Answer **all** questions.

**1** (a) The polynomial $f(x)$ is defined by $f(x) = 3x^3 + 2x^2 - 7x + 2$.

(i) Find $f(1)$. *(1 mark)*

(ii) Show that $f(-2) = 0$. *(1 mark)*

(iii) Hence, or otherwise, show that

$$\frac{(x-1)(x+2)}{3x^3 + 2x^2 - 7x + 2} = \frac{1}{ax+b}$$

where $a$ and $b$ are integers. *(3 marks)*

(b) The polynomial $g(x)$ is defined by $g(x) = 3x^3 + 2x^2 - 7x + d$.

When $g(x)$ is divided by $(3x - 1)$, the remainder is 2. Find the value of $d$. *(3 marks)*

**2** A curve is defined by the parametric equations

$$x = 3 - 4t \qquad y = 1 + \frac{2}{t}$$

(a) Find $\frac{dy}{dx}$ in terms of $t$. *(4 marks)*

(b) Find the equation of the tangent to the curve at the point where $t = 2$, giving your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers. *(4 marks)*

(c) Verify that the cartesian equation of the curve can be written as

$$(x-3)(y-1) + 8 = 0$$ *(3 marks)*

**3** It is given that $3\cos\theta - 2\sin\theta = R\cos(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$.

(a) Find the value of $R$. *(1 mark)*

(b) Show that $\alpha \approx 33.7°$. *(2 marks)*

(c) Hence write down the maximum value of $3\cos\theta - 2\sin\theta$ and find a **positive** value of $\theta$ at which this maximum value occurs. *(3 marks)*

**4** On 1 January 1900, a sculpture was valued at £80.

When the sculpture was sold on 1 January 1956, its value was £5000.

The value, £$V$, of the sculpture is modelled by the formula $V = A k^t$, where $t$ is the time in years since 1 January 1900 and $A$ and $k$ are constants.

(a) Write down the value of $A$. *(1 mark)*

(b) Show that $k \approx 1.07664$. *(3 marks)*

(c) Use this model to:

(i) show that the value of the sculpture on 1 January 2006 will be greater than £200 000; *(2 marks)*

(ii) find the year in which the value of the sculpture will first exceed £800 000. *(3 marks)*

**5** (a) (i) Obtain the binomial expansion of $(1-x)^{-1}$ up to and including the term in $x^2$. *(2 marks)*

(ii) Hence, or otherwise, show that

$$\frac{1}{3-2x} \approx \frac{1}{3} + \frac{2}{9}x + \frac{4}{27}x^2$$

for small values of $x$. *(3 marks)*

(b) Obtain the binomial expansion of $\dfrac{1}{(1-x)^2}$ up to and including the term in $x^2$. *(2 marks)*

(c) Given that $\dfrac{2x^2-3}{(3-2x)(1-x)^2}$ can be written in the form $\dfrac{A}{(3-2x)} + \dfrac{B}{(1-x)} + \dfrac{C}{(1-x)^2}$, find the values of $A$, $B$ and $C$. *(5 marks)*

(d) Hence find the binomial expansion of $\dfrac{2x^2-3}{(3-2x)(1-x)^2}$ up to and including the term in $x^2$. *(3 marks)*

**Turn over for the next question**

**6** (a) Express $\cos 2x$ in the form $a\cos^2 x + b$, where $a$ and $b$ are constants. *(2 marks)*

(b) Hence show that $\int_0^{\frac{\pi}{2}} \cos^2 x \, dx = \frac{\pi}{a}$, where $a$ is an integer. *(5 marks)*

**7** The quadrilateral $ABCD$ has vertices $A(2, 1, 3)$, $B(6, 5, 3)$, $C(6, 1, -1)$ and $D(2, -3, -1)$.

The line $l_1$ has vector equation $\mathbf{r} = \begin{bmatrix} 6 \\ 1 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$.

(a) (i) Find the vector $\overline{AB}$. *(2 marks)*

(ii) Show that the line $AB$ is parallel to $l_1$. *(1 mark)*

(iii) Verify that $D$ lies on $l_1$. *(2 marks)*

(b) The line $l_2$ passes through $D(2, -3, -1)$ and $M(4, 1, 1)$.

(i) Find the vector equation of $l_2$. *(2 marks)*

(ii) Find the angle between $l_2$ and $AC$. *(3 marks)*

**8** (a) Solve the differential equation

$$\frac{dx}{dt} = -2(x-6)^{\frac{1}{2}}$$

to find $t$ in terms of $x$, given that $x = 70$ when $t = 0$. *(6 marks)*

(b) Liquid fuel is stored in a tank. At time $t$ minutes, the depth of fuel in the tank is $x$ cm. Initially there is a depth of 70 cm of fuel in the tank. There is a tap 6 cm above the bottom of the tank. The flow of fuel out of the tank is modelled by the differential equation

$$\frac{dx}{dt} = -2(x-6)^{\frac{1}{2}}$$

(i) Explain what happens when $x = 6$. *(1 mark)*

(ii) Find how long it will take for the depth of fuel to fall from 70 cm to 22 cm. *(2 marks)*

**END OF QUESTIONS**

**Practice 2**

Leave blank

**1.**

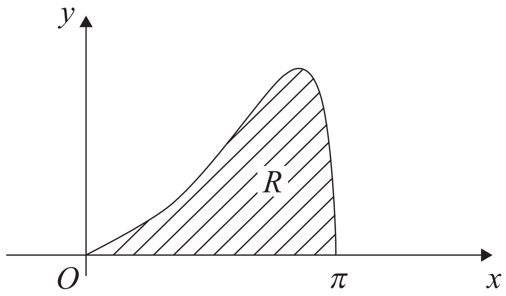


**Figure 1**

The curve shown in Figure 1 has equation $y = e^x\sqrt{(\sin x)}$ , $0 \leqslant x \leqslant \pi$ . The finite region $R$ bounded by the curve and the $x$-axis is shown shaded in Figure 1.

(a) Complete the table below with the values of $y$ corresponding to $x = \frac{\pi}{4}$ and $\frac{\pi}{2}$, giving your answers to 5 decimal places.

| $x$ | 0 | $\frac{\pi}{4}$ | $\frac{\pi}{2}$ | $\frac{3\pi}{4}$ | $\pi$ |
|---|---|---|---|---|---|
| $y$ | 0 | | | 8.87207 | 0 |

**(2)**

(b) Use the trapezium rule, with all the values in the completed table, to obtain an estimate for the area of the region $R$. Give your answer to 4 decimal places.

**(4)**

Leave blank

**Question 1 continued**

**(Total 6 marks)**

**Q1**

Leave blank

**2.** (a) Use the binomial theorem to expand

$$(8-3x)^{\frac{1}{3}}, \qquad |x| < \tfrac{8}{3},$$

in ascending powers of $x$, up to and including the term in $x^3$, giving each term as a simplified fraction.

**(5)**

(b) Use your expansion, with a suitable value of $x$, to obtain an approximation to $\sqrt[3]{(7.7)}$. Give your answer to 7 decimal places.

**(2)**

Leave blank

**Question 2 continued**

Q2

**(Total 7 marks)**

Leave blank

**3.**

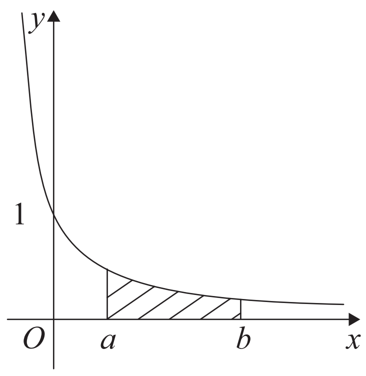


**Figure 2**

The curve shown in Figure 2 has equation $y = \frac{1}{(2x+1)}$. The finite region bounded by the curve, the $x$-axis and the lines $x = a$ and $x = b$ is shown shaded in Figure 2. This region is rotated through 360° about the $x$-axis to generate a solid of revolution.

Find the volume of the solid generated. Express your answer as a single simplified fraction, in terms of $a$ and $b$.

**(5)**

Leave blank

**Question 3 continued**

Q3

**(Total 5 marks)**



**Turn over**

Leave blank

**4.** (i) Find $\int \ln(\frac{x}{2})\,dx$ .

**(4)**

(ii) Find the exact value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x\,dx$.

**(5)**

Leave blank

**Question 4 continued**

**(Total 9 marks)**

Q4

Leave blank

**5.** A curve is described by the equation

$$x^3 - 4y^2 = 12xy.$$

(a) Find the coordinates of the two points on the curve where $x = -8$.
**(3)**

(b) Find the gradient of the curve at each of these points.
**(6)**

Leave blank

**Question 5 continued**

Q5

**(Total 9 marks)**



**Turn over**

Leave blank

**6.** The points $A$ and $B$ have position vectors $2\mathbf{i} + 6\mathbf{j} - \mathbf{k}$ and $3\mathbf{i} + 4\mathbf{j} + \mathbf{k}$ respectively.

The line $l_1$ passes through the points $A$ and $B$.

(a) Find the vector $\overrightarrow{AB}$.

**(2)**

(b) Find a vector equation for the line $l_1$.

**(2)**

A second line $l_2$ passes through the origin and is parallel to the vector $\mathbf{i} + \mathbf{k}$. The line $l_1$ meets the line $l_2$ at the point $C$.

(c) Find the acute angle between $l_1$ and $l_2$.

**(3)**

(d) Find the position vector of the point $C$.

**(4)**

Leave blank

**Question 6 continued**



Turn over

Leave blank

**Question 6 continued**

Leave blank

**Question 6 continued**

Q6

**(Total 11 marks)**

Leave blank

**7.**

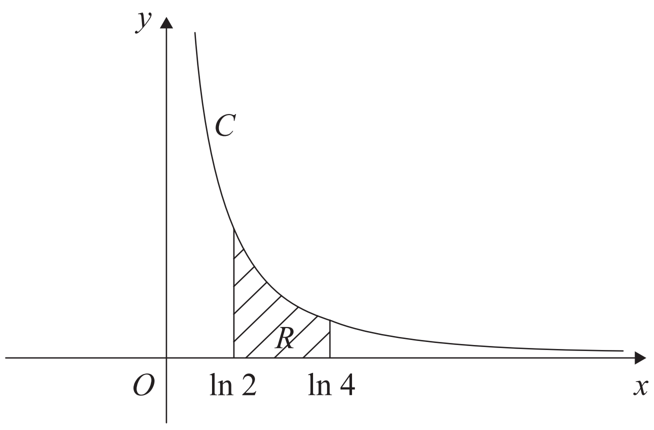


**Figure 3**

The curve $C$ has parametric equations

$$x = \ln(t+2), \quad y = \frac{1}{(t+1)}, \quad t > -1.$$

The finite region $R$ between the curve $C$ and the $x$-axis, bounded by the lines with equations $x = \ln 2$ and $x = \ln 4$, is shown shaded in Figure 3.

(a) Show that the area of $R$ is given by the integral

$$\int_0^2 \frac{1}{(t+1)(t+2)}\,\mathrm{d}t.$$

**(4)**

(b) Hence find an exact value for this area.

**(6)**

(c) Find a cartesian equation of the curve $C$, in the form $y = \mathrm{f}(x)$.

**(4)**

(d) State the domain of values for $x$ for this curve.

**(1)**

Leave blank

**Question 7 continued**

Leave blank

**Question 7 continued**

Leave blank

**Question 7 continued**

**(Total 15 marks)**

**Q7**

Leave blank

**8.** Liquid is pouring into a large vertical circular cylinder at a constant rate of 1600 $cm^3 s^{-1}$ and is leaking out of a hole in the base, at a rate proportional to the square root of the height of the liquid already in the cylinder. The area of the circular cross section of the cylinder is 4000 $cm^2$.

(a) Show that at time $t$ seconds, the height $h$ cm of liquid in the cylinder satisfies the differential equation

$$\frac{dh}{dt} = 0.4 - k\sqrt{h}, \text{ where } k \text{ is a positive constant.}$$

**(3)**

When $h = 25$, water is leaking out of the hole at 400 $cm^3 s^{-1}$.

(b) Show that $k = 0.02$

**(1)**

(c) Separate the variables of the differential equation

$$\frac{dh}{dt} = 0.4 - 0.02\sqrt{h},$$

to show that the time taken to fill the cylinder from empty to a height of 100 cm is given by

$$\int_0^{100} \frac{50}{20 - \sqrt{h}}\, dh.$$

**(2)**

Using the substitution $h = (20 - x)^2$, or otherwise,

(d) find the exact value of $\int_0^{100} \frac{50}{20 - \sqrt{h}}\, dh$.

**(6)**

(e) Hence find the time taken to fill the cylinder from empty to a height of 100 cm, giving your answer in minutes and seconds to the nearest second.

**(1)**

Leave blank

**Question 8 continued**



**Turn over**

Leave blank

**Question 8 continued**

Leave blank

**Question 8 continued**

Q8

**(Total 13 marks)**

**TOTAL FOR PAPER: 75 MARKS**

**END**

Answer **all** questions.

**1** A curve is defined by the parametric equations

$$x = 1 + 2t, \quad y = 1 - 4t^2$$

(a) (i) Find $\frac{dx}{dt}$ and $\frac{dy}{dt}$. *(2 marks)*

(ii) Hence find $\frac{dy}{dx}$ in terms of $t$. *(2 marks)*

(b) Find an equation of the normal to the curve at the point where $t = 1$. *(4 marks)*

(c) Find a cartesian equation of the curve. *(3 marks)*

**2** The polynomial $f(x)$ is defined by $f(x) = 2x^3 - 7x^2 + 13$.

(a) Use the Remainder Theorem to find the remainder when $f(x)$ is divided by $(2x - 3)$. *(2 marks)*

(b) The polynomial $g(x)$ is defined by $g(x) = 2x^3 - 7x^2 + 13 + d$, where $d$ is a constant.

Given that $(2x - 3)$ is a factor of $g(x)$, show that $d = -4$. *(2 marks)*

(c) Express $g(x)$ in the form $(2x - 3)(x^2 + ax + b)$. *(2 marks)*

**3** (a) Express $\cos 2x$ in terms of $\sin x$. *(1 mark)*

(b) (i) Hence show that $3\sin x - \cos 2x = 2\sin^2 x + 3\sin x - 1$ for all values of $x$. *(2 marks)*

(ii) Solve the equation $3\sin x - \cos 2x = 1$ for $0° < x < 360°$. *(4 marks)*

(c) Use your answer from part (a) to find $\int \sin^2 x \, dx$. *(2 marks)*

**4** (a) (i) Express $\dfrac{3x-5}{x-3}$ in the form $A+\dfrac{B}{x-3}$, where $A$ and $B$ are integers. *(2 marks)*

(ii) Hence find $\displaystyle\int \frac{3x-5}{x-3}\,\mathrm{d}x$. *(2 marks)*

(b) (i) Express $\dfrac{6x-5}{4x^2-25}$ in the form $\dfrac{P}{2x+5}+\dfrac{Q}{2x-5}$, where $P$ and $Q$ are integers. *(3 marks)*

(ii) Hence find $\displaystyle\int \frac{6x-5}{4x^2-25}\,\mathrm{d}x$. *(3 marks)*

**5** (a) Find the binomial expansion of $(1+x)^{\frac{1}{3}}$ up to the term in $x^2$. *(2 marks)*

(b) (i) Show that $(8+3x)^{\frac{1}{3}} \approx 2+\frac{1}{4}x-\frac{1}{32}x^2$ for small values of $x$. *(3 marks)*

(ii) Hence show that $\sqrt[3]{9} \approx \frac{599}{288}$. *(2 marks)*

**6** The points $A$, $B$ and $C$ have coordinates $(3,-2,4)$, $(5,4,0)$ and $(11,6,-4)$ respectively.

(a) (i) Find the vector $\overrightarrow{BA}$. *(2 marks)*

(ii) Show that the size of angle $ABC$ is $\cos^{-1}\left(-\frac{5}{7}\right)$. *(5 marks)*

(b) The line $l$ has equation $\mathbf{r}=\begin{bmatrix}8\\-3\\2\end{bmatrix}+\lambda\begin{bmatrix}1\\3\\-2\end{bmatrix}$.

(i) Verify that $C$ lies on $l$. *(2 marks)*

(ii) Show that $AB$ is parallel to $l$. *(1 mark)*

(c) The quadrilateral $ABCD$ is a parallelogram. Find the coordinates of $D$. *(3 marks)*

**Turn over for the next question**

**7** (a) Use the identity

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

to express $\tan 2x$ in terms of $\tan x$. *(2 marks)*

(b) Show that

$$2 - 2\tan x - \frac{2\tan x}{\tan 2x} = (1 - \tan x)^2$$

for all values of $x$, $\tan 2x \neq 0$. *(4 marks)*

**8** (a) (i) Solve the differential equation $\frac{dy}{dt} = y \sin t$ to obtain $y$ in terms of $t$. *(4 marks)*

(ii) Given that $y = 50$ when $t = \pi$, show that $y = 50e^{-(1+\cos t)}$. *(3 marks)*

(b) A wave machine at a leisure pool produces waves. The height of the water, $y$ cm, above a fixed point at time $t$ seconds is given by the differential equation

$$\frac{dy}{dt} = y \sin t$$

(i) Given that this height is 50 cm after $\pi$ seconds, find, to the nearest centimetre, the height of the water after 6 seconds. *(2 marks)*

(ii) Find $\frac{d^2y}{dt^2}$ and hence verify that the water reaches a maximum height after $\pi$ seconds. *(4 marks)*

**END OF QUESTIONS**

**Practice 4**

Leave blank

**1.** A curve $C$ has the equation $y^2 - 3y = x^3 + 8$.

(a) Find $\frac{dy}{dx}$ in terms of $x$ and $y$.

**(4)**

(b) Hence find the gradient of $C$ at the point where $y = 3$.

**(3)**

Leave blank

**Question 1 continued**

Q1

**(Total 7 marks)**



**Turn over**

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**2.**

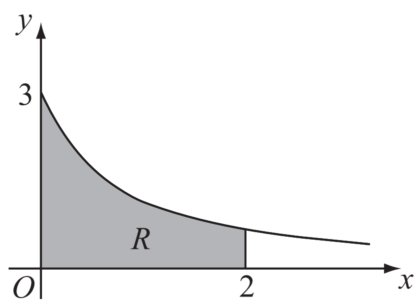


**Figure 1**

Figure 1 shows part of the curve $y = \dfrac{3}{\sqrt{(1+4x)}}$. The region $R$ is bounded by the curve, the $x$-axis, and the lines $x = 0$ and $x = 2$, as shown shaded in Figure 1.

(a) Use integration to find the area of $R$.

**(4)**

The region $R$ is rotated 360° about the $x$-axis.

(b) Use integration to find the exact value of the volume of the solid formed.

**(5)**

Leave blank

**Question 2 continued**

Leave blank

**Question 2 continued**

Leave blank

**Question 2 continued**

**(Total 9 marks)**

**Q2**

Leave blank

**3.**
$$f(x)=\frac{27x^2+32x+16}{(3x+2)^2(1-x)}, \quad |x|<\frac{2}{3}$$

Given that f($x$) can be expressed in the form

$$f(x)=\frac{A}{(3x+2)}+\frac{B}{(3x+2)^2}+\frac{C}{(1-x)},$$

(a) find the values of $B$ and $C$ and show that $A$ = 0.

**(4)**

(b) Hence, or otherwise, find the series expansion of f($x$), in ascending powers of $x$, up to and including the term in $x^2$. Simplify each term.

**(6)**

(c) Find the percentage error made in using the series expansion in part (b) to estimate the value of f(0.2). Give your answer to 2 significant figures.

**(4)**

Leave blank

**Question 3 continued**

Leave blank

**Question 3 continued**

Leave blank

**Question 3 continued**

Q3

**(Total 14 marks)**



**Turn over**

Leave blank

**4.** With respect to a fixed origin $O$ the lines $l_1$ and $l_2$ are given by the equations

$$l_1: \quad \mathbf{r} = \begin{pmatrix} 11 \\ 2 \\ 17 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} \qquad l_2: \quad \mathbf{r} = \begin{pmatrix} -5 \\ 11 \\ p \end{pmatrix} + \mu \begin{pmatrix} q \\ 2 \\ 2 \end{pmatrix}$$

where $\lambda$ and $\mu$ are parameters and $p$ and $q$ are constants. Given that $l_1$ and $l_2$ are perpendicular,

(a) show that $q = -3$.
**(2)**

Given further that $l_1$ and $l_2$ intersect, find

(b) the value of $p$,
**(6)**

(c) the coordinates of the point of intersection.
**(2)**

The point $A$ lies on $l_1$ and has position vector $\begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix}$. The point $C$ lies on $l_2$.

Given that a circle, with centre $C$, cuts the line $l_1$ at the points $A$ and $B$,

(d) find the position vector of $B$.
**(3)**

Leave blank

**Question 4 continued**



**Turn over**

Leave blank

**Question 4 continued**

Leave blank

**Question 4 continued**

**(Total 13 marks)**

**Q4**

Leave blank

**5.**

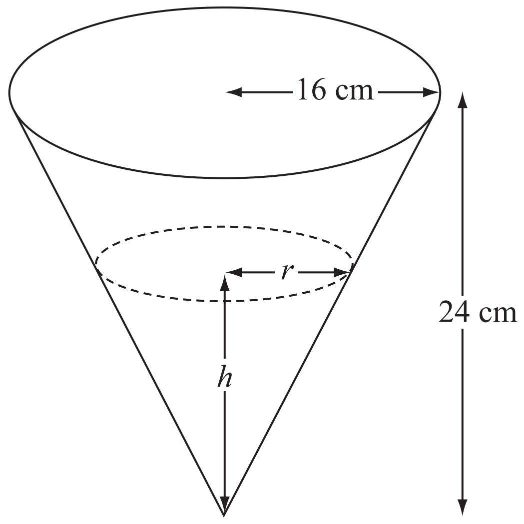


**Figure 2**

A container is made in the shape of a hollow inverted right circular cone. The height of the container is 24 cm and the radius is 16 cm, as shown in Figure 2. Water is flowing into the container. When the height of water is $h$ cm, the surface of the water has radius $r$ cm and the volume of water is $V$ cm$^3$.

(a) Show that $V = \dfrac{4\pi h^3}{27}$.

**(2)**

[*The volume V of a right circular cone with vertical height h and base radius r is given by the formula* $V = \dfrac{1}{3}\pi r^2 h$ .]

Water flows into the container at a rate of 8 cm$^3$ s$^{-1}$.

(b) Find, in terms of $\pi$, the rate of change of $h$ when $h = 12$.

**(5)**

Leave blank

**Question 5 continued**



**Turn over**

Leave blank

**Question 5 continued**

Leave blank

**Question 5 continued**

**(Total 7 marks)**

**Q5**



**Turn over**

Leave blank

**6.** (a) Find $\int \tan^2 x \, dx$.

**(2)**

(b) Use integration by parts to find $\int \frac{1}{x^3} \ln x \, dx$.

**(4)**

(c) Use the substitution $u = 1 + e^x$ to show that

$$\int \frac{e^{3x}}{1+e^x} \, dx = \frac{1}{2}e^{2x} - e^x + \ln(1+e^x) + k,$$

where $k$ is a constant.

**(7)**

Leave blank

**Question 6 continued**

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**Question 6 continued**

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**Question 6 continued**

Q6

**(Total 13 marks)**



**Turn over**

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**7.**

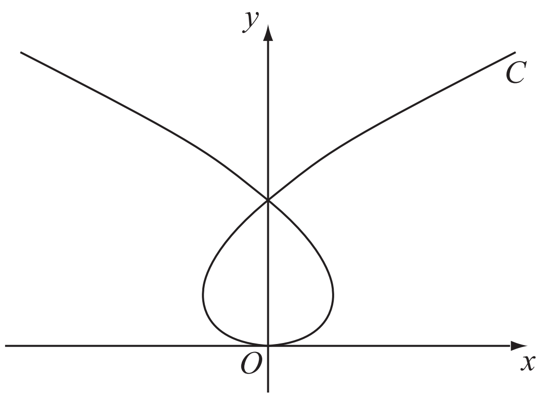


**Figure 3**

The curve $C$ shown in Figure 3 has parametric equations

$$x = t^3 - 8t, \quad y = t^2$$

where $t$ is a parameter. Given that the point $A$ has parameter $t = -1$,

(a) find the coordinates of $A$.

**(1)**

The line $l$ is the tangent to $C$ at $A$.

(b) Show that an equation for $l$ is $2x - 5y - 9 = 0$.

**(5)**

The line $l$ also intersects the curve at the point $B$.

(c) Find the coordinates of $B$.

**(6)**

Leave blank

**Question 7 continued**



**Turn over**

Leave blank

**Question 7 continued**

Leave blank

**Question 7 continued**

Q7

**(Total 12 marks)**

**TOTAL FOR PAPER: 75 MARKS**

**END**

Answer **all** questions.

**1** (a) Given that $\dfrac{3}{9-x^2}$ can be expressed in the form $k\left(\dfrac{1}{3+x}+\dfrac{1}{3-x}\right)$, find the value of the rational number $k$. *(2 marks)*

(b) Show that $\displaystyle\int_1^2 \frac{3}{9-x^2}\,dx = \frac{1}{2}\ln\left(\frac{a}{b}\right)$, where $a$ and $b$ are integers. *(3 marks)*

**2** (a) The polynomial $f(x)$ is defined by $f(x) = 2x^3 + 3x^2 - 18x + 8$.

(i) Use the Factor Theorem to show that $(2x-1)$ is a factor of $f(x)$. *(2 marks)*

(ii) Write $f(x)$ in the form $(2x-1)(x^2+px+q)$, where $p$ and $q$ are integers. *(2 marks)*

(iii) Simplify the algebraic fraction $\dfrac{4x^2+16x}{2x^3+3x^2-18x+8}$. *(2 marks)*

(b) Express the algebraic fraction $\dfrac{2x^2}{(x+5)(x-3)}$ in the form $A + \dfrac{B+Cx}{(x+5)(x-3)}$, where $A$, $B$ and $C$ are integers. *(4 marks)*

**3** (a) Obtain the binomial expansion of $(1+x)^{\frac{1}{2}}$ up to and including the term in $x^2$. *(2 marks)*

(b) Hence obtain the binomial expansion of $\sqrt{1+\frac{3}{2}x}$ up to and including the term in $x^2$. *(2 marks)*

(c) Hence show that $\sqrt{\dfrac{2+3x}{8}} \approx a + bx + cx^2$ for small values of $x$, where $a$, $b$ and $c$ are constants to be found. *(2 marks)*

**4** David is researching changes in the selling price of houses. One particular house was sold on 1 January 1885 for £20. Sixty years later, on 1 January 1945, it was sold for £2000. David proposes a model

$$P = Ak^t$$

for the selling price, £$P$, of this house, where $t$ is the time in years after 1 January 1885 and $A$ and $k$ are constants.

(a) (i) Write down the value of $A$. *(1 mark)*

(ii) Show that, to six decimal places, $k = 1.079775$. *(2 marks)*

(iii) Use the model, with this value of $k$, to estimate the selling price of this house on 1 January 2008. Give your answer to the nearest £1000. *(2 marks)*

(b) For another house, which was sold for £15 on 1 January 1885, David proposes the model

$$Q = 15 \times 1.082709^t$$

for the selling price, £$Q$, of this house $t$ years after 1 January 1885. Calculate the year in which, according to these models, these two houses would have had the same selling price. *(4 marks)*

**5** A curve is defined by the parametric equations $x = 2t + \dfrac{1}{t^2}$, $y = 2t - \dfrac{1}{t^2}$.

(a) At the point $P$ on the curve, $t = \dfrac{1}{2}$.

(i) Find the coordinates of $P$. *(2 marks)*

(ii) Find an equation of the tangent to the curve at $P$. *(5 marks)*

(b) Show that the cartesian equation of the curve can be written as

$$(x - y)(x + y)^2 = k$$

where $k$ is an integer. *(3 marks)*

**Turn over for the next question**

**6** A curve has equation $3xy - 2y^2 = 4$.

Find the gradient of the curve at the point (2, 1). *(5 marks)*

**7** (a) (i) Express $6\sin\theta + 8\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give your value for $\alpha$ to the nearest 0.1°. *(2 marks)*

(ii) Hence solve the equation $6\sin 2x + 8\cos 2x = 7$, giving all solutions to the nearest 0.1° in the interval $0° < x < 360°$. *(4 marks)*

(b) (i) Prove the identity $\dfrac{\sin 2x}{1 - \cos 2x} = \dfrac{1}{\tan x}$. *(4 marks)*

(ii) Hence solve the equation

$$\frac{\sin 2x}{1 - \cos 2x} = \tan x$$

giving all solutions in the interval $0° < x < 360°$. *(4 marks)*

**8** Solve the differential equation

$$\frac{dy}{dx} = \frac{3\cos 3x}{y}$$

given that $y = 2$ when $x = \dfrac{\pi}{2}$. Give your answer in the form $y^2 = f(x)$. *(5 marks)*

**9** The points $A$ and $B$ lie on the line $l_1$ and have coordinates (2, 5, 1) and (4, 1, −2) respectively.

(a) (i) Find the vector $\overrightarrow{AB}$. *(2 marks)*

(ii) Find a vector equation of the line $l_1$, with parameter $\lambda$. *(1 mark)*

(b) The line $l_2$ has equation $\mathbf{r} = \begin{bmatrix} 1 \\ -3 \\ -1 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$.

(i) Show that the point $P(-2, -3, 5)$ lies on $l_2$. *(2 marks)*

(ii) The point $Q$ lies on $l_1$ and is such that $PQ$ is perpendicular to $l_2$. Find the coordinates of $Q$. *(6 marks)*

**END OF QUESTIONS**

Answer **all** questions.

**1** (a) The polynomial $f(x)$ is defined by $f(x) = 4x^3 - 7x - 3$.

(i) Find $f(-1)$. *(1 mark)*

(ii) Use the Factor Theorem to show that $2x + 1$ is a factor of $f(x)$. *(2 marks)*

(iii) Simplify the algebraic fraction $\dfrac{4x^3 - 7x - 3}{2x^2 + 3x + 1}$. *(3 marks)*

(b) The polynomial $g(x)$ is defined by $g(x) = 4x^3 - 7x + d$. When $g(x)$ is divided by $2x + 1$, the remainder is 2. Find the value of $d$. *(2 marks)*

**2** (a) Express $\sin x - 3\cos x$ in the form $R\sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$. Give your value of $\alpha$ in radians to two decimal places. *(3 marks)*

(b) Hence:

(i) write down the minimum value of $\sin x - 3\cos x$; *(1 mark)*

(ii) find the value of $x$ in the interval $0 < x < 2\pi$ at which this minimum value occurs, giving your value of $x$ in radians to two decimal places. *(2 marks)*

**3** (a) (i) Express $\dfrac{2x + 7}{x + 2}$ in the form $A + \dfrac{B}{x + 2}$, where $A$ and $B$ are integers. *(2 marks)*

(ii) Hence find $\displaystyle\int \frac{2x + 7}{x + 2}\,dx$. *(2 marks)*

(b) (i) Express $\dfrac{28 + 4x^2}{(1 + 3x)(5 - x)^2}$ in the form $\dfrac{P}{1 + 3x} + \dfrac{Q}{5 - x} + \dfrac{R}{(5 - x)^2}$, where $P$, $Q$ and $R$ are constants. *(5 marks)*

(ii) Hence find $\displaystyle\int \frac{28 + 4x^2}{(1 + 3x)(5 - x)^2}\,dx$. *(4 marks)*

**4** (a) (i) Find the binomial expansion of $(1-x)^{\frac{1}{2}}$ up to and including the term in $x^2$. *(2 marks)*

(ii) Hence obtain the binomial expansion of $\sqrt{4-x}$ up to and including the term in $x^2$. *(3 marks)*

(b) Use your answer to part (a)(ii) to find an approximate value for $\sqrt{3}$. Give your answer to three decimal places. *(2 marks)*

**5** (a) Express $\sin 2x$ in terms of $\sin x$ and $\cos x$. *(1 mark)*

(b) Solve the equation

$$5\sin 2x + 3\cos x = 0$$

giving all solutions in the interval $0° \leqslant x \leqslant 360°$ to the nearest 0.1°, where appropriate. *(4 marks)*

(c) Given that $\sin 2x + \cos 2x = 1 + \sin x$ and $\sin x \neq 0$, show that $2(\cos x - \sin x) = 1$. *(4 marks)*

**6** A curve is defined by the equation $x^2y + y^3 = 2x + 1$.

(a) Find the gradient of the curve at the point $(2, 1)$. *(6 marks)*

(b) Show that the $x$-coordinate of any stationary point on this curve satisfies the equation

$$\frac{1}{x^3} = x + 1$$

*(4 marks)*

**Turn over for the next question**

**7** (a) A differential equation is given by $\frac{dx}{dt} = -kte^{\frac{1}{2}x}$, where $k$ is a positive constant.

(i) Solve the differential equation. *(3 marks)*

(ii) Hence, given that $x = 6$ when $t = 0$, show that $x = -2\ln\left(\frac{kt^2}{4} + e^{-3}\right)$. *(3 marks)*

(b) The population of a colony of insects is decreasing according to the model $\frac{dx}{dt} = -kte^{\frac{1}{2}x}$, where $x$ thousands is the number of insects in the colony after time $t$ minutes. Initially, there were 6000 insects in the colony.

Given that $k = 0.004$, find:

(i) the population of the colony after 10 minutes, giving your answer to the nearest hundred; *(2 marks)*

(ii) the time after which there will be no insects left in the colony, giving your answer to the nearest 0.1 of a minute. *(2 marks)*

**8** The points $A$ and $B$ have coordinates $(2, 1, -1)$ and $(3, 1, -2)$ respectively. The angle $OBA$ is $\theta$, where $O$ is the origin.

(a) (i) Find the vector $\overrightarrow{AB}$. *(2 marks)*

(ii) Show that $\cos\theta = \frac{5}{2\sqrt{7}}$. *(4 marks)*

(b) The point $C$ is such that $\overrightarrow{OC} = 2\overrightarrow{OB}$. The line $l$ is parallel to $\overrightarrow{AB}$ and passes through the point $C$. Find a vector equation of $l$. *(2 marks)*

(c) The point $D$ lies on $l$ such that angle $ODC = 90°$. Find the coordinates of $D$. *(4 marks)*

**END OF QUESTIONS**