC*** about the axis is $\frac{16ml^2}{3}$. *(4 marks)*

(b) Particles of masses $4m$, $3m$, $2m$ and $m$ are fixed at the vertices $A$, $B$, $C$ and $D$ respectively.

Show that the moment of inertia of the whole system about the axis through $A$ is $\frac{136ml^2}{3}$. *(6 marks)*

(c) The system is released from rest with $AD$ horizontal and $B$ vertically **above** $A$. Find, in terms of $g$ and $l$, the angular velocity of the system when $B$ is vertically **below** $A$. *(7 marks)*

**END OF QUESTIONS**

**Practice 2**

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**1.** [*In this question* **i** *and* **j** *are unit vectors due east and due north respectively*.]

A ship $P$ is moving with velocity $(5\mathbf{i} - 4\mathbf{j})$ km h$^{-1}$ and a ship $Q$ is moving with velocity $(3\mathbf{i} + 7\mathbf{j})$ km h$^{-1}$. Find the direction that ship $Q$ appears to be moving in, to an observer on ship $P$, giving your answer as a bearing.

**(5)**

Leave blank

**Question 1 continued**

**(Total 5 marks)**

**Q1**

Leave blank

**2.** Two small smooth spheres $A$ and $B$ have equal radii. The mass of $A$ is $2m$ kg and the mass of $B$ is $m$ kg. The spheres are moving on a smooth horizontal plane and they collide. Immediately before the collision the velocity of $A$ is $(2\mathbf{i} - 2\mathbf{j})$ m s$^{-1}$ and the velocity of $B$ is $(-3\mathbf{i} - \mathbf{j})$ m s$^{-1}$. Immediately after the collision the velocity of $A$ is $(\mathbf{i} - 3\mathbf{j})$ m s$^{-1}$. Find the speed of $B$ immediately after the collision.

**(5)**

Leave blank

**Question 2 continued**

**Q2**

**(Total 5 marks)**



**Turn over**

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**3.** At time $t = 0$, a particle of mass $m$ is projected vertically downwards with speed $U$ from a point above the ground. At time $t$ the speed of the particle is $v$ and the magnitude of the air resistance is modelled as being $mkv$, where $k$ is a constant.

Given that $U < \frac{g}{2k}$, find, in terms of $k$, $U$ and $g$, the time taken for the particle to double its speed.

**(8)**

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**Question 3 continued**

**Q3**

**(Total 8 marks)**



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**4.**

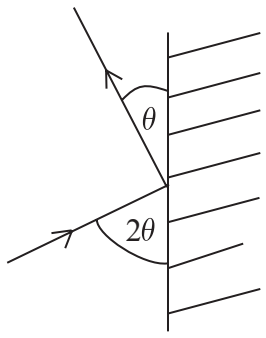


**Figure 1**

A small smooth ball $B$, moving on a horizontal plane, collides with a fixed vertical wall. Immediately before the collision the angle between the direction of motion of $B$ and the wall is $2\theta$, where $0^o < \theta < 45^o$. Immediately after the collision the angle between the direction of motion of $B$ and the wall is $\theta$, as shown in Figure 1. Given that the coefficient of restitution between $B$ and the wall is $\frac{3}{8}$, find the value of $\tan\theta$.

**(8)**

Leave blank

**Question 4 continued**

**(Total 8 marks)**

**Q4**

Leave blank

**5.** A light elastic spring has natural length $l$ and modulus of elasticity $mg$. One end of the spring is fixed to a point $O$ on a rough horizontal table. The other end is attached to a particle $P$ of mass $m$ which is at rest on the table with $OP = l$. At time $t = 0$ the particle is projected with speed $\sqrt{(gl)}$ along the table in the direction $OP$. At time $t$ the displacement of $P$ from its initial position is $x$ and its speed is $v$. The motion of $P$ is subject to air resistance of magnitude $2mv\omega$, where $\omega = \sqrt{\frac{g}{l}}$. The coefficient of friction between $P$ and the table is 0.5.

(a) Show that, until $P$ first comes to rest,

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2\omega\frac{\mathrm{d}x}{\mathrm{d}t} + \omega^2 x = -0.5g.$$

**(6)**

(b) Find $x$ in terms of $t$, $l$ and $\omega$.

**(6)**

(c) Hence find, in terms of $\omega$, the time taken for $P$ to first come to instantaneous rest.

**(3)**

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**Question 5 continued**

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**Question 5 continued**

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**Question 5 continued**

**(Total 15 marks)**

**Q5**

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**6.**

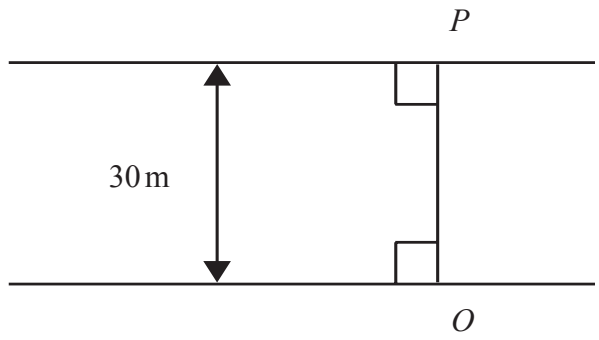


**Figure 2**

A river is 30 m wide and flows between two straight parallel banks. At each point of the river, the direction of flow is parallel to the banks. At time $t = 0$, a boat leaves a point $O$ on one bank and moves in a straight line across the river to a point $P$ on the opposite bank. Its path $OP$ is perpendicular to both banks and $OP = 30$ m, as shown in Figure 2. The speed of flow of the river, $r$ m s$^{-1}$, at a point on OP which is at a distance $x$ m from $O$, is modelled as

$$r = \frac{1}{10}x, \quad 0 \leqslant x \leqslant 30.$$

The speed of the boat relative to the water is constant at 5 m s$^{-1}$. At time $t$ seconds the boat is at a distance $x$ m from $O$ and is moving with speed $v$ m s$^{-1}$ in the direction $OP$.

(a) Show that

$$100v^2 = 2500 - x^2.$$

**(3)**

(b) Hence show that

$$\frac{d^2x}{dt^2} + \frac{x}{100} = 0.$$

**(4)**

(c) Find the total time taken for the boat to cross the river from $O$ to $P$.

**(9)**

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**Question 6 continued**

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**Question 6 continued**

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**Question 6 continued**

**(Total 16 marks)**

**Q6**



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**7.**

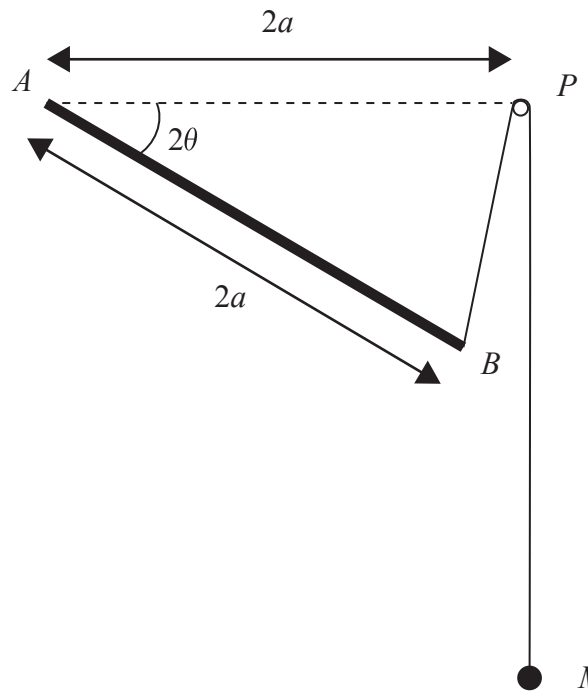


**Figure 3**

A uniform rod $AB$, of length $2a$ and mass $kM$ where $k$ is a constant, is free to rotate in a vertical plane about the fixed point $A$. One end of a light inextensible string of length $6a$ is attached to the end $B$ of the rod and passes over a small smooth pulley which is fixed at the point $P$. The line $AP$ is horizontal and of length $2a$. The other end of the string is attached to a particle of mass $M$ which hangs vertically below the point $P$, as shown in Figure 3. The angle $PAB$ is $2\theta$ , where $0^o \leqslant \theta \leqslant 180^o$.

(a) Show that the potential energy of the system is

$$Mga(4\sin\theta - k\sin 2\theta) + \text{constant}.$$

**(5)**

The system has a position of equilibrium when $\cos\theta = \frac{3}{4}$ .

(b) Find the value of $k$.

**(5)**

(c) Hence find the value of $\cos\theta$ at the other position of equilibrium.

**(3)**

(d) Determine the stability of each of the two positions of equilibrium.

**(5)**

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**Question 7 continued**

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**Question 7 continued**

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**Question 7 continued**



**Turn over**

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**Question 7 continued**

Q7

**(Total 18 marks)**

**TOTAL FOR PAPER: 75 MARKS**

**END**

Answer **all** questions.

**1** Anne has made a simple pendulum which has a period of 4 seconds.

Show that the length of Anne's pendulum is $\frac{4g}{\pi^2}$. *(3 marks)*

**2** A particle moves with simple harmonic motion on a straight line between the points $P$ and $Q$. The amplitude of this motion is 0.3 m.

When the particle is 0.06 m from $P$, its speed is $0.9\,\text{m}\,\text{s}^{-1}$.

(a) Show that the period of motion is $\frac{2\pi}{5}$ seconds. *(5 marks)*

(b) Find the magnitude of the maximum acceleration of the particle. *(2 marks)*

**3** A particle moves so that, at time $t$, its polar coordinates $(r, \theta)$ with respect to a fixed origin, $O$, are such that $r = \frac{a}{1 + 5\cos\theta}$, where $a$ is a constant.

(a) At the point $A$, the value of $r$ is a minimum.

Show that at this point $r = \frac{a}{6}$. *(2 marks)*

(b) Show that $\dot{r} = \frac{5r^2}{a}\dot{\theta}\sin\theta$. *(3 marks)*

(c) Show that $\ddot{r} = \frac{5a\dot{\theta}^2}{36}$ when the particle is at $A$. *(6 marks)*

(d) The radial acceleration of the particle is $-\frac{\lambda}{r^2}$, where $\lambda$ is a constant.

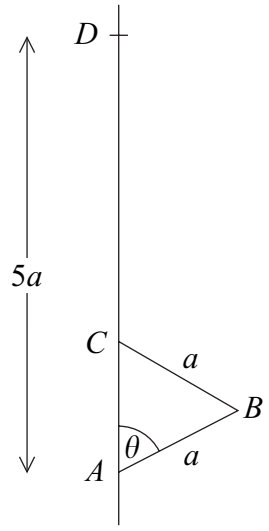
Find the speed of the particle at $A$ in terms of $\lambda$ and $a$. *(5 marks)*

**4** A smooth rod passes through two points $A$ and $D$, where $D$ is a distance $5a$ vertically above $A$.

Two uniform rods, $AB$ and $BC$, each of mass $2m$ and length $a$, are smoothly pivoted at $B$. The rod $AB$ is smoothly pivoted at $A$, and the rod $BC$ has the end $C$ attached to a light smooth ring that can move freely on the vertical rod $AD$.

The ring, attached to the rod $BC$ at $C$, is joined to the point $D$ by a light spring, which has modulus of elasticity $6mg$, and natural length $3a$.

The rod $AB$ is inclined at an angle $\theta$ to the vertical with the ring above $A$, as shown in the diagram.




(a) Show that $V$, the total potential energy of the system, is given by

$$V = 4mga(1 - \cos\theta + \cos^2\theta)$$

where the gravitational potential energy is taken to be zero at the level of $A$. *(6 marks)*

(b) Find the two values of $\theta$ in the range $0 \leqslant \theta \leqslant \frac{\pi}{2}$ for which the system is in equilibrium. *(4 marks)*

(c) Determine, for each of these values of $\theta$, whether the system is in stable or unstable equilibrium. *(4 marks)*

**Turn over for the next question**

**5** A car travels along a straight horizontal road. At time $t$, its speed is $v$ and the total resistance to motion has magnitude $kv$, where $k$ is a constant. The car is powered by a rocket, which ejects burnt fuel backwards at a constant rate $\lambda$ and at a constant speed $V$ relative to the car.

The initial mass of the car and the fuel is $M$.

(a) By considering linear momentum, show that the acceleration of the car along the road is

$$\frac{\lambda V - kv}{M - \lambda t}$$ *(7 marks)*

(b) The initial speed of the car is zero. Find its speed at time $t$. *(6 marks)*

(c) Initially, the mass of the fuel is 75% of the mass $M$. When $t = T_0$, all of the fuel has been burnt. Find $T_0$ in terms of $M$ and $\lambda$. *(3 marks)*

**6** A particle $P$, of mass $2m$, is suspended from a fixed point $O$ by a light elastic string of natural length $a$ and modulus of elasticity $8mn^2a$, where $n$ is a positive constant. The particle is released from rest when $t = 0$ at a point $A$, where $A$ is vertically below $O$ and $OA = a$.

When the particle is moving with speed $v$, it experiences air resistance of magnitude $4mnv$.

(a) The displacement of $P$ below $A$ at time $t$ is $x$. Show that $x$ satisfies the equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2n\frac{\mathrm{d}x}{\mathrm{d}t} + 4n^2x = g$$ *(3 marks)*

(b) Find $x$ in terms of $n$, $g$ and $t$. *(12 marks)*

(c) Hence show that the first time the particle comes to rest is when $t = \dfrac{\pi}{\sqrt{3}n}$. *(4 marks)*

**END OF QUESTIONS**

**Practice 4**

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**1.** [*In this question* **i** *and* **j** *are horizontal unit vectors.*]

A small bead of mass 0.5 kg is threaded on a smooth horizontal wire. The bead is initially at rest at the point with position vector (**i** – 6**j**) m. A constant horizontal force **P** N then acts on the bead causing it to move along the wire. The bead passes through the point with position vector (7**i** – 14**j**) m with speed $2\sqrt{7}$ m s$^{-1}$.

Given that **P** is parallel to (6**i** + **j**), find **P**.

**(6)**

Leave blank

**Question 1 continued**

**Q1**

**(Total 6 marks)**

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**2.** The velocity $\mathbf{v}$ m s$^{-1}$ of a particle $P$ at time $t$ seconds satisfies the vector differential equation

$$\frac{d\mathbf{v}}{dt} + 4\mathbf{v} = 0 \ .$$

The position vector of $P$ at time $t$ seconds is $\mathbf{r}$ metres.

Given that at $t = 0$, $\mathbf{r} = (\mathbf{i} - \mathbf{j})$ and $\mathbf{v} = (-8\mathbf{i} + 4\mathbf{j})$, find $\mathbf{r}$ at time $t$ seconds.

**(7)**

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**Question 2 continued**

**(Total 7 marks)**

**Q2**

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**3.** A system of forces consists of two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ acting on a rigid body.

$\mathbf{F}_1 = (-2\mathbf{i} + \mathbf{j} - \mathbf{k})$ N and acts at the point with position vector $\mathbf{r}_1 = (\mathbf{i} - \mathbf{j} + \mathbf{k})$ m.

$\mathbf{F}_2 = (3\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ N and acts at the point with position vector $\mathbf{r}_2 = (4\mathbf{i} - \mathbf{j} - 2\mathbf{k})$ m.

Given that the system is equivalent to a single force **R** N, acting at the point with position vector $(5\mathbf{i} + \mathbf{j} - \mathbf{k})$ m, together with a couple **G** N m, find

(a) **R** ,

**(2)**

(b) the magnitude of **G**.

**(9)**

Leave blank

**Question 3 continued**

**(Total 11 marks)**

**Q3**

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**4.** At time $t = 0$ a rocket is launched from rest vertically upwards. The rocket propels itself upwards by expelling burnt fuel vertically downwards with constant speed $U$ m s$^{-1}$ relative to the rocket. The initial mass of the rocket is $M_0$ kg. At time $t$ seconds, where $t < 2$, its mass is $M_0(1-\frac{1}{2}t)$ kg, and it is moving upwards with speed $v$ m s$^{-1}$.

(a) Show that

$$\frac{dv}{dt} = \frac{U}{(2-t)} - 9.8$$

**(7)**

(b) Hence show that $U > 19.6$

**(2)**

(c) Find, in terms of $U$, the speed of the rocket one second after its launch.

**(5)**

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**Question 4 continued**

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**Question 4 continued**

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**Question 4 continued**

**(Total 14 marks)**

**Q4**



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**5.**

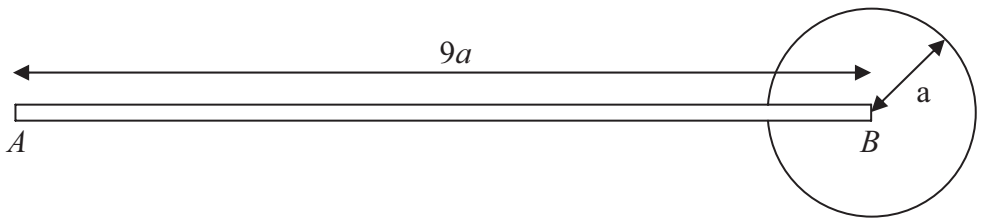


**Figure 1**

A pendulum $P$ is modelled as a uniform rod $AB$, of length $9a$ and mass $m$, rigidly fixed to a uniform circular disc of radius $a$ and mass $2m$. The end $B$ of the rod is attached to the centre of the disc, and the rod lies in the plane of the disc, as shown in Figure 1. The pendulum is free to rotate in a vertical plane about a fixed smooth horizontal axis $L$ which passes through the end $A$ and is perpendicular to the plane of the disc.

(a) Show that the moment of inertia of $P$ about $L$ is $190ma^2$.

**(4)**

The pendulum makes small oscillations about $L$.

(b) By writing down an equation of motion for $P$, find the approximate period of these small oscillations.

**(7)**

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**Question 5 continued**

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**Question 5 continued**

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**Question 5 continued**

**(Total 11 marks)**

**Q5**

Leave blank

**6.** A uniform solid right circular cylinder has mass $M$, height $h$ and radius $a$. Find, using integration, its moment of inertia about a diameter of one of its circular ends.

[*You may assume without proof that the moment of inertia of a uniform circular disc, of mass m and radius a, about a diameter is* $\frac{1}{4}ma^2$.]

**(10)**

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**Question 6 continued**

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**Question 6 continued**

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**Question 6 continued**

**(Total 10 marks)**

**Q6**

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**7.** A uniform square lamina $ABCD$, of mass $2m$ and side $3a\sqrt{2}$, is free to rotate in a vertical plane about a fixed smooth horizontal axis $L$ which passes through $A$ and is perpendicular to the plane of the lamina. The moment of inertia of the lamina about $L$ is $24ma^2$.

The lamina is at rest with $C$ vertically above $A$. At time $t = 0$ the lamina is slightly displaced. At time $t$ the lamina has rotated through an angle $\theta$.

(a) Show that

$$2a\left(\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2 = g(1-\cos\theta).$$

**(4)**

(b) Show that, at time $t$, the magnitude of the component of the force acting on the lamina at $A$, in a direction perpendicular to $AC$, is $\frac{1}{2}mg\sin\theta$.

**(7)**

When the lamina reaches the position with $C$ vertically below $A$, it receives an impulse which acts at $C$, in the plane of the lamina and in a direction which is perpendicular to the line $AC$. As a result of this impulse the lamina is brought immediately to rest.

(c) Find the magnitude of the impulse.

**(5)**

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**Question 7 continued**

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**Question 7 continued**

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**Question 7 continued**

Q7

**(Total 16 marks)**

**TOTAL FOR PAPER: 75 MARKS**

**END**

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**1.**

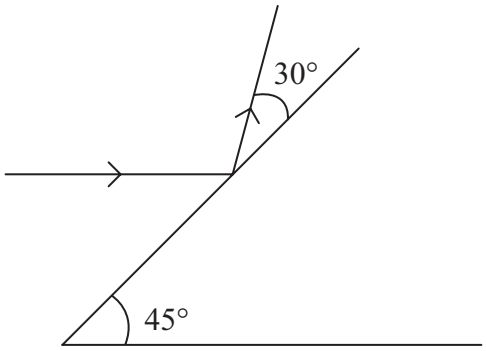


**Figure 1**

A fixed smooth plane is inclined to the horizontal at an angle of 45°. A particle $P$ is moving horizontally and strikes the plane. Immediately before the impact, $P$ is moving in a vertical plane containing a line of greatest slope of the inclined plane. Immediately after the impact, $P$ is moving in a direction which makes an angle of 30° with the inclined plane, as shown in Figure 1.

Find the fraction of the kinetic energy of $P$ which is lost in the impact.

**(6)**

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**Question 1 continued**

**(Total 6 marks)**

**Q1**

Leave blank

**2.** At time $t = 0$, a particle $P$ of mass $m$ is projected vertically upwards with speed $\sqrt{\frac{g}{k}}$, where $k$ is a constant. At time $t$ the speed of $P$ is $v$. The particle $P$ moves against air resistance whose magnitude is modelled as being $mkv^2$ when the speed of $P$ is $v$. Find, in terms of $k$, the distance travelled by $P$ until its speed first becomes half of its initial speed.

**(9)**

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**Question 2 continued**

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**Question 2 continued**

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**Question 2 continued**

**Q2**

**(Total 9 marks)**

Leave blank

**3.** At noon a motorboat $P$ is 2 km north-west of another motorboat $Q$. The motorboat $P$ is moving due south at 20 m $s^{-1}$. The motorboat $Q$ is pursuing motorboat $P$ at a speed of 12 m $s^{-1}$ and sets a course in order to get as close to motorboat $P$ as possible.

(a) Find the course set by $Q$, giving your answer as a bearing to the nearest degree.

**(4)**

(b) Find the shortest distance between $P$ and $Q$.

**(3)**

(c) Find the distance travelled by $Q$ from its position at noon to the point of closest approach.

**(5)**

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**Question 3 continued**

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**Question 3 continued**

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**Question 3 continued**

**(Total 12 marks)**

**Q3**

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**4.**

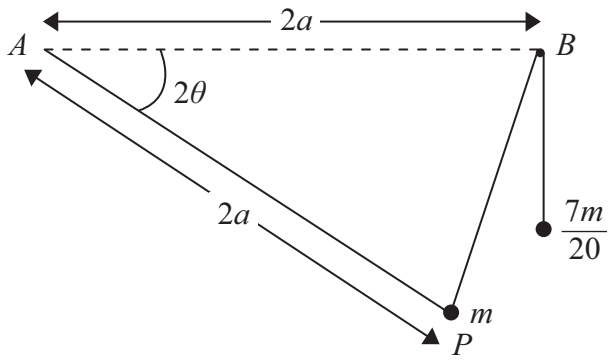


**Figure 2**

A light inextensible string of length $2a$ has one end attached to a fixed point $A$. The other end of the string is attached to a particle $P$ of mass $m$. A second light inextensible string of length $L$, where $L > \frac{12a}{5}$, has one of its ends attached to $P$ and passes over a small smooth peg fixed at a point $B$. The line $AB$ is horizontal and $AB = 2a$. The other end of the second string is attached to a particle of mass $\frac{7}{20}m$, which hangs vertically below $B$, as shown in Figure 2.

(a) Show that the potential energy of the system, when the angle $PAB = 2\theta$, is

$$\tfrac{1}{5}mga(7\sin\theta - 10\sin 2\theta) + \text{constant}.$$

**(4)**

(b) Show that there is only one value of $\cos\theta$ for which the system is in equilibrium and find this value.

**(8)**

(c) Determine the stability of the position of equilibrium.

**(4)**

Leave blank

**Question 4 continued**

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**Question 4 continued**

Leave blank

**Question 4 continued**

**(Total 16 marks)**

**Q4**

Leave blank

**5.** Two small smooth spheres $A$ and $B$, of mass 2 kg and 1 kg respectively, are moving on a smooth horizontal plane when they collide. Immediately before the collision the velocity of $A$ is $(\mathbf{i} + 2\mathbf{j})$ m s$^{-1}$ and the velocity of $B$ is $-2\mathbf{i}$ m s$^{-1}$. Immediately after the collision the velocity of $A$ is $\mathbf{j}$ m s$^{-1}$.

(a) Show that the velocity of $B$ immediately after the collision is $2\mathbf{j}$ m s$^{-1}$.

**(3)**

(b) Find the impulse of $B$ on $A$ in the collision, giving your answer as a vector, and hence show that the line of centres is parallel to $\mathbf{i} + \mathbf{j}$.

**(4)**

(c) Find the coefficient of restitution between $A$ and $B$.

**(6)**

Leave blank

**Question 5 continued**

Leave blank

**Question 5 continued**

Leave blank

**Question 5 continued**

**(Total 13 marks)**

**Q5**

Leave blank

**6.** A light elastic spring $AB$ has natural length $2a$ and modulus of elasticity $2mn^2a$, where $n$ is a constant. A particle $P$ of mass $m$ is attached to the end $A$ of the spring. At time $t = 0$, the spring, with $P$ attached, lies at rest and unstretched on a smooth horizontal plane. The other end $B$ of the spring is then pulled along the plane in the direction $AB$ with constant acceleration $f$. At time $t$ the extension of the spring is $x$.

(a) Show that $\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} + n^2x = f$.

**(6)**

(b) Find $x$ in terms of $n$, $f$ and $t$.

**(8)**

Hence find

(c) the maximum extension of the spring,

**(3)**

(d) the speed of $P$ when the spring first reaches its maximum extension.

**(2)**

Leave blank

**Question 6 continued**

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**Question 6 continued**

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**Question 6 continued**

Q6

**(Total 19 marks)**

**TOTAL FOR PAPER: 75 MARKS**

**END**

**Practice 6**

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**1.** At time $t = 0$, a particle $P$ of mass 3 kg is at rest at the point $A$ with position vector $(\mathbf{j} - 3\mathbf{k})$ m. Two constant forces $\mathbf{F}_1$ and $\mathbf{F}_2$ then act on the particle $P$ and it passes through the point $B$ with position vector $(8\mathbf{i} - 3\mathbf{j} + 5\mathbf{k})$ m.

Given that $\mathbf{F}_1 = (4\mathbf{i} - 2\mathbf{j} + 5\mathbf{k})$ N and $\mathbf{F}_2 = (8\mathbf{i} - 4\mathbf{j} + 7\mathbf{k})$ N and that $\mathbf{F}_1$ and $\mathbf{F}_2$ are the *only* two forces acting on $P$, find the velocity of $P$ as it passes through $B$, giving your answer as a vector.

**(7)**

Leave blank

**Question 1 continued**

**Q1**

**(Total 7 marks)**



**Turn over**

Leave blank

**2.** At time $t$ seconds, the position vector of a particle $P$ is $\mathbf{r}$ metres, where $\mathbf{r}$ satisfies the vector differential equation

$$\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2} + 4\mathbf{r} = \mathrm{e}^{2t}\mathbf{j}.$$

When $t = 0$, $P$ has position vector $(\mathbf{i} + \mathbf{j})$ m and velocity $2\mathbf{i}$ m s$^{-1}$.

Find an expression for $\mathbf{r}$ in terms of $t$.

**(11)**

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**Question 2 continued**

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**Question 2 continued**

**Question 2 continued**

Leave blank

**(Total 11 marks)**

**Q2**

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**3.** A spaceship is moving in a straight line in deep space and needs to increase its speed. This is done by ejecting fuel backwards from the spaceship at a constant speed $c$ relative to the spaceship. When the speed of the spaceship is $v$, its mass is $m$.

(a) Show that, while the spaceship is ejecting fuel,

$$\frac{\mathrm{d}v}{\mathrm{d}m} = -\frac{c}{m}.$$

**(5)**

The initial mass of the spaceship is $m_0$ and at time $t$ the mass of the spaceship is given by $m = m_0(1 - kt)$, where $k$ is a positive constant.

(b) Find the acceleration of the spaceship at time $t$.

**(4)**

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**Question 3 continued**

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**Question 3 continued**

**Question 3 continued**

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**Q3**

**(Total 9 marks)**

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**4.**

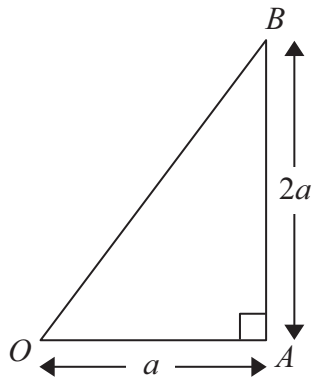


**Figure 1**

A uniform lamina of mass $M$ is in the shape of a right-angled triangle $OAB$. The angle $OAB$ is 90°, $OA = a$ and $AB = 2a$, as shown in Figure 1.

(a) Prove, using integration, that the moment of inertia of the lamina $OAB$ about the edge $OA$ is $\frac{2}{3}Ma^2$.

(You may assume without proof that the moment of inertia of a uniform rod of mass $m$ and length $2l$ about an axis through one end and perpendicular to the rod is $\frac{4}{3}ml^2$.)

**(6)**

The lamina $OAB$ is free to rotate about a fixed smooth horizontal axis along the edge $OA$ and hangs at rest with $B$ vertically below $A$. The lamina is then given a horizontal impulse of magnitude $J$. The impulse is applied to the lamina at the point $B$, in a direction which is perpendicular to the plane of the lamina. Given that the lamina first comes to instantaneous rest after rotating through an angle of 120°,

(b) find an expression for $J$, in terms of $M$, $a$ and $g$.

**(7)**

Leave blank

**Question 4 continued**

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**Question 4 continued**

**Question 4 continued**

Leave blank

**(Total 13 marks)**

**Q4**

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**5.** Two forces $\mathbf{F}_1 = (2\mathbf{i} + \mathbf{j})$ N and $\mathbf{F}_2 = (-2\mathbf{j} - \mathbf{k})$ N act on a rigid body. The force $\mathbf{F}_1$ acts at the point with position vector $\mathbf{r}_1 = (3\mathbf{i} + \mathbf{j} + \mathbf{k})$ m and the force $\mathbf{F}_2$ acts at the point with position vector $\mathbf{r}_2 = (\mathbf{i} - 2\mathbf{j})$ m. A third force $\mathbf{F}_3$ acts on the body such that $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ are in equilibrium.

(a) Find the magnitude of $\mathbf{F}_3$.

**(4)**

(b) Find a vector equation of the line of action of $\mathbf{F}_3$.

**(8)**

The force $\mathbf{F}_3$ is replaced by a fourth force $\mathbf{F}_4$, acting through the origin $O$, such that $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_4$ are equivalent to a couple.

(c) Find the magnitude of this couple.

**(4)**

Leave blank

**Question 5 continued**

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**Question 5 continued**

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**Question 5 continued**

**(Total 16 marks)**

**Q5**

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**6.** A pendulum consists of a uniform rod $AB$, of length $4a$ and mass $2m$, whose end $A$ is rigidly attached to the centre $O$ of a uniform square lamina $PQRS$, of mass $4m$ and side $a$. The rod $AB$ is perpendicular to the plane of the lamina. The pendulum is free to rotate about a fixed smooth horizontal axis $L$ which passes through $B$. The axis $L$ is perpendicular to $AB$ and parallel to the edge $PQ$ of the square.

(a) Show that the moment of inertia of the pendulum about $L$ is $75ma^2$.

**(4)**

The pendulum is released from rest when $BA$ makes an angle $\alpha$ with the downward vertical through $B$, where $\tan \alpha = \frac{7}{24}$. When $BA$ makes an angle $\theta$ with the downward vertical through $B$, the magnitude of the component, in the direction $AB$, of the force exerted by the axis $L$ on the pendulum is $X$.

(b) Find an expression for $X$ in terms of $m$, $g$ and $\theta$.

**(9)**

Using the approximation $\theta \approx \sin \theta$,

(c) find an estimate of the time for the pendulum to rotate through an angle $\alpha$ from its initial rest position.

**(6)**

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**Question 6 continued**

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**Question 6 continued**

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**Question 6 continued**

**Question 6 continued**

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**Q6**

**(Total 19 marks)**

**TOTAL FOR PAPER: 75 MARKS**

**END**