

NOTICE TO CUSTOMER:

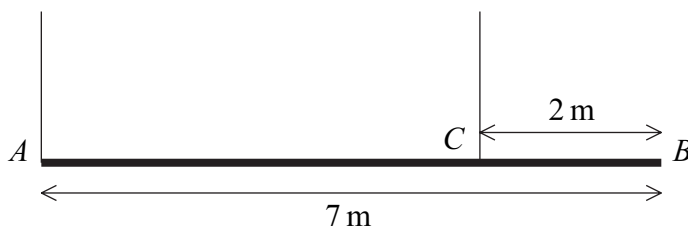
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Answer **all** questions.

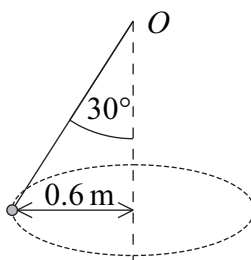
- 1 A uniform beam, AB , has mass 20 kg and length 7 metres. A rope is attached to the beam at A . A second rope is attached to the beam at the point C , which is 2 metres from B . Both of the ropes are vertical. The beam is in equilibrium in a horizontal position, as shown in the diagram.



Find the tensions in the two ropes.

(6 marks)

- 2 A particle, of mass 2 kg, is attached to one end of a light inextensible string. The other end is fixed to the point O . The particle is set into motion, so that it describes a horizontal circle of radius 0.6 metres, with the string at an angle of 30° to the vertical. The centre of the circle is vertically below O .



- (a) Show that the tension in the string is 22.6 N, correct to three significant figures.

(3 marks)

- (b) Find the speed of the particle.

(4 marks)

3 A particle moves in a straight line and at time t has velocity v , where

$$v = 2t - 12e^{-t}, \quad t \geq 0$$

- (a) (i) Find an expression for the acceleration of the particle at time t . (2 marks)
- (ii) State the range of values of the acceleration of the particle. (3 marks)
- (b) When $t = 0$, the particle is at the origin.

Find an expression for the displacement of the particle from the origin at time t . (4 marks)

4 A car has a maximum speed of 42 m s^{-1} when it is moving on a horizontal road. When the speed of the car is $v \text{ m s}^{-1}$, it experiences a resistance force of magnitude $30v$ newtons.

- (a) Show that the maximum power of the car is 52920 W . (2 marks)
- (b) The car has mass 1200 kg . It travels, from rest, up a slope inclined at 5° to the horizontal.

(i) Show that, when the car is travelling at its maximum speed $V \text{ m s}^{-1}$ up the slope,

$$V^2 + 392 \sin 5^\circ V - 1764 = 0 \quad (4 \text{ marks})$$

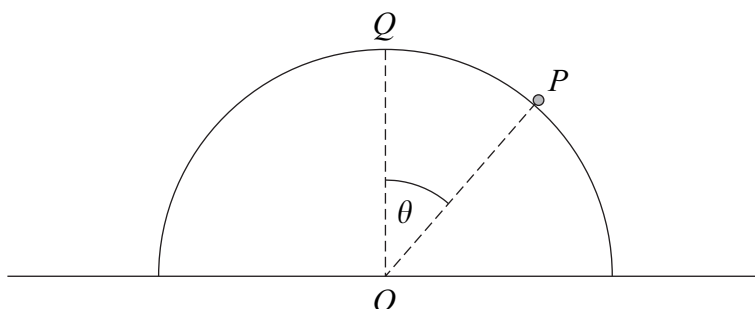
(ii) Hence find V . (2 marks)

5 A car, of mass 1600 kg , is travelling along a straight horizontal road at a speed of 20 m s^{-1} when the driving force is removed. The car then freewheels and experiences a resistance force. The resistance force has magnitude $40v$ newtons, where $v \text{ m s}^{-1}$ is the speed of the car after it has been freewheeling for t seconds.

Find an expression for v in terms of t . (7 marks)

Turn over for the next question

- 6 A particle P , of mass m kg, is placed at the point Q on the top of a smooth upturned hemisphere of radius 3 metres and centre O . The plane face of the hemisphere is fixed to a horizontal table. The particle is set into motion with an initial horizontal velocity of 2 m s^{-1} . When the particle is on the surface of the hemisphere, the angle between OP and OQ is θ and the particle has speed $v \text{ m s}^{-1}$.



- (a) Show that $v^2 = 4 + 6g(1 - \cos \theta)$. (4 marks)
- (b) Find the value of θ when the particle leaves the hemisphere. (5 marks)
- 7 A particle, of mass 10 kg, is attached to one end of a light elastic string of natural length 0.4 metres and modulus of elasticity 100 N. The other end of the string is fixed to the point O .
- (a) Find the length of the elastic string when the particle hangs in equilibrium directly below O . (2 marks)
- (b) The particle is pulled down and held at a point P , which is 1 metre vertically below O .
Show that the elastic potential energy of the string when the particle is in this position is 45 J. (2 marks)
- (c) The particle is released from rest at the point P . In the subsequent motion, the particle has speed $v \text{ m s}^{-1}$ when it is x metres **below** O .
- (i) Show that, while the string is taut,
- $$v^2 = 39.6x - 25x^2 - 14.6$$
- (7 marks)
- (ii) Find the value of x when the particle comes to rest for the first time after being released, given that the string is still taut. (3 marks)

END OF QUESTIONS

2. At time t seconds ($t \geq 0$), a particle P has position vector \mathbf{p} metres, with respect to a fixed origin O , where

$$\mathbf{p} = (3t^2 - 6t + 4)\mathbf{i} + (3t^3 - 4t)\mathbf{j}.$$

Find

(a) the velocity of P at time t seconds, (2)

(b) the value of t when P is moving parallel to the vector \mathbf{i} . (3)

When $t = 1$, the particle P receives an impulse of $(2\mathbf{i} - 6\mathbf{j})$ N s. Given that the mass of P is 0.5 kg,

(c) find the velocity of P immediately after the impulse. (4)

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Question 2 continued

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(Total 9 marks)

Q2

5

Turn over



3. A car of mass 1000 kg is moving at a constant speed of 16 m s^{-1} up a straight road inclined at an angle θ to the horizontal. The rate of working of the engine of the car is 20 kW and the resistance to motion from non-gravitational forces is modelled as a constant force of magnitude 550 N.

(a) Show that $\sin \theta = \frac{1}{14}$.

(5)

When the car is travelling up the road at 16 m s^{-1} , the engine is switched off. The car comes to rest, without braking, having moved a distance y metres from the point where the engine was switched off. The resistance to motion from non-gravitational forces is again modelled as a constant force of magnitude 550 N.

(b) Find the value of y .

(4)

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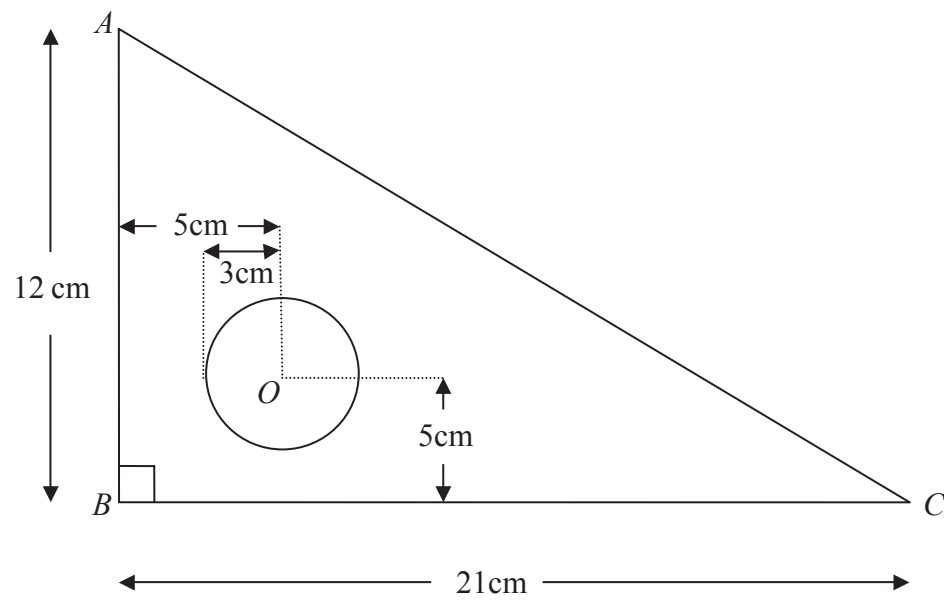


Figure 1

A set square S is made by removing a circle of centre O and radius 3 cm from a triangular piece of wood. The piece of wood is modelled as a uniform triangular lamina ABC , with $\angle ABC = 90^\circ$, $AB = 12$ cm and $BC = 21$ cm. The point O is 5 cm from AB and 5 cm from BC , as shown in Figure 1.

(a) Find the distance of the centre of mass of S from

- (i) AB ,
- (ii) BC .

(9)

The set square is freely suspended from C and hangs in equilibrium.

(b) Find, to the nearest degree, the angle between CB and the vertical.

(3)

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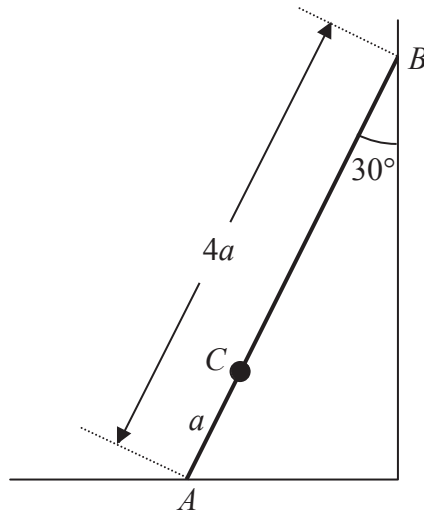


Figure 2

A ladder AB , of mass m and length $4a$, has one end A resting on rough horizontal ground. The other end B rests against a smooth vertical wall. A load of mass $3m$ is fixed on the ladder at the point C , where $AC = a$. The ladder is modelled as a uniform rod in a vertical plane perpendicular to the wall and the load is modelled as a particle. The ladder rests in limiting equilibrium making an angle of 30° with the wall, as shown in Figure 2.

Find the coefficient of friction between the ladder and the ground.

(10)

Question 5 continued

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Question 5 continued

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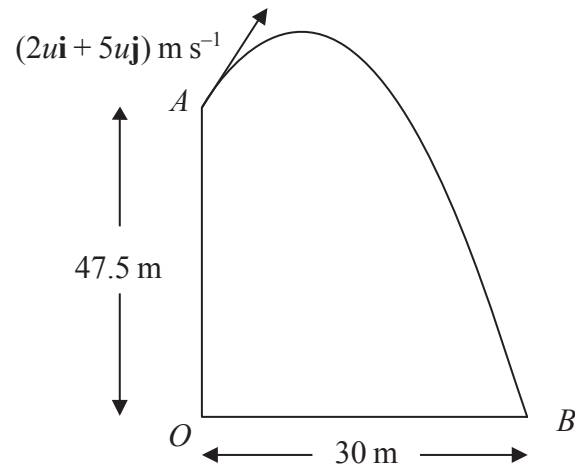


Figure 3

[In this question, the unit vectors \mathbf{i} and \mathbf{j} are in a vertical plane, \mathbf{i} being horizontal and \mathbf{j} being vertical.]

A particle P is projected from the point A which has position vector $47.5\mathbf{j}$ metres with respect to a fixed origin O . The velocity of projection of P is $(2u\mathbf{i} + 5u\mathbf{j}) \text{ m s}^{-1}$. The particle moves freely under gravity passing through the point B with position vector $30\mathbf{i}$ metres, as shown in Figure 3.

- (a) Show that the time taken for P to move from A to B is 5 s. (6)
- (b) Find the value of u . (2)
- (c) Find the speed of P at B . (5)

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7. A particle P of mass $2m$ is moving with speed $2u$ in a straight line on a smooth horizontal plane. A particle Q of mass $3m$ is moving with speed u in the same direction as P . The particles collide directly. The coefficient of restitution between P and Q is $\frac{1}{2}$.

(a) Show that the speed of Q immediately after the collision is $\frac{8}{5}u$. (5)

(b) Find the total kinetic energy lost in the collision. (5)

After the collision between P and Q , the particle Q collides directly with a particle R of mass m which is at rest on the plane. The coefficient of restitution between Q and R is e .

(c) Calculate the range of values of e for which there will be a second collision between P and Q . (7)

Lined area for writing answers to questions (a), (b), and (c).

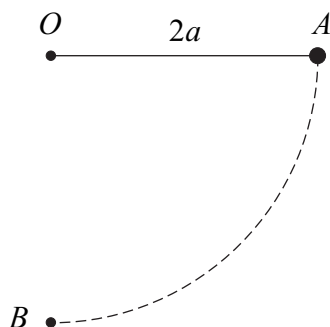
Answer **all** questions.

- 1 A lift containing miners moves vertically from rest at one level to rest at a higher level.

The total mass of the lift and the miners is 800 kg. The vertical distance between the two levels is 200 metres.

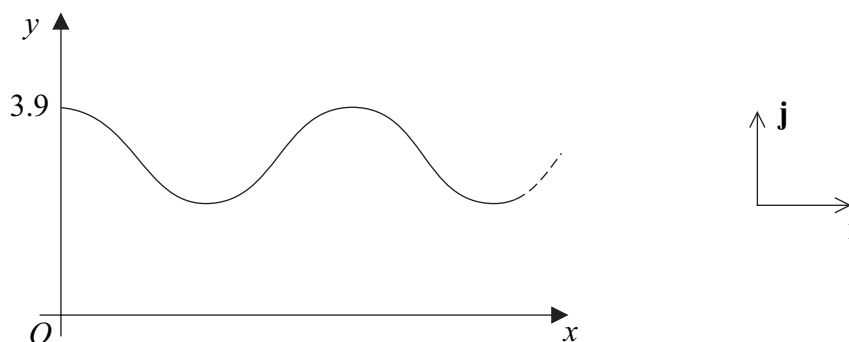
Find the work done in raising the lift and the miners from the lower level to the higher level.
(3 marks)

- 2 A light inextensible string has length $2a$. One end of the string is attached to a fixed point O and a particle of mass m is attached to the other end. Initially, the particle is held at the point A with the string taut and horizontal. The particle is then released from rest and moves in a circular path. Subsequently, it passes through the point B , which is directly below O . The points O , A and B are as shown in the diagram.



- (a) Show that the speed of the particle at B is $2\sqrt{ag}$. (3 marks)
- (b) Find the tension in the string as the particle passes through B . Give your answer in terms of m and g . (3 marks)

- 3 Jane is on a ride in a theme park. Part of the curved path of the ride is shown in the diagram.



Jane's position vector, \mathbf{r} metres, at time t seconds is given by

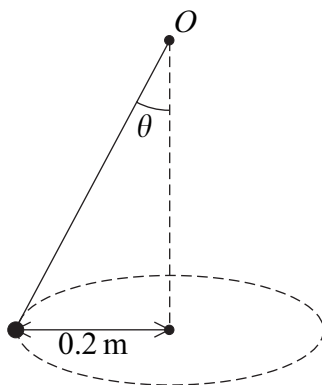
$$\mathbf{r} = 1.2t\mathbf{i} + (3 + 0.9 \cos t)\mathbf{j}$$

where the perpendicular unit vectors \mathbf{i} and \mathbf{j} are directed horizontally and vertically upwards respectively.

- (a) Find an expression for Jane's velocity at time t . (2 marks)
- (b) (i) Find an expression for Jane's speed at time t . (2 marks)
(ii) Find Jane's maximum speed during the ride. (2 marks)

Turn over for the next question

- 4 A particle is attached to one end of a light inextensible string. The other end of the string is attached to a fixed point O . The particle is set into motion, so that it describes a horizontal circle whose centre is vertically below O . The angle between the string and the vertical is θ , as shown in the diagram.



- (a) The particle completes 40 revolutions every minute.

Show that the angular speed of the particle is $\frac{4\pi}{3}$ radians per second. *(2 marks)*

- (b) The radius of the circle is 0.2 metres.

Find, in terms of π , the magnitude of the acceleration of the particle. *(2 marks)*

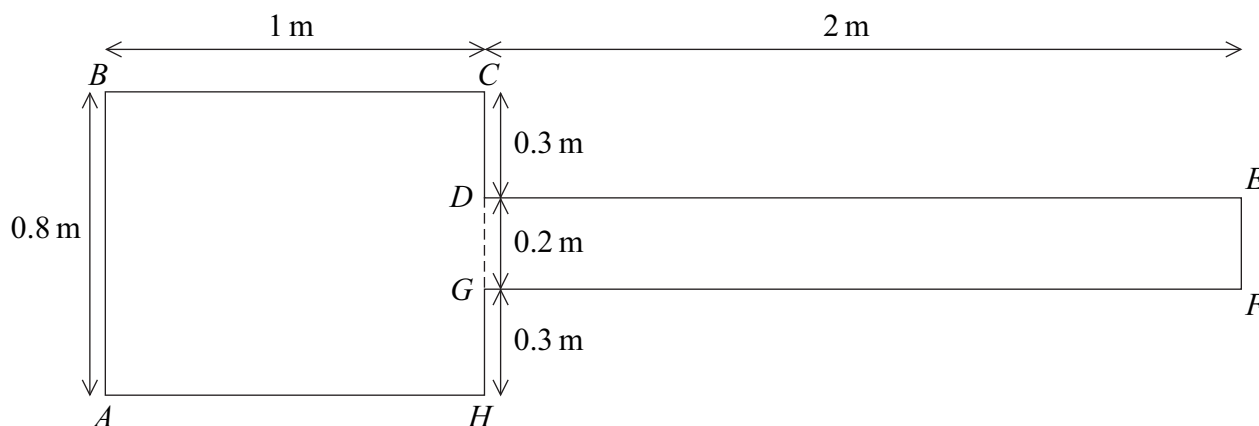
- (c) The mass of the particle is m kg and the tension in the string is T newtons.

(i) Draw a diagram showing the forces acting on the particle. *(1 mark)*

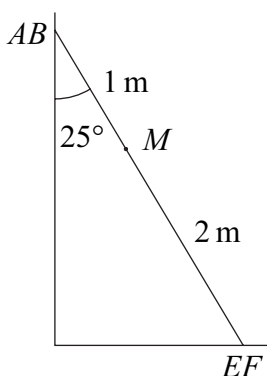
(ii) Explain why $T \cos \theta = mg$. *(1 mark)*

(iii) Find the value of θ , giving your answer to the nearest degree. *(5 marks)*

- 5 A sign advertising a gym consists of two rectangles $ABCH$ and $DEFG$ fixed rigidly together. The sign can be modelled as a uniform lamina, as shown in the diagram.



- (a) The centre of mass of the sign is at the point M . Show that M lies on the line CH .
(4 marks)
- (b) The sign is placed with its side EF on rough horizontal ground and its side AB against a smooth vertical wall. The sign rests in equilibrium at an angle of 25° with the **vertical**, as shown in the diagram.



The weight of the sign is 90 newtons.

- (i) By taking moments, show that the normal reaction force between the sign and the wall is $60 \tan 25^\circ$ newtons.
(3 marks)
- (ii) The coefficient of friction between the sign and the ground is μ .

Show that $\mu \geq \frac{2}{3} \tan 25^\circ$.
(4 marks)

6 A motorcycle has a maximum power of 72 kilowatts. The motorcycle and its rider are travelling along a straight horizontal road. When they are moving at a speed of $V \text{ m s}^{-1}$, they experience a total resistance force of magnitude kV newtons, where k is a constant.

(a) The maximum speed of the motorcycle and its rider is 60 m s^{-1} .

Show that $k = 20$.

(3 marks)

(b) When the motorcycle is travelling at 20 m s^{-1} , the rider allows the motorcycle to freewheel so that the only horizontal force acting is the resistance force. When the motorcycle has been freewheeling for t seconds, its speed is $v \text{ m s}^{-1}$ and the magnitude of the resistance force is $20v$ newtons.

The mass of the motorcycle and its rider is 500 kg.

(i) Show that $\frac{dv}{dt} = -\frac{v}{25}$.

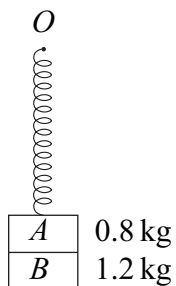
(2 marks)

(ii) Hence find the time that it takes for the speed of the motorcycle to reduce from 20 m s^{-1} to 10 m s^{-1} .

(6 marks)

7 Two small blocks, A and B , of masses 0.8 kg and 1.2 kg respectively, are stuck together. A spring has natural length 0.5 metres and modulus of elasticity 49 N . One end of the spring is attached to the top of the block A and the other end of the spring is attached to a fixed point O .

- (a) The system hangs in equilibrium with the blocks stuck together, as shown in the diagram.



Find the extension of the spring. (3 marks)

- (b) Show that the elastic potential energy of the spring when the system is in equilibrium is 1.96 J . (2 marks)
- (c) The system is hanging in this equilibrium position when block B falls off and block A begins to move vertically upwards.

Block A next comes to rest when the spring is **compressed** by x metres.

- (i) Show that x satisfies the equation

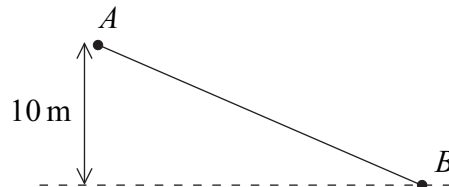
$$x^2 + 0.16x - 0.008 = 0 \quad (5\text{ marks})$$

- (ii) Find the value of x . (2 marks)

END OF QUESTIONS

Answer **all** questions.

- 1 A child, of mass 35 kg, slides down a slide in a water park. The child, starting from rest, slides from the point A to the point B , which is 10 metres vertically below the level of A , as shown in the diagram.



- (a) In a simple model, all resistance forces are ignored.

Use an energy method to find the speed of the child at B . (3 marks)

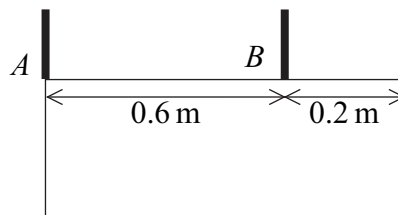
- (b) State one resistance force that has been ignored in answering part (a). (1 mark)

- (c) In fact, when the child slides down the slide, she reaches B with a speed of 12 m s^{-1} .

Given that the slide is 20 metres long and the sum of the resistance forces has a constant magnitude of F newtons, use an energy method to find the value of F .

(4 marks)

- 2 A hotel sign consists of a uniform rectangular lamina of weight W . The sign is suspended in equilibrium in a vertical plane by two vertical light chains attached to the sign at the points A and B , as shown in the diagram. The edge containing A and B is horizontal.



The tensions in the chains attached at A and B are T_A and T_B respectively.

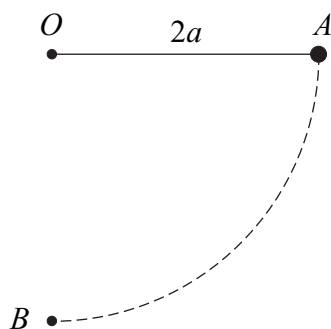
- (a) Draw a diagram to show the forces acting on the sign. (1 mark)

- (b) Find T_A and T_B in terms of W . (4 marks)

- (c) Explain how you have used the fact that the lamina is uniform in answering part (b).

(1 mark)

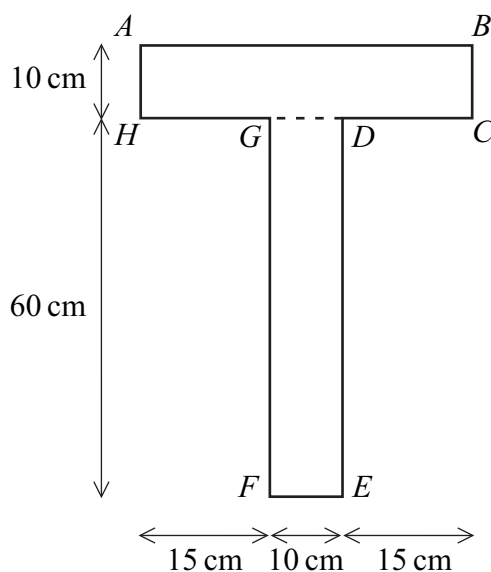
- 3 A light inextensible string has length $2a$. One end of the string is attached to a fixed point O and a particle of mass m is attached to the other end. Initially, the particle is held at the point A with the string taut and horizontal. The particle is then released from rest and moves in a circular path. Subsequently, it passes through the point B , which is directly below O . The points O , A and B are as shown in the diagram.



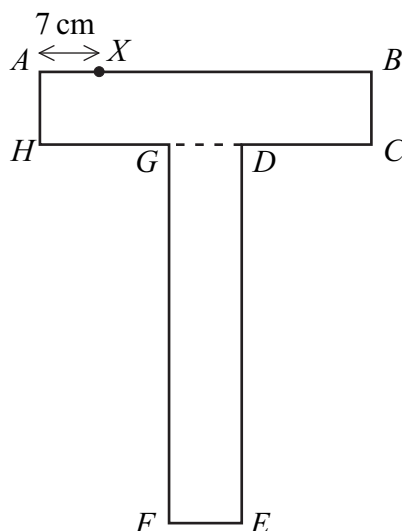
- (a) Show that the speed of the particle at B is $2\sqrt{ag}$. (3 marks)
- (b) Find the tension in the string as the particle passes through B . Give your answer in terms of m and g . (3 marks)

Turn over for the next question

- 4 A uniform T-shaped lamina is formed by rigidly joining two rectangles $ABCH$ and $DEFG$, as shown in the diagram.



- (a) Show that the centre of mass of the lamina is 26 cm from the edge AB . (4 marks)
- (b) Explain why the centre of mass of the lamina is 5 cm from the edge GF . (1 mark)
- (c) The point X is on the edge AB and is 7 cm from A , as shown in the diagram below.



The lamina is freely suspended from X and hangs in equilibrium.

Find the angle between the edge AB and the vertical, giving your answer to the nearest degree. (4 marks)

5 Tom is on a fairground ride.

Tom's position vector, \mathbf{r} metres, at time t seconds is given by

$$\mathbf{r} = 2 \cos t \mathbf{i} + 2 \sin t \mathbf{j} + (10 - 0.4t)\mathbf{k}$$

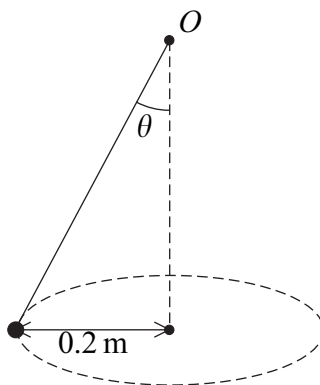
The perpendicular unit vectors \mathbf{i} and \mathbf{j} are in the horizontal plane and the unit vector \mathbf{k} is directed vertically upwards.

- (a) (i) Find Tom's position vector when $t = 0$. *(1 mark)*
- (ii) Find Tom's position vector when $t = 2\pi$. *(1 mark)*
- (iii) Write down the first **two** values of t for which Tom is directly below his starting point. *(2 marks)*
- (b) Find an expression for Tom's velocity at time t . *(3 marks)*
- (c) Tom has mass 25 kg.

Show that the resultant force acting on Tom during the motion has constant magnitude.
State the magnitude of the resultant force. *(5 marks)*

Turn over for the next question

- 6 A particle is attached to one end of a light inextensible string. The other end of the string is attached to a fixed point O . The particle is set into motion, so that it describes a horizontal circle whose centre is vertically below O . The angle between the string and the vertical is θ , as shown in the diagram.



- (a) The particle completes 40 revolutions every minute.

Show that the angular speed of the particle is $\frac{4\pi}{3}$ radians per second. *(2 marks)*

- (b) The radius of the circle is 0.2 metres.

Find, in terms of π , the magnitude of the acceleration of the particle. *(2 marks)*

- (c) The mass of the particle is m kg and the tension in the string is T newtons.

(i) Draw a diagram showing the forces acting on the particle. *(1 mark)*

(ii) Explain why $T \cos \theta = mg$. *(1 mark)*

(iii) Find the value of θ , giving your answer to the nearest degree. *(5 marks)*

7 A motorcycle has a maximum power of 72 kilowatts. The motorcycle and its rider are travelling along a straight horizontal road. When they are moving at a speed of $V \text{ m s}^{-1}$, they experience a total resistance force of magnitude kV newtons, where k is a constant.

(a) The maximum speed of the motorcycle and its rider is 60 m s^{-1} .

Show that $k = 20$.

(3 marks)

(b) When the motorcycle is travelling at 20 m s^{-1} , the rider allows the motorcycle to freewheel so that the only horizontal force acting is the resistance force. When the motorcycle has been freewheeling for t seconds, its speed is $v \text{ m s}^{-1}$ and the magnitude of the resistance force is $20v$ newtons.

The mass of the motorcycle and its rider is 500 kg.

(i) Show that $\frac{dv}{dt} = -\frac{v}{25}$.

(2 marks)

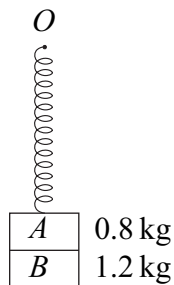
(ii) Hence find the time that it takes for the speed of the motorcycle to reduce from 20 m s^{-1} to 10 m s^{-1} .

(6 marks)

Turn over for the next question

8 Two small blocks, A and B , of masses 0.8 kg and 1.2 kg respectively, are stuck together. A spring has natural length 0.5 metres and modulus of elasticity 49 N. One end of the spring is attached to the top of the block A and the other end of the spring is attached to a fixed point O .

- (a) The system hangs in equilibrium with the blocks stuck together, as shown in the diagram.



Find the extension of the spring. (3 marks)

- (b) Show that the elastic potential energy of the spring when the system is in equilibrium is 1.96 J. (2 marks)

- (c) The system is hanging in this equilibrium position when block B falls off and block A begins to move vertically upwards.

Block A next comes to rest when the spring is **compressed** by x metres.

- (i) Show that x satisfies the equation

$$x^2 + 0.16x - 0.008 = 0 \quad (5 \text{ marks})$$

- (ii) Find the value of x . (2 marks)

END OF QUESTIONS

Question 1 continued

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(Total 5 marks)

Q1

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Turn over

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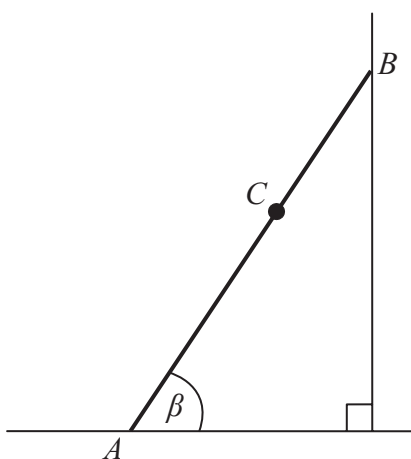


Figure 1

Figure 1 shows a ladder AB , of mass 25 kg and length 4 m, resting in equilibrium with one end A on rough horizontal ground and the other end B against a smooth vertical wall. The ladder is in a vertical plane perpendicular to the wall. The coefficient of friction between the ladder and the ground is $\frac{11}{25}$. The ladder makes an angle β with the ground. When Reece, who has mass 75 kg, stands at the point C on the ladder, where $AC = 2.8$ m, the ladder is on the point of slipping. The ladder is modelled as a uniform rod and Reece is modelled as a particle.

- (a) Find the magnitude of the frictional force of the ground on the ladder. (3)
- (b) Find, to the nearest degree, the value of β . (6)
- (c) State how you have used the modelling assumption that Reece is a particle. (1)

3. A block of mass 10 kg is pulled along a straight horizontal road by a constant horizontal force of magnitude 70 N in the direction of the road. The block moves in a straight line passing through two points A and B on the road, where $AB = 50$ m. The block is modelled as a particle and the road is modelled as a rough plane. The coefficient of friction between the block and the road is $\frac{4}{7}$.

(a) Calculate the work done against friction in moving the block from A to B . (4)

The block passes through A with a speed of 2 m s^{-1} .

(b) Find the speed of the block at B . (4)

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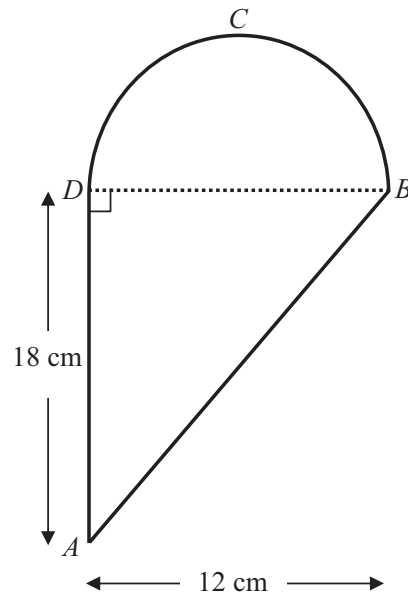


Figure 2

A uniform lamina $ABCD$ is made by joining a uniform triangular lamina ABD to a uniform semi-circular lamina DBC , of the same material, along the edge BD , as shown in Figure 2. Triangle ABD is right-angled at D and $AD = 18$ cm. The semi-circle has diameter BD and $BD = 12$ cm.

- (a) Show that, to 3 significant figures, the distance of the centre of mass of the lamina $ABCD$ from AD is 4.69 cm. (4)

Given that the centre of mass of a uniform semicircular lamina, radius r , is at a distance $\frac{4r}{3\pi}$ from the centre of the bounding diameter,

- (b) find, in cm to 3 significant figures, the distance of the centre of mass of the lamina $ABCD$ from BD . (4)

The lamina is freely suspended from B and hangs in equilibrium.

- (c) Find, to the nearest degree, the angle which BD makes with the vertical. (4)

Question 5 continued

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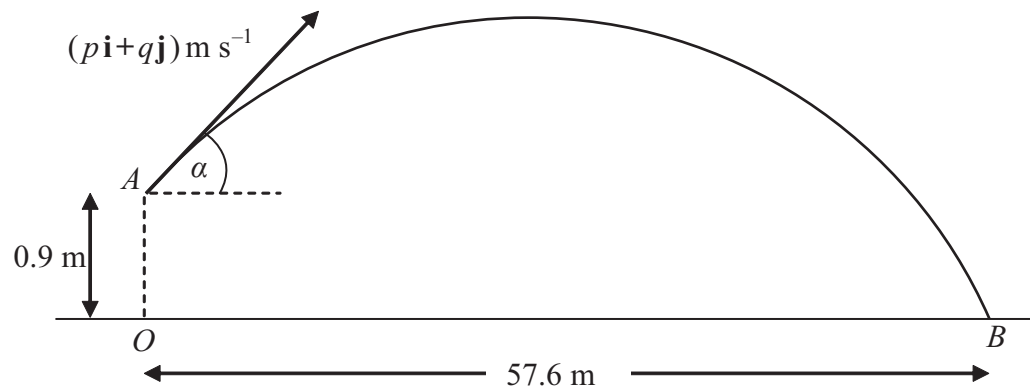


Figure 3

A cricket ball is hit from a point A with velocity of $(p\mathbf{i} + q\mathbf{j}) \text{ m s}^{-1}$, at an angle α above the horizontal. The unit vectors \mathbf{i} and \mathbf{j} are respectively horizontal and vertically upwards. The point A is 0.9 m vertically above the point O , which is on horizontal ground.

The ball takes 3 seconds to travel from A to B , where B is on the ground and $OB = 57.6 \text{ m}$, as shown in Figure 3. By modelling the motion of the cricket ball as that of a particle moving freely under gravity,

- (a) find the value of p , (2)
- (b) show that $q = 14.4$, (3)
- (c) find the initial speed of the cricket ball, (2)
- (d) find the exact value of $\tan \alpha$. (1)
- (e) Find the length of time for which the cricket ball is at least 4 m above the ground. (6)
- (f) State an additional physical factor which may be taken into account in a refinement of the above model to make it more realistic. (1)

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Question 6 continued

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7. A particle P of mass $3m$ is moving in a straight line with speed $2u$ on a smooth horizontal table. It collides directly with another particle Q of mass $2m$ which is moving with speed u in the opposite direction to P . The coefficient of restitution between P and Q is e .

(a) Show that the speed of Q immediately after the collision is $\frac{1}{5}(9e + 4)u$. (5)

The speed of P immediately after the collision is $\frac{1}{2}u$.

(b) Show that $e = \frac{1}{4}$. (4)

The collision between P and Q takes place at the point A . After the collision Q hits a smooth fixed vertical wall which is at right-angles to the direction of motion of Q . The distance from A to the wall is d .

(c) Show that P is a distance $\frac{3}{5}d$ from the wall at the instant when Q hits the wall. (4)

Particle Q rebounds from the wall and moves so as to collide directly with particle P at the point B . Given that the coefficient of restitution between Q and the wall is $\frac{1}{5}$,

(d) find, in terms of d , the distance of the point B from the wall. (4)



Question 7 continued

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