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Answer **all** questions.

1 (a) Find the roots of the equation $m^2 + 2m + 2 = 0$ in the form $a + \mathrm{i}b$. *(2 marks)*

(b) (i) Find the general solution of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2\frac{\mathrm{d}y}{\mathrm{d}x} + 2y = 4x$$

(6 marks)

(ii) Hence express y in terms of x, given that $y = 1$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 2$ when $x = 0$. *(4 marks)*

2 (a) Find $\int_0^a x\mathrm{e}^{-2x}\,\mathrm{d}x$, where $a > 0$. *(5 marks)*

(b) Write down the value of $\lim_{a\to\infty} a^k\mathrm{e}^{-2a}$, where k is a positive constant. *(1 mark)*

(c) Hence find $\int_0^\infty x\mathrm{e}^{-2x}\,\mathrm{d}x$. *(2 marks)*

3 (a) Show that $y = x^3 - x$ is a particular integral of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{2xy}{x^2 - 1} = 5x^2 - 1$$

(3 marks)

(b) By differentiating $(x^2 - 1)y = c$ implicitly, where y is a function of x and c is a constant, show that $y = \frac{c}{x^2 - 1}$ is a solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{2xy}{x^2 - 1} = 0$$

(3 marks)

(c) Hence find the general solution of

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{2xy}{x^2 - 1} = 5x^2 - 1$$

(2 marks)

4 (a) Use the series expansion

$$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \dots$$

to write down the first four terms in the expansion, in ascending powers of x, of $\ln(1-x)$. *(1 mark)*

(b) The function f is defined by

$$f(x) = e^{\sin x}$$

Use Maclaurin's theorem to show that when $f(x)$ is expanded in ascending powers of x:

(i) the first three terms are

$$1 + x + \frac{1}{2}x^2$$ *(6 marks)*

(ii) the coefficient of x^3 is zero. *(3 marks)*

(c) Find

$$\lim_{x \to 0} \frac{e^{\sin x} - 1 + \ln(1-x)}{x^2 \sin x}$$ *(4 marks)*

Turn over for the next question

5 (a) The function $y(x)$ satisfies the differential equation

$$\frac{dy}{dx} = f(x, y)$$

where $$f(x,y) = x \ln x + \frac{y}{x}$$

and $$y(1) = 1$$

(i) Use the Euler formula

$$y_{r+1} = y_r + h\,f(x_r, y_r)$$

with $h = 0.1$, to obtain an approximation to $y(1.1)$. *(3 marks)*

(ii) Use the formula

$$y_{r+1} = y_{r-1} + 2h\,f(x_r, y_r)$$

with your answer to part (a)(i) to obtain an approximation to $y(1.2)$, giving your answer to three decimal places. *(4 marks)*

(b) (i) Show that $\frac{1}{x}$ is an integrating factor for the first-order differential equation

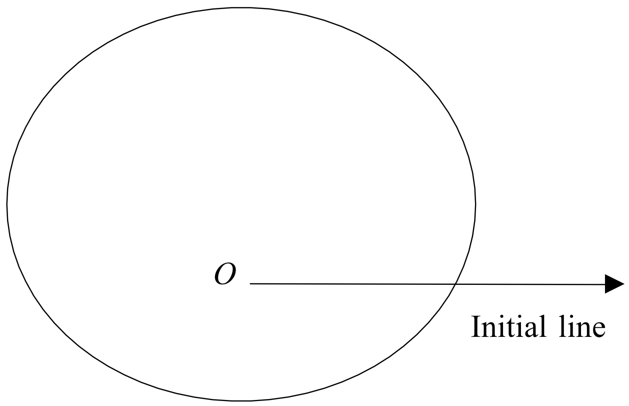
$$\frac{dy}{dx} - \frac{1}{x}y = x \ln x$$ *(3 marks)*

(ii) Solve this differential equation, given that $y = 1$ when $x = 1$. *(6 marks)*

(iii) Calculate the value of y when $x = 1.2$, giving your answer to three decimal places. *(1 mark)*

6 (a) A circle C_1 has cartesian equation $x^2 + (y - 6)^2 = 36$. Show that the polar equation of C_1 is $r = 12 \sin \theta$. *(4 marks)*

(b) A curve C_2 with polar equation $r = 2 \sin \theta + 5$, $0 \leqslant \theta \leqslant 2\pi$ is shown in the diagram.



Calculate the area bounded by C_2. *(6 marks)*

(c) The circle C_1 intersects the curve C_2 at the points P and Q. Find, in surd form, the area of the quadrilateral $OPMQ$, where M is the centre of the circle and O is the pole. *(6 marks)*

END OF QUESTIONS

Practice 2

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1. $$\frac{dy}{dx} = x^2 + 2\sin y$$

It is given that $y = 1$ at $x = 0$.

Use the approximation $\frac{y_1 - y_0}{h} = \left(\frac{dy}{dx}\right)_0$ with a step length of $h = 0.1$ to find estimates of y at $x = 0.1$ and at $x = 0.2$, giving your answers to 4 decimal places.

(5)

Leave blank

Question 1 continued

(Total 5 marks)

Q1

Turn over

Leave blank

2. Given that $y = x^3 \ln x$,

(a) find $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$.

(5)

(b) Find the Taylor series expansion of $x^3 \ln x$ in ascending powers of $(x-1)$ up to and including the term in $(x-1)^3$.

(3)

Leave blank

Question 2 continued

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Question 2 continued

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Question 2 continued

(Total 8 marks)

Q2

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3. (a) Use De Moivre's theorem to show that

$$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$$

(5)

(b) Hence find the two positive solutions of

$$32x^5 - 40x^3 + 10x + 1 = 0,$$

giving your answers to 3 decimal places.

(6)

Leave blank

Question 3 continued

Leave blank

Question 3 continued

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Question 3 continued

Q3

(Total 11 marks)

Turn over

Leave blank

4. For $n \in \mathbb{Z}^+$, show, using mathematical induction, that

(i) $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}^n = \begin{pmatrix} 1 & 0 & 0 \\ n & 1 & 0 \\ n(n+2) & 2n & 1 \end{pmatrix}$,

(5)

(ii) $(2^{3n+1} + 5)$ is divisible by 7.

(5)

Leave blank

Question 4 continued

Turn over

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Question 4 continued

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Question 4 continued

(Total 10 marks)

Q4

Leave blank

5. Given that

$$\mathbf{a} = \mathbf{i} + 7\mathbf{j} + 9\mathbf{k} \quad \text{and} \quad \mathbf{b} = -\mathbf{i} + 3\mathbf{j} + \mathbf{k},$$

(a) show that $\mathbf{a} \times \mathbf{b} = c(2\mathbf{i} + \mathbf{j} - \mathbf{k})$, and state the value of the constant c.
(2)

The plane Π_1 passes through the point (3, 1, 3) and the vector $\mathbf{a} \times \mathbf{b}$ is perpendicular to Π_1.

(b) Find a cartesian equation for the plane Π_1.
(2)

The line l_1 has equation $\mathbf{r} = \mathbf{i} - 2\mathbf{k} + \lambda\mathbf{a}$.

(c) Show that the line l_1 lies in the plane Π_1.
(2)

The line l_2 has equation $\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + \mu\mathbf{b}$.
The line l_2 lies in a plane Π_2, which is parallel to the plane Π_1.

(d) Find a cartesian equation of the plane Π_2.
(2)

(e) Find the distance between the planes Π_1 and Π_2.
(3)

Leave blank

Question 5 continued

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Question 5 continued

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Question 5 continued

(Total 11 marks)

Q5

Turn over

Leave blank

6.

$$\mathbf{M} = \begin{pmatrix} 11 & -5\sqrt{3} \\ -5\sqrt{3} & 1 \end{pmatrix}$$

Given that λ_1 and λ_2 are the eigenvalues of **M** and $\lambda_1 > \lambda_2$,

(a) show that $\lambda_1 = 16$ and find the value of λ_2 .

(4)

(b) Find eigenvectors corresponding to the eigenvalues λ_1 and λ_2 .

(4)

Given that there is an orthogonal matrix **P** such that $\mathbf{P}^{-1}\mathbf{MP}$ is the diagonal matrix **D**, where $\mathbf{D} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$,

(c) find the matrix **P**,

(2)

(d) verify that $\mathbf{P}^{-1}\mathbf{MP} = \mathbf{D}$.

(4)

Leave blank

Question 6 continued

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Question 6 continued

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Question 6 continued

Q6

(Total 14 marks)

Leave blank

7. The point P represents the complex number z in an Argand diagram.
Point P moves on the curve C given by the equation

$$|z-4+4\text{i}| = 2|z-1+\text{i}|$$

(a) Show that C is a circle whose equation may be written $|z| = k$, giving the exact value of k.
(5)

(b) Draw an Argand diagram showing the circle C and the points representing the complex numbers 1 – i and 4 – 4i.
(3)

(c) For the points on the circle C, find the maximum and minimum values of $|z-4+4\text{i}|$.
(3)

The transformation T from the z-plane to the w-plane is given by $w = z + \dfrac{8}{z}$.

(d) Show that T maps the curve C onto a line segment in the w-plane and define this line segment by giving its equation and the coordinates of its end points.
(5)

Leave blank

Question 7 continued

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Question 7 continued

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Question 7 continued

Q7

(Total 16 marks)

TOTAL FOR PAPER: 75 MARKS

END

Answer **all** questions.

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1 The function $y(x)$ satisfies the differential equation

$$\frac{dy}{dx} = f(x, y)$$

where $$f(x, y) = \ln(1 + x^2 + y)$$

and $$y(1) = 0.6$$

(a) Use the Euler formula

$$y_{r+1} = y_r + h\,f(x_r, y_r)$$

with $h = 0.05$, to obtain an approximation to $y(1.05)$, giving your answer to four decimal places. *(3 marks)*

(b) Use the improved Euler formula

$$y_{r+1} = y_r + \tfrac{1}{2}(k_1 + k_2)$$

where $k_1 = h\,f(x_r, y_r)$ and $k_2 = h\,f(x_r + h, y_r + k_1)$ and $h = 0.05$, to obtain an approximation to $y(1.05)$, giving your answer to four decimal places. *(6 marks)*

2 A curve has polar equation $r(1 - \sin\theta) = 4$. Find its cartesian equation in the form $y = f(x)$. *(6 marks)*

3 (a) Show that x^2 is an integrating factor for the first-order differential equation

$$\frac{dy}{dx} + \frac{2}{x}y = 3(x^3 + 1)^{\frac{1}{2}}$$ *(3 marks)*

(b) Solve this differential equation, given that $y = 1$ when $x = 2$. *(6 marks)*

4 (a) Explain why $\int_0^{e} \frac{\ln x}{\sqrt{x}}\,dx$ is an improper integral. *(1 mark)*

(b) Use integration by parts to find $\int x^{-\frac{1}{2}} \ln x \, dx$. *(3 marks)*

(c) Show that $\int_0^{e} \frac{\ln x}{\sqrt{x}}\,dx$ exists and find its value. *(4 marks)*

5 Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 6 + 5\sin x$$ *(12 marks)*

6 The function f is defined by $f(x) = (1+2x)^{\frac{1}{2}}$.

(a) (i) Find $f'''(x)$. *(4 marks)*

(ii) Using Maclaurin's theorem, show that, for small values of x,

$$f(x) \approx 1 + x - \tfrac{1}{2}x^2 + \tfrac{1}{2}x^3$$ *(4 marks)*

(b) Use the expansion of e^x together with the result in part (a)(ii) to show that, for small values of x,

$$e^x(1+2x)^{\frac{1}{2}} \approx 1 + 2x + x^2 + kx^3$$

where k is a rational number to be found. *(3 marks)*

(c) Write down the first four terms in the expansion, in ascending powers of x, of e^{2x}. *(1 mark)*

(d) Find

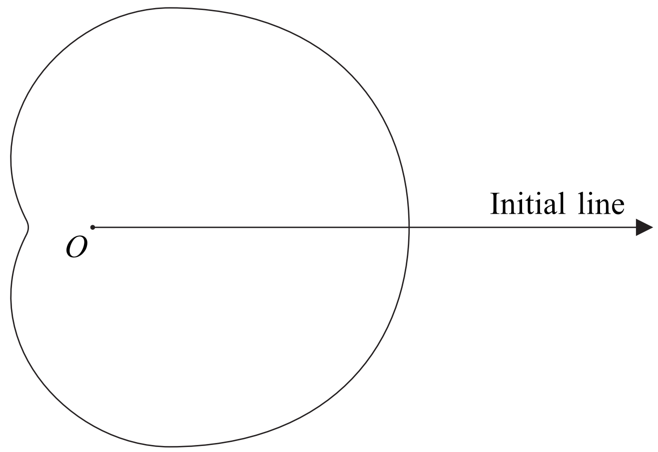
$$\lim_{x\to 0} \frac{e^x(1+2x)^{\frac{1}{2}} - e^{2x}}{1-\cos x}$$ *(4 marks)*

Turn over for the next question

7 A curve C has polar equation

$$r = 6 + 4\cos\theta, \qquad -\pi \leqslant \theta \leqslant \pi$$

The diagram shows a sketch of the curve C, the pole O and the initial line.



(a) Calculate the area of the region bounded by the curve C. *(6 marks)*

(b) The point P is the point on the curve C for which $\theta = \frac{2\pi}{3}$.

The point Q is the point on C for which $\theta = \pi$.

Show that QP is parallel to the line $\theta = \frac{\pi}{2}$. *(4 marks)*

(c) The line PQ intersects the curve C again at a point R.

The line RO intersects C again at a point S.

(i) Find, in surd form, the length of PS. *(4 marks)*

(ii) Show that the angle OPS is a right angle. *(1 mark)*

END OF QUESTIONS

Practice 4

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1. The variable y satisfies the differential equation

$$\frac{dy}{dx} = x + \cos y.$$

It is given that $y = 0.6$ at $x = 0$.

Use the approximation $\left(\frac{dy}{dx}\right)_0 \approx \frac{y_1 - y_0}{h}$, with a step length of 0.05, to estimate the values of y at $x = 0.05$ and $x = 0.1$, giving your answers to four decimal places.

(6)

Leave blank

Question 1 continued

Q1

(Total 6 marks)

Turn over

Leave blank

2.

$$\mathbf{M} = \begin{pmatrix} 1 & p & 2 \\ 0 & 3 & q \\ 2 & p & 1 \end{pmatrix},$$

where p and q are constants.

Given that $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ is an eigenvector of $\mathbf{M}$,

(a) show that $q = 4p$.

(3)

Given also that $\lambda = 5$ is an eigenvalue of $\mathbf{M}$, and $p < 0$ and $q < 0$, find

(b) the values of p and q,

(4)

(c) an eigenvector corresponding to the eigenvalue $\lambda = 5$.

(3)

Leave blank

Question 2 continued

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Question 2 continued

Leave blank

Question 2 continued

Q2

(Total 10 marks)

Turn over

Leave blank

3.

$$(x^2+1)\frac{d^2y}{dx^2}=2y^2+(1-2x)\frac{dy}{dx}. \qquad \text{(I)}$$

(a) By differentiating equation (I) with respect to x, show that

$$(x^2+1)\frac{d^3y}{dx^3}=(1-4x)\frac{d^2y}{dx^2}+(4y-2)\frac{dy}{dx}.$$

(3)

Given that $y = 1$ and $\frac{dy}{dx} = 1$ at $x = 0$,

(b) find the series solution for y, in ascending powers of x, up to and including the term in x^3.

(4)

(c) Use your series to estimate the value of y at $x = -0.5$, giving your answer to two decimal places.

(1)

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Q3

(Total 8 marks)

Leave blank

4. The point P represents a complex number z on an Argand diagram such that

$$|z - 3| = 2|z|.$$

(a) Show that, as z varies, the locus of P is a circle, and give the coordinates of the centre and the radius of the circle.
(5)

The point Q represents a complex number z on an Argand diagram such that

$$|z + 3| = |z - \mathrm{i}\sqrt{3}|.$$

(b) Sketch, on the same Argand diagram, the locus of P and the locus of Q as z varies.
(5)

(c) On your diagram shade the region which satisfies

$$|z - 3| \geqslant 2|z| \quad \text{and} \quad |z + 3| \geqslant |z - \mathrm{i}\sqrt{3}|.$$

(2)

Leave blank

Question 4 continued

Turn over

Leave blank

Question 4 continued

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Question 4 continued

Q4

(Total 12 marks)

Turn over

Leave blank

5. $$\mathbf{A} = \begin{pmatrix} k & -2 \\ 1-k & k \end{pmatrix}, \text{ where } k \text{ is constant.}$$

A transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ is represented by the matrix $\mathbf{A}$.

(a) Find the value of k for which the line $y = 2x$ is mapped onto itself under T.
(3)

(b) Show that $\mathbf{A}$ is non-singular for all values of k.
(3)

(c) Find $\mathbf{A}^{-1}$ in terms of k.
(2)

A point P is mapped onto a point Q under T.

The point Q has position vector $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$ relative to an origin O.

Given that $k = 3$,

(d) find the position vector of P.
(3)

Leave blank

Question 5 continued

Turn over

Leave blank

Question 5 continued

Leave blank

Question 5 continued

(Total 11 marks)

Q5

Leave blank

6. De Moivre's theorem states that

$$(\cos\theta + \mathrm{i}\sin\theta)^n = \cos n\theta + \mathrm{i}\sin n\theta \text{ for } n \in \mathbb{R}.$$

(a) Use induction to prove de Moivre's theorem for $n \in \mathbb{Z}^+$.

(5)

(b) Show that

$$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta .$$

(5)

(c) Hence show that $2\cos\frac{\pi}{10}$ is a root of the equation

$$x^4 - 5x^2 + 5 = 0 .$$

(3)

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Question 6 continued

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Question 6 continued

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Question 6 continued

Q6

(Total 13 marks)

Turn over

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7.

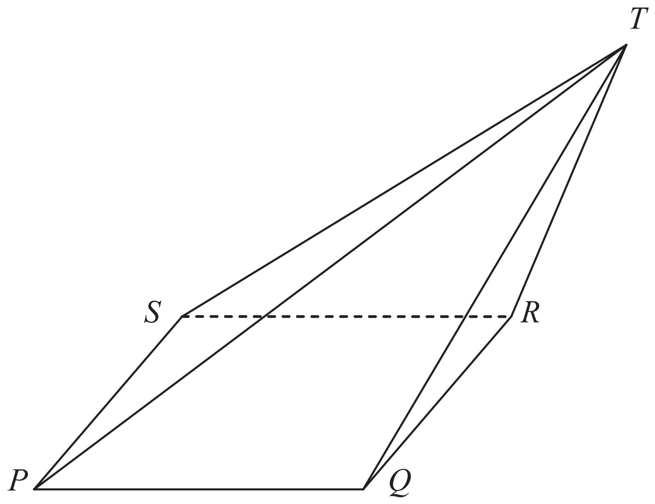

Figure 1

Figure 1 shows a pyramid *PQRST* with base *PQRS*.

The coordinates of *P*, *Q* and *R* are *P* (1, 0, –1), *Q* (2, –1, 1) and *R* (3, –3, 2).

Find

(a) $\overrightarrow{PQ} \times \overrightarrow{PR}$,

(3)

(b) a vector equation for the plane containing the face *PQRS*, giving your answer in the form **r . n** = *d*.

(2)

The plane *Π* contains the face *PST*. The vector equation of *Π* is **r . (i** – 2**j** – 5**k)** = 6.

(c) Find cartesian equations of the line through *P* and *S*.

(5)

(d) Hence show that *PS* is parallel to *QR*.

(2)

Given that *PQRS* is a parallelogram and that *T* has coordinates (5, 2, –1),

(e) find the volume of the pyramid *PQRST*.

(3)

Leave blank

Question 7 continued

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Question 7 continued

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Question 7 continued

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Question 7 continued

Q7

(Total 15 marks)

TOTAL FOR PAPER: 75 MARKS

END

Answer **all** questions.

1 The function $y(x)$ satisfies the differential equation

$$\frac{dy}{dx} = f(x, y)$$

where $$f(x, y) = x^2 - y^2$$

and $$y(2) = 1$$

(a) Use the Euler formula

$$y_{r+1} = y_r + hf(x_r, y_r)$$

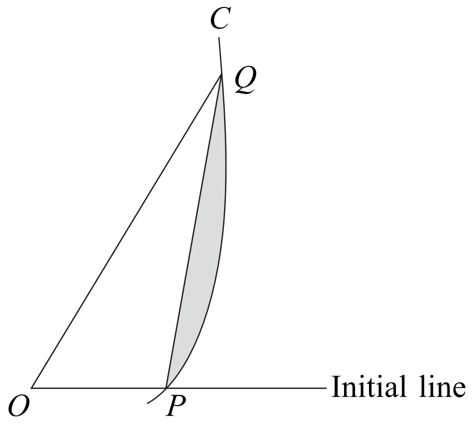
with $h = 0.1$, to obtain an approximation to $y(2.1)$. *(3 marks)*

(b) Use the formula

$$y_{r+1} = y_{r-1} + 2hf(x_r, y_r)$$

with your answer to part (a), to obtain an approximation to $y(2.2)$. *(3 marks)*

2 The diagram shows a sketch of part of the curve C whose polar equation is $r = 1 + \tan\theta$. The point O is the pole.



The points P and Q on the curve are given by $\theta = 0$ and $\theta = \frac{\pi}{3}$ respectively.

(a) Show that the area of the region bounded by the curve C and the lines OP and OQ is

$$\frac{1}{2}\sqrt{3} + \ln 2$$ *(6 marks)*

(b) Hence find the area of the shaded region bounded by the line PQ and the arc PQ of C. *(3 marks)*

3 (a) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 5$$ *(6 marks)*

(b) Hence express y in terms of x, given that $y = 2$ and $\frac{dy}{dx} = 3$ when $x = 0$. *(4 marks)*

4 (a) Explain why $\int_1^{\infty} xe^{-3x}\,dx$ is an improper integral. *(1 mark)*

(b) Find $\int xe^{-3x}\,dx$. *(3 marks)*

(c) Hence evaluate $\int_1^{\infty} xe^{-3x}\,dx$, showing the limiting process used. *(3 marks)*

5 By using an integrating factor, find the solution of the differential equation

$$\frac{dy}{dx} + \frac{4x}{x^2+1}\,y = x$$

given that $y = 1$ when $x = 0$. Give your answer in the form $y = f(x)$. *(9 marks)*

6 A curve C has polar equation

$$r^2 \sin 2\theta = 8$$

(a) Find the cartesian equation of C in the form $y = f(x)$. *(3 marks)*

(b) Sketch the curve C. *(1 mark)*

(c) The line with polar equation $r = 2\sec\theta$ intersects C at the point A. Find the polar coordinates of A. *(4 marks)*

7 (a) (i) Write down the expansion of $\ln(1+2x)$ in ascending powers of x up to and including the term in x^3. *(2 marks)*

(ii) State the range of values of x for which this expansion is valid. *(1 mark)*

(b) (i) Given that $y = \ln\cos x$, find $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$. *(4 marks)*

(ii) Find the value of $\frac{d^4y}{dx^4}$ when $x = 0$. *(3 marks)*

(iii) Hence, by using Maclaurin's theorem, show that the first two non-zero terms in the expansion, in ascending powers of x, of $\ln\cos x$ are

$$-\frac{x^2}{2} - \frac{x^4}{12}$$ *(2 marks)*

(c) Find

$$\lim_{x\to 0}\left[\frac{x\ln(1+2x)}{x^2 - \ln\cos x}\right]$$ *(3 marks)*

8 (a) Given that $x = \mathrm{e}^t$ and that y is a function of x, show that:

(i) $x\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}y}{\mathrm{d}t}$; *(3 marks)*

(ii) $x^2\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = \dfrac{\mathrm{d}^2y}{\mathrm{d}t^2} - \dfrac{\mathrm{d}y}{\mathrm{d}t}$. *(3 marks)*

(b) Hence find the general solution of the differential equation

$$x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 6x\frac{\mathrm{d}y}{\mathrm{d}x} + 6y = 0$$ *(5 marks)*

END OF QUESTIONS

Answer **all** questions.

1 The function $y(x)$ satisfies the differential equation

$$\frac{dy}{dx} = f(x, y)$$

where $$f(x, y) = \frac{x^2 + y^2}{x + y}$$

and $$y(1) = 3$$

(a) Use the Euler formula

$$y_{r+1} = y_r + hf(x_r, y_r)$$

with $h = 0.2$, to obtain an approximation to $y(1.2)$. *(3 marks)*

(b) Use the improved Euler formula

$$y_{r+1} = y_r + \frac{1}{2}(k_1 + k_2)$$

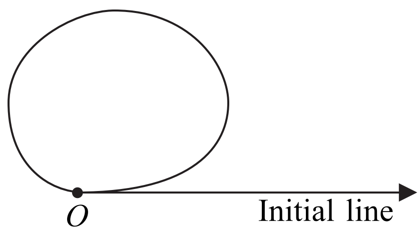
where $k_1 = hf(x_r, y_r)$ and $k_2 = hf(x_r + h, y_r + k_1)$ and $h = 0.2$, to obtain an approximation to $y(1.2)$, giving your answer to four decimal places. *(5 marks)*

2 (a) Show that $\frac{1}{x^2}$ is an integrating factor for the first-order differential equation

$$\frac{dy}{dx} - \frac{2}{x}y = x$$ *(3 marks)*

(b) Hence find the general solution of this differential equation, giving your answer in the form $y = f(x)$. *(4 marks)*

3 The diagram shows a sketch of a loop, the pole O and the initial line.



The polar equation of the loop is

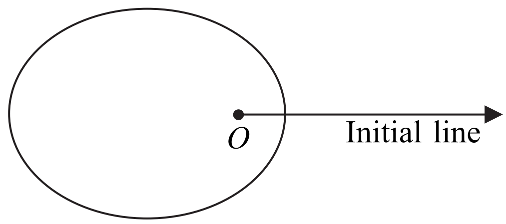
$$r = (2 + \cos\theta)\sqrt{\sin\theta}, \quad 0 \leqslant \theta \leqslant \pi$$

Find the area enclosed by the loop. *(6 marks)*

4 (a) Use integration by parts to show that $\int \ln x \, \mathrm{d}x = x\ln x - x + c$, where c is an arbitrary constant. *(2 marks)*

(b) Hence evaluate $\int_0^1 \ln x \, \mathrm{d}x$, showing the limiting process used. *(4 marks)*

5 The diagram shows a sketch of a curve C, the pole O and the initial line.



The curve C has polar equation

$$r = \frac{2}{3 + 2\cos\theta}, \quad 0 \leqslant \theta \leqslant 2\pi$$

(a) Verify that the point L with polar coordinates $(2, \pi)$ lies on C. *(1 mark)*

(b) The circle with polar equation $r = 1$ intersects C at the points M and N.

(i) Find the polar coordinates of M and N. *(3 marks)*

(ii) Find the area of triangle LMN. *(4 marks)*

(c) Find a cartesian equation of C, giving your answer in the form $9y^2 = \mathrm{f}(x)$. *(5 marks)*

Turn over for the next question

6 The function f is defined by $f(x) = e^{2x}(1+3x)^{-\frac{2}{3}}$.

(a) (i) Use the series expansion for e^x to write down the first four terms in the series expansion of e^{2x}. *(2 marks)*

(ii) Use the binomial series expansion of $(1+3x)^{-\frac{2}{3}}$ and your answer to part (a)(i) to show that the first three non-zero terms in the series expansion of $f(x)$ are $1 + 3x^2 - 6x^3$. *(5 marks)*

(b) (i) Given that $y = \ln(1 + 2\sin x)$, find $\dfrac{d^2y}{dx^2}$. *(4 marks)*

(ii) By using Maclaurin's theorem, show that, for small values of x,

$$\ln(1 + 2\sin x) \approx 2x - 2x^2$$ *(2 marks)*

(c) Find

$$\lim_{x \to 0} \frac{1 - f(x)}{x \ln(1 + 2\sin x)}$$ *(3 marks)*

7 (a) Given that $x = e^t$ and that y is a function of x, show that

$$x^2 \frac{d^2y}{dx^2} = \frac{d^2y}{dt^2} - \frac{dy}{dt}$$ *(7 marks)*

(b) Hence show that the substitution $x = e^t$ transforms the differential equation

$$x^2 \frac{d^2y}{dx^2} - 4x\frac{dy}{dx} = 10$$

into

$$\frac{d^2y}{dt^2} - 5\frac{dy}{dt} = 10$$ *(2 marks)*

(c) Find the general solution of the differential equation $\dfrac{d^2y}{dt^2} - 5\dfrac{dy}{dt} = 10$. *(5 marks)*

(d) Hence solve the differential equation $x^2 \dfrac{d^2y}{dx^2} - 4x\dfrac{dy}{dx} = 10$, given that $y = 0$ and $\dfrac{dy}{dx} = 8$ when $x = 1$. *(5 marks)*

END OF QUESTIONS