Answer **all** questions.

1 (a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ when $y = \tan 3x$. *(2 marks)*

(b) Given that $y = \dfrac{3x+1}{2x+1}$, show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{(2x+1)^2}$. *(3 marks)*

2 Use Simpson's rule with 5 ordinates (4 strips) to find an approximation to

$$\int_1^3 \frac{1}{\sqrt{1+x^3}}\,\mathrm{d}x$$

giving your answer to three significant figures. *(4 marks)*

3 (a) (i) Given that $\mathrm{f}(x) = x^4 + 2x$, find $\mathrm{f}'(x)$. *(1 mark)*

(ii) Hence, or otherwise, find $\displaystyle\int \frac{2x^3+1}{x^4+2x}\,\mathrm{d}x$. *(2 marks)*

(b) (i) Use the substitution $u = 2x + 1$ to show that

$$\int x\sqrt{2x+1}\,\mathrm{d}x = \frac{1}{4}\int\left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right)\mathrm{d}u$$ *(3 marks)*

(ii) Hence show that $\displaystyle\int_0^4 x\sqrt{2x+1}\,\mathrm{d}x = 19.9$ correct to three significant figures. *(4 marks)*

4 It is given that $2\operatorname{cosec}^2 x = 5 - 5\cot x$.

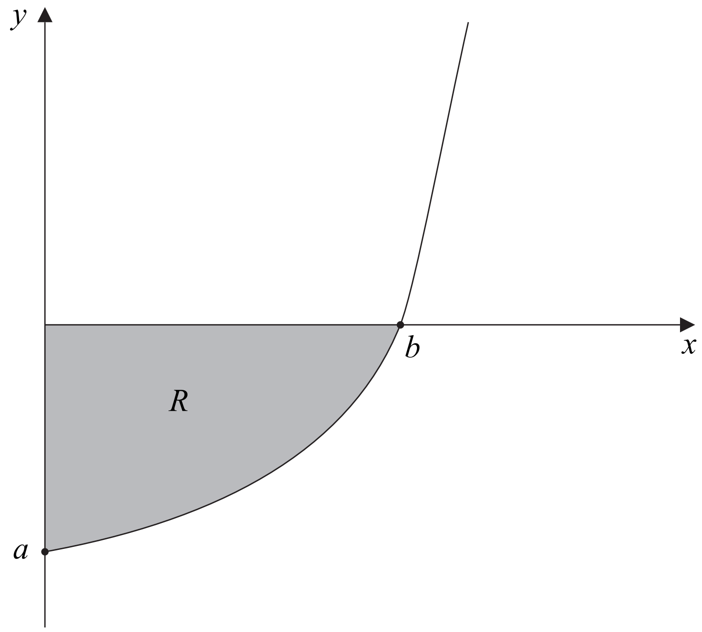
(a) Show that the equation $2\operatorname{cosec}^2 x = 5 - 5\cot x$ can be written in the form

$$2\cot^2 x + 5\cot x - 3 = 0$$ *(2 marks)*

(b) Hence show that $\tan x = 2$ or $\tan x = -\frac{1}{3}$. *(2 marks)*

(c) Hence, or otherwise, solve the equation $2\operatorname{cosec}^2 x = 5 - 5\cot x$, giving all values of x in radians to one decimal place in the interval $-\pi < x \leqslant \pi$. *(3 marks)*

5 The diagram shows part of the graph of $y = e^{2x} - 9$. The graph cuts the coordinate axes at $(0, a)$ and $(b, 0)$.



(a) State the value of a, and show that $b = \ln 3$. *(3 marks)*

(b) Show that $y^2 = e^{4x} - 18e^{2x} + 81$. *(1 mark)*

(c) The shaded region R is rotated through 360° about the x-axis. Find the volume of the solid formed, giving your answer in the form $\pi(p \ln 3 + q)$, where p and q are integers. *(6 marks)*

(d) Sketch the curve with equation $y = |e^{2x} - 9|$ for $x \geqslant 0$. *(2 marks)*

Turn over for the next question

6 [**Figure 1**, printed on the insert, is provided for use in this question.]

The curve $y = x^3 + 4x - 3$ intersects the x-axis at the point A where $x = \alpha$.

(a) Show that α lies between 0.5 and 1.0. *(2 marks)*

(b) Show that the equation $x^3 + 4x - 3 = 0$ can be rearranged into the form $x = \dfrac{3 - x^3}{4}$.

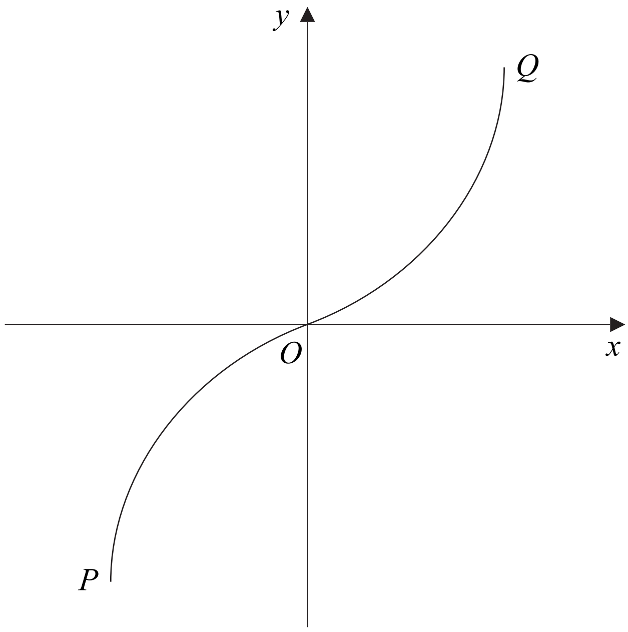
(1 mark)

(c) (i) Use the iteration $x_{n+1} = \dfrac{3 - x_n{}^3}{4}$ with $x_1 = 0.5$ to find x_3, giving your answer to two decimal places. *(3 marks)*

(ii) The sketch on **Figure 1** shows parts of the graphs of $y = \dfrac{3 - x^3}{4}$ and $y = x$, and the position of x_1.

On **Figure 1**, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of x_2 and x_3 on the x-axis. *(3 marks)*

7 (a) The sketch shows the graph of $y = \sin^{-1} x$.



Write down the coordinates of the points P and Q, the end-points of the graph. *(2 marks)*

(b) Sketch the graph of $y = -\sin^{-1}(x - 1)$. *(3 marks)*

8 The functions f and g are defined with their respective domains by

$$f(x) = x^2 \qquad \text{for all real values of } x$$

$$g(x) = \frac{1}{x+2} \qquad \text{for real values of } x, \quad x \neq -2$$

(a) State the range of f. *(1 mark)*

(b) (i) Find $fg(x)$. *(1 mark)*

(ii) Solve the equation $fg(x) = 4$. *(4 marks)*

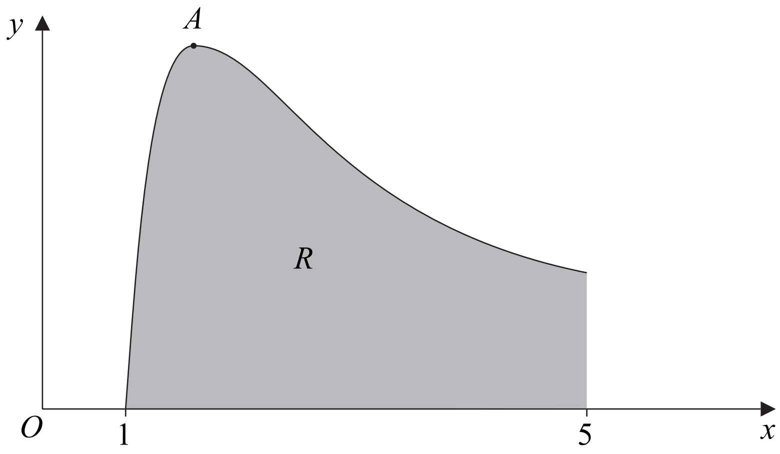
(c) (i) Explain why the function f does **not** have an inverse. *(1 mark)*

(ii) The inverse of g is g^{-1}. Find $g^{-1}(x)$. *(3 marks)*

9 (a) Given that $y = x^{-2} \ln x$, show that $\dfrac{dy}{dx} = \dfrac{1 - 2\ln x}{x^3}$. *(4 marks)*

(b) Using integration by parts, find $\displaystyle\int x^{-2} \ln x \, dx$. *(4 marks)*

(c) The sketch shows the graph of $y = x^{-2} \ln x$.



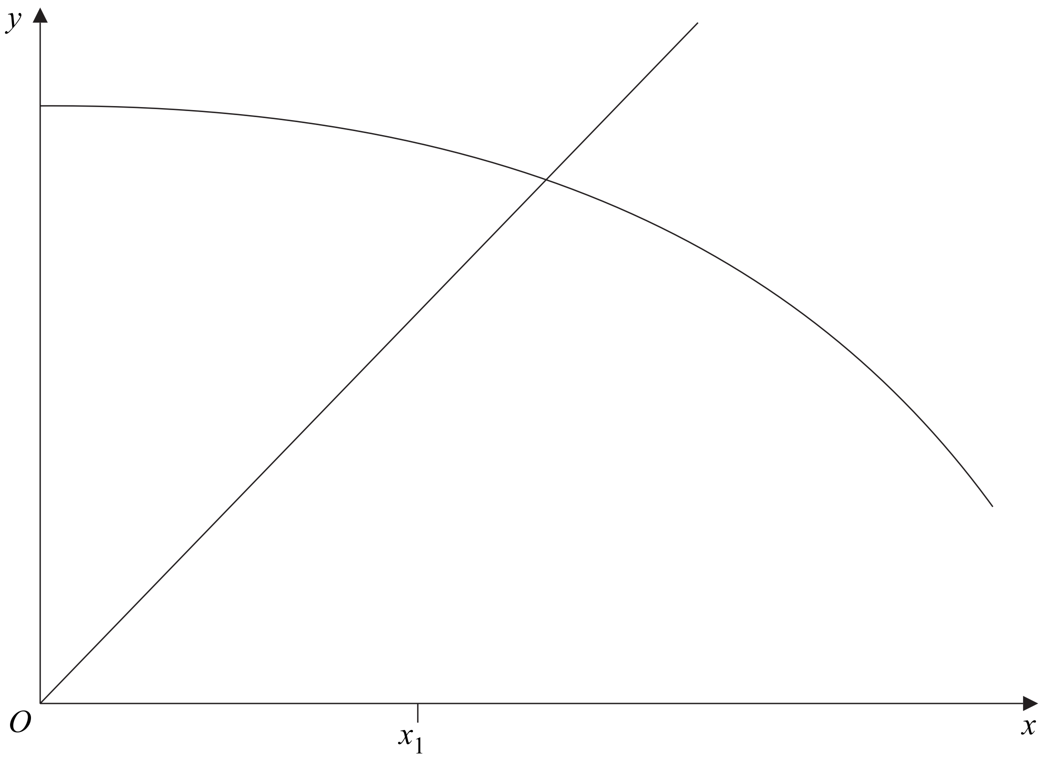
(i) Using the answer to part (a), find, in terms of e, the x-coordinate of the stationary point A. *(2 marks)*

(ii) The region R is bounded by the curve, the x-axis and the line $x = 5$. Using your answer to part (b), show that the area of R is

$$\frac{1}{5}(4 - \ln 5)$$ *(3 marks)*

END OF QUESTIONS

Figure 1 (for Question 6)



Practice 2

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1. Given that

$$\frac{2x^4 - 3x^2 + x + 1}{(x^2 - 1)} \equiv (ax^2 + bx + c) + \frac{dx + e}{(x^2 - 1)},$$

find the values of the constants a, b, c, d and e.

(4)

Leave blank

Question 1 continued

Q1

(Total 4 marks)

Leave blank

2. A curve C has equation

$$y = e^{2x} \tan x, \quad x \neq (2n+1)\frac{\pi}{2}.$$

(a) Show that the turning points on C occur where $\tan x = -1$.

(6)

(b) Find an equation of the tangent to C at the point where $x = 0$.

(2)

Leave blank

Question 2 continued

Q2

(Total 8 marks)

Leave blank

3. $$f(x) = \ln(x+2) - x + 1, \quad x > -2, x \in \mathbb{R}.$$

(a) Show that there is a root of $f(x) = 0$ in the interval $2 < x < 3$.

(2)

(b) Use the iterative formula

$$x_{n+1} = \ln(x_n + 2) + 1, \;\; x_0 = 2.5$$

to calculate the values of x_1, x_2 and x_3 giving your answers to 5 decimal places.

(3)

(c) Show that $x = 2.505$ is a root of $f(x) = 0$ correct to 3 decimal places.

(2)

Leave blank

Question 3 continued

Q3

(Total 7 marks)

Turn over

Leave blank

4.

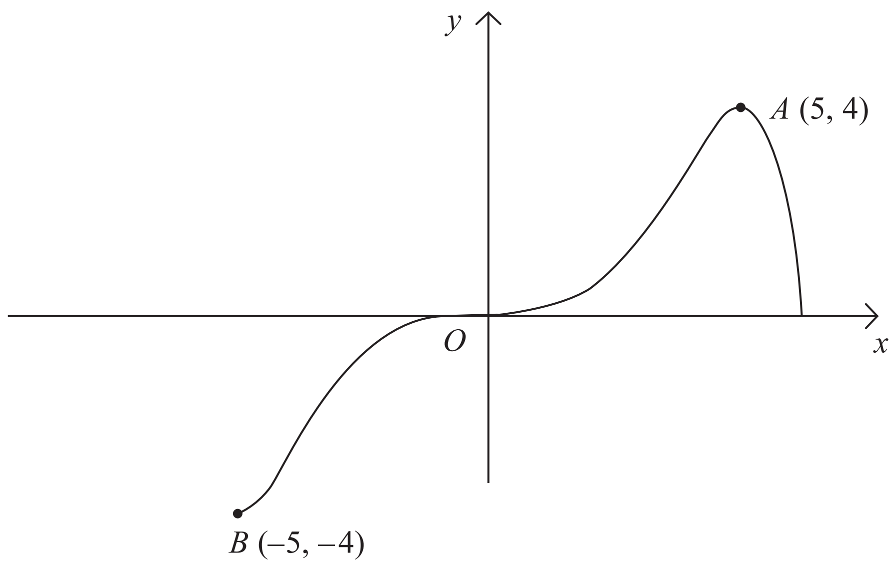


Figure 1

Figure 1 shows a sketch of the curve with equation $y = f(x)$.
The curve passes through the origin O and the points $A(5, 4)$ and $B(-5, -4)$.

In separate diagrams, sketch the graph with equation

(a) $y = |f(x)|$,

(3)

(b) $y = f(|x|)$,

(3)

(c) $y = 2f(x+1)$.

(4)

On each sketch, show the coordinates of the points corresponding to A and B.

Leave blank

Question 4 continued

Turn over

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Q4

(Total 10 marks)

Leave blank

5. The radioactive decay of a substance is given by

$$R = 1000e^{-ct}, \qquad t \geqslant 0.$$

where R is the number of atoms at time t years and c is a positive constant.

(a) Find the number of atoms when the substance started to decay.
(1)

It takes 5730 years for half of the substance to decay.

(b) Find the value of c to 3 significant figures.
(4)

(c) Calculate the number of atoms that will be left when $t = 22\ 920$.
(2)

(d) In the space provided on page 13, sketch the graph of R against t .
(2)

Leave blank

Question 5 continued

Q5

(Total 9 marks)

Turn over

Leave blank

6. (a) Use the double angle formulae and the identity

$$\cos(A+B) \equiv \cos A \cos B - \sin A \sin B$$

to obtain an expression for $\cos 3x$ in terms of powers of $\cos x$ only.

(4)

(b) (i) Prove that

$$\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} \equiv 2\sec x, \qquad x \neq (2n+1)\frac{\pi}{2} .$$

(4)

(ii) Hence find, for $0 < x < 2\pi$, all the solutions of

$$\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} = 4 .$$

(3)

Leave blank

Question 6 continued

Turn over

Leave blank

Question 6 continued

Leave blank

Question 6 continued

(Total 11 marks)

Q6

Leave blank

7. A curve C has equation

$$y = 3\sin 2x + 4\cos 2x,\ -\pi \leqslant x \leqslant \pi .$$

The point $A(0, 4)$ lies on C.

(a) Find an equation of the normal to the curve C at A.
(5)

(b) Express y in the form $R\sin(2x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 3 significant figures.
(4)

(c) Find the coordinates of the points of intersection of the curve C with the x-axis. Give your answers to 2 decimal places.
(4)

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Leave blank

Question 7 continued

(Total 13 marks)

Q7

Leave blank

8. The functions f and g are defined by

$$f : x \mapsto 1 - 2x^3, \ \ x \in \mathbb{R}$$

$$g : x \mapsto \frac{3}{x} - 4, \ \ x > 0, \ \ x \in \mathbb{R}$$

(a) Find the inverse function f^{-1}.

(2)

(b) Show that the composite function gf is

$$gf : x \mapsto \frac{8x^3 - 1}{1 - 2x^3}.$$

(4)

(c) Solve $gf(x) = 0$.

(2)

(d) Use calculus to find the coordinates of the stationary point on the graph of $y = gf(x)$.

(5)

Leave blank

Question 8 continued

Turn over

Leave blank

Question 8 continued

Q8

(Total 13 marks)

TOTAL FOR PAPER: 75 MARKS

END

Answer **all** questions.

1 Use the mid-ordinate rule with four strips of equal width to find an estimate for $\int_1^5 \frac{1}{1+\ln x}\,\mathrm{d}x$, giving your answer to three significant figures. *(4 marks)*

2 Describe a sequence of **two** geometrical transformations that maps the graph of $y = \sec x$ onto the graph of $y = 1 + \sec 3x$. *(4 marks)*

3 The functions f and g are defined with their respective domains by

$$\mathrm{f}(x) = 3 - x^2, \quad \text{for all real values of } x$$

$$\mathrm{g}(x) = \frac{2}{x+1}, \quad \text{for real values of } x,\ x \neq -1$$

(a) Find the range of f. *(2 marks)*

(b) The inverse of g is g^{-1}.

(i) Find $\mathrm{g}^{-1}(x)$. *(3 marks)*

(ii) State the range of g^{-1}. *(1 mark)*

(c) The composite function gf is denoted by h.

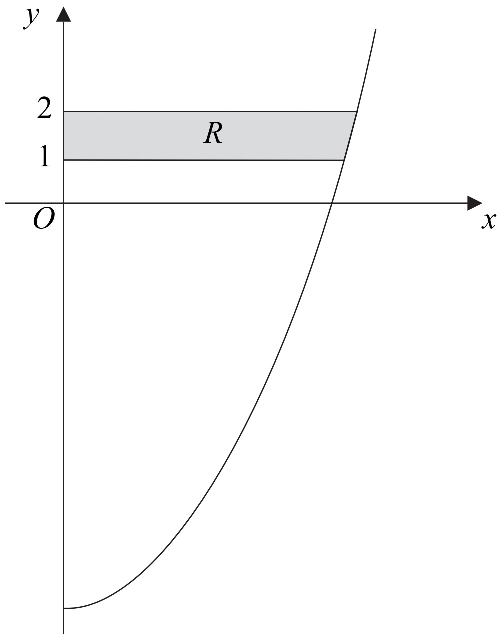
(i) Find $\mathrm{h}(x)$, simplifying your answer. *(2 marks)*

(ii) State the greatest possible domain of h. *(1 mark)*

4 (a) Use integration by parts to find $\int x \sin x \, dx$. *(4 marks)*

(b) Using the substitution $u = x^2 + 5$, or otherwise, find $\int x\sqrt{x^2+5} \, dx$. *(4 marks)*

(c) The diagram shows the curve $y = x^2 - 9$ for $x \geqslant 0$.



The shaded region R is bounded by the curve, the lines $y = 1$ and $y = 2$, and the y-axis.

Find the exact value of the volume of the solid generated when the region R is rotated through 360° **about the *y*-axis**. *(4 marks)*

5 (a) (i) Show that the equation

$$2\cot^2 x + 5\operatorname{cosec} x = 10$$

can be written in the form $2\operatorname{cosec}^2 x + 5\operatorname{cosec} x - 12 = 0$. *(2 marks)*

(ii) Hence show that $\sin x = -\frac{1}{4}$ or $\sin x = \frac{2}{3}$. *(3 marks)*

(b) Hence, or otherwise, solve the equation

$$2\cot^2(\theta - 0.1) + 5\operatorname{cosec}(\theta - 0.1) = 10$$

giving all values of θ in radians to two decimal places in the interval $-\pi < \theta < \pi$. *(3 marks)*

6 (a) Find $\dfrac{dy}{dx}$ when:

(i) $y = (4x^2 + 3x + 2)^{10}$; *(2 marks)*

(ii) $y = x^2 \tan x$. *(2 marks)*

(b) (i) Find $\dfrac{dx}{dy}$ when $x = 2y^3 + \ln y$. *(1 mark)*

(ii) Hence find an equation of the tangent to the curve $x = 2y^3 + \ln y$ at the point $(2,1)$. *(3 marks)*

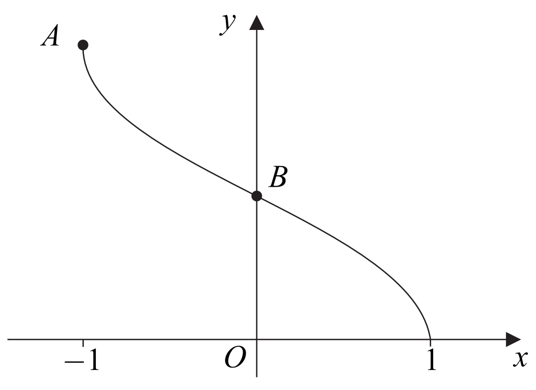
7 (a) Sketch the graph of $y = |2x|$. *(1 mark)*

(b) On a separate diagram, sketch the graph of $y = 4 - |2x|$, indicating the coordinates of the points where the graph crosses the coordinate axes. *(3 marks)*

(c) Solve $4 - |2x| = x$. *(3 marks)*

(d) Hence, or otherwise, solve the inequality $4 - |2x| > x$. *(2 marks)*

8 The diagram shows the curve $y = \cos^{-1} x$ for $-1 \leqslant x \leqslant 1$.

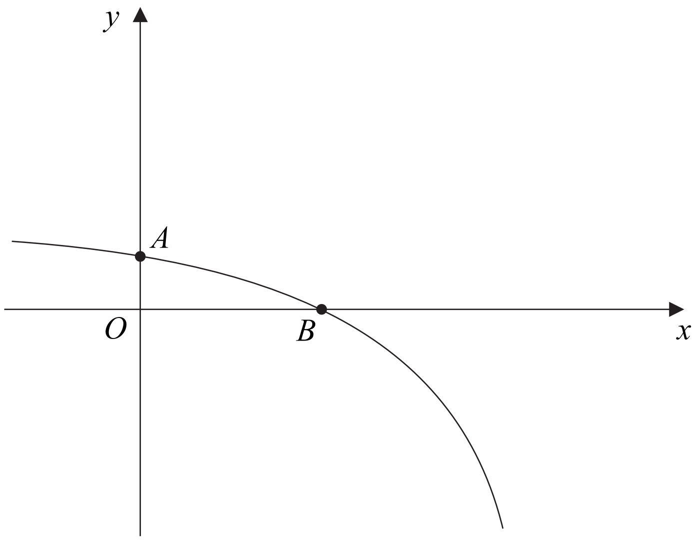


(a) Write down the exact coordinates of the points A and B. *(2 marks)*

(b) The equation $\cos^{-1} x = 3x + 1$ has only one root. Given that the root of this equation is α, show that $0.1 \leqslant \alpha \leqslant 0.2$. *(2 marks)*

(c) Use the iteration $x_{n+1} = \frac{1}{3}(\cos^{-1} x_n - 1)$ with $x_1 = 0.1$ to find the values of x_2, x_3 and x_4, giving your answers to three decimal places. *(3 marks)*

9 The sketch shows the graph of $y = 4 - e^{2x}$. The curve crosses the y-axis at the point A and the x-axis at the point B.



(a) (i) Find $\int (4 - e^{2x})\,dx$. *(2 marks)*

(ii) Hence show that $\int_0^{\ln 2} (4 - e^{2x})\,dx = 4\ln 2 - \frac{3}{2}$. *(2 marks)*

(b) (i) Write down the y-coordinate of A. *(1 mark)*

(ii) Show that $x = \ln 2$ at B. *(2 marks)*

(c) Find the equation of the normal to the curve $y = 4 - e^{2x}$ at the point B. *(4 marks)*

(d) Find the area of the region enclosed by the curve $y = 4 - e^{2x}$, the normal to the curve at B and the y-axis. *(3 marks)*

END OF QUESTIONS

Practice 4

Leave blank

1. (a) Find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point where $x = 2$ on the curve with equation

$$y = x^2\sqrt{(5x - 1)}.$$

(6)

(b) Differentiate $\frac{\sin 2x}{x^2}$ with respect to x.

(4)

Leave blank

Question 1 continued

(Total 10 marks)

Q1

Leave blank

2. $$\mathrm{f}(x) = \frac{2x+2}{x^2-2x-3} - \frac{x+1}{x-3}$$

(a) Express $\mathrm{f}(x)$ as a single fraction in its simplest form.

(4)

(b) Hence show that $\mathrm{f}'(x) = \dfrac{2}{(x-3)^2}$

(3)

Leave blank

Question 2 continued

Turn over

Leave blank

Question 2 continued

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Question 2 continued

Q2

(Total 7 marks)

Turn over

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3.

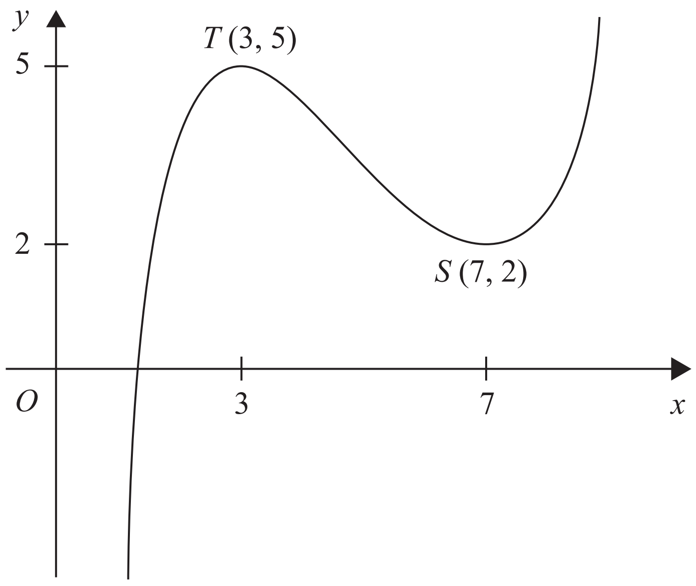


Figure 1

Figure 1 shows the graph of $y = \mathrm{f}(x)$, $1 < x < 9$.
The points $T(3, 5)$ and $S(7, 2)$ are turning points on the graph.

Sketch, on separate diagrams, the graphs of

(a) $y = 2\mathrm{f}(x) - 4$, **(3)**

(b) $y = |\mathrm{f}(x)|$. **(3)**

Indicate on each diagram the coordinates of any turning points on your sketch.

Leave blank

Question 3 continued

Q3

(Total 6 marks)

Turn over

Leave blank

4. Find the equation of the tangent to the curve $x = \cos(2y + \pi)$ at $\left(0, \frac{\pi}{4}\right)$.

Give your answer in the form $y = ax + b$, where a and b are constants to be found.

(6)

Leave blank

Question 4 continued

Q4

(Total 6 marks)

Leave blank

5. The functions f and g are defined by

$$\mathrm{f} : x \mapsto 3x + \ln x, \quad x > 0, \quad x \in \mathbb{R}$$
$$\mathrm{g} : x \mapsto \mathrm{e}^{x^2}, \quad x \in \mathbb{R}$$

(a) Write down the range of g.

(1)

(b) Show that the composite function fg is defined by

$$\mathrm{fg} : x \mapsto x^2 + 3\mathrm{e}^{x^2}, \quad x \in \mathbb{R}\,.$$

(2)

(c) Write down the range of fg.

(1)

(d) Solve the equation $\dfrac{\mathrm{d}}{\mathrm{d}x}\left[\mathrm{fg}(x)\right] = x(x\mathrm{e}^{x^2} + 2)$.

(6)

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Leave blank

Question 5 continued

(Total 10 marks)

Q5

Turn over

Leave blank

6. (a) (i) By writing $3\theta = (2\theta + \theta)$, show that

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta.$$

(4)

(ii) Hence, or otherwise, for $0 < \theta < \frac{\pi}{3}$, solve

$$8\sin^3\theta - 6\sin\theta + 1 = 0.$$

Give your answers in terms of π.

(5)

(b) Using $\sin(\theta - \alpha) = \sin\theta\cos\alpha - \cos\theta\sin\alpha$, or otherwise, show that

$$\sin 15° = \frac{1}{4}(\sqrt{6} - \sqrt{2}).$$

(4)

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

(Total 13 marks)

Q6

Leave blank

7. $$f(x) = 3xe^x - 1$$

The curve with equation $y = f(x)$ has a turning point P.

(a) Find the exact coordinates of P.

(5)

The equation $f(x) = 0$ has a root between $x = 0.25$ and $x = 0.3$

(b) Use the iterative formula

$$x_{n+1} = \frac{1}{3}e^{-x_n}$$

with $x_0 = 0.25$ to find, to 4 decimal places, the values of x_1, x_2 and x_3.

(3)

(c) By choosing a suitable interval, show that a root of $f(x) = 0$ is $x = 0.2576$ correct to 4 decimal places.

(3)

Leave blank

Question 7 continued

Turn over

Leave blank

Question 7 continued

Leave blank

Question 7 continued

(Total 11 marks)

Q7

Leave blank

8. (a) Express $3\cos\theta + 4\sin\theta$ in the form $R\cos(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90°$.

(4)

(b) Hence find the maximum value of $3\cos\theta + 4\sin\theta$ and the smallest positive value of θ for which this maximum occurs.

(3)

The temperature, $f(t)$, of a warehouse is modelled using the equation

$$f(t) = 10 + 3\cos(15t)° + 4\sin(15t)°,$$

where t is the time in hours from midday and $0 \leqslant t < 24$.

(c) Calculate the minimum temperature of the warehouse as given by this model.

(2)

(d) Find the value of t when this minimum temperature occurs.

(3)

Leave blank

Question 8 continued

Turn over

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Q8

(Total 12 marks)

TOTAL FOR PAPER: 75 MARKS

END

Answer **all** questions.

1 (a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ when:

(i) $y = (2x^2 - 5x + 1)^{20}$; *(2 marks)*

(ii) $y = x\cos x$. *(2 marks)*

(b) Given that

$$y = \frac{x^3}{x-2}$$

show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{kx^2(x-3)}{(x-2)^2}$$

where k is a positive integer. *(3 marks)*

2 (a) Solve the equation $\cot x = 2$, giving all values of x in the interval $0 \leqslant x \leqslant 2\pi$ in radians to two decimal places. *(2 marks)*

(b) Show that the equation $\operatorname{cosec}^2 x = \frac{3\cot x + 4}{2}$ can be written as

$$2\cot^2 x - 3\cot x - 2 = 0$$ *(2 marks)*

(c) Solve the equation $\operatorname{cosec}^2 x = \frac{3\cot x + 4}{2}$, giving all values of x in the interval $0 \leqslant x \leqslant 2\pi$ in radians to two decimal places. *(4 marks)*

3 The equation

$$x + (1 + 3x)^{\frac{1}{4}} = 0$$

has a single root, α.

(a) Show that α lies between -0.33 and -0.32. *(2 marks)*

(b) Show that the equation $x + (1 + 3x)^{\frac{1}{4}} = 0$ can be rearranged into the form

$$x = \tfrac{1}{3}(x^4 - 1)$$ *(2 marks)*

(c) Use the iteration $x_{n+1} = \dfrac{(x_n{}^4 - 1)}{3}$ with $x_1 = -0.3$ to find x_4, giving your answer to three significant figures. *(3 marks)*

4 The functions f and g are defined with their respective domains by

$$\mathrm{f}(x) = x^3, \qquad \text{for all real values of } x$$

$$\mathrm{g}(x) = \frac{1}{x-3}, \qquad \text{for real values of } x,\ x \neq 3$$

(a) State the range of f. *(1 mark)*

(b) (i) Find $\mathrm{fg}(x)$. *(1 mark)*

(ii) Solve the equation $\mathrm{fg}(x) = 64$. *(3 marks)*

(c) (i) The inverse of g is g^{-1}. Find $\mathrm{g}^{-1}(x)$. *(3 marks)*

(ii) State the range of g^{-1}. *(1 mark)*

5 (a) (i) Given that $y = 2x^2 - 8x + 3$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. *(1 mark)*

(ii) Hence, or otherwise, find

$$\int_4^6 \frac{x-2}{2x^2 - 8x + 3}\,\mathrm{d}x$$

giving your answer in the form $k \ln 3$, where k is a rational number. *(4 marks)*

(b) Use the substitution $u = 3x - 1$ to find $\int x\sqrt{3x-1}\,\mathrm{d}x$, giving your answer in terms of x. *(4 marks)*

Turn over for the next question

6 (a) Sketch the curve with equation $y = \operatorname{cosec} x$ for $0 < x < \pi$. *(2 marks)*

(b) Use the mid-ordinate rule with four strips to find an estimate for $\int_{0.1}^{0.5} \operatorname{cosec} x \, dx$, giving your answer to three significant figures. *(4 marks)*

7 (a) Describe a sequence of **two** geometrical transformations that maps the graph of $y = x^2$ onto the graph of $y = 4x^2 - 5$. *(4 marks)*

(b) Sketch the graph of $y = |4x^2 - 5|$, indicating the coordinates of the point where the curve crosses the y-axis. *(3 marks)*

(c) (i) Solve the equation $|4x^2 - 5| = 4$. *(3 marks)*

(ii) Hence, or otherwise, solve the inequality $|4x^2 - 5| \geqslant 4$. *(2 marks)*

8 (a) Given that $e^{-2x} = 3$, find the exact value of x. *(2 marks)*

(b) Use integration by parts to find $\int x e^{-2x} \, dx$. *(4 marks)*

(c) A curve has equation $y = e^{-2x} + 6x$.

(i) Find the exact values of the coordinates of the stationary point of the curve. *(4 marks)*

(ii) Determine the nature of the stationary point. *(2 marks)*

(iii) The region R is bounded by the curve $y = e^{-2x} + 6x$, the x-axis and the lines $x = 0$ and $x = 1$.

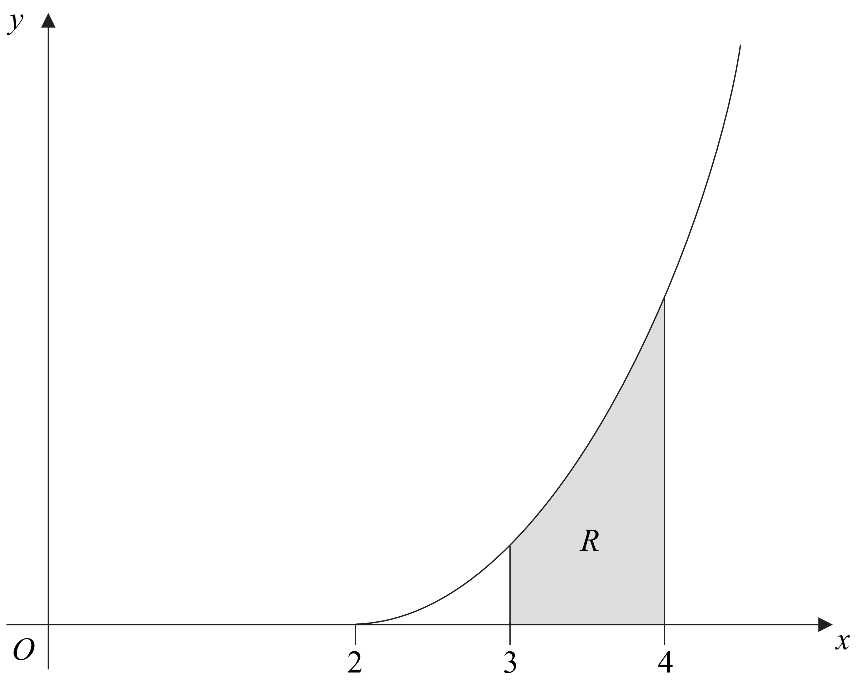
Find the volume of the solid formed when R is rotated through 2π radians about the x-axis, giving your answer to three significant figures. *(5 marks)*

END OF QUESTIONS

Answer **all** questions.

1 Use Simpson's rule with 5 ordinates (4 strips) to find an approximation to $\int_1^9 \frac{1}{1+\sqrt{x}}\,dx$, giving your answer to three significant figures. *(4 marks)*

2 The diagram shows the curve with equation $y = \sqrt{(x-2)^5}$ for $x \geqslant 2$.



The shaded region R is bounded by the curve $y = \sqrt{(x-2)^5}$, the x-axis and the lines $x = 3$ and $x = 4$.

Find the exact value of the volume of the solid formed when the region R is rotated through 360° about the x-axis. *(4 marks)*

3 [**Figure 1**, printed on the insert, is provided for use in this question.]

The curve with equation $y = x^3 + 5x - 4$ intersects the x-axis at the point A, where $x = \alpha$.

(a) Show that α lies between 0.5 and 1. *(2 marks)*

(b) Show that the equation $x^3 + 5x - 4 = 0$ can be rearranged into the form

$$x = \frac{1}{5}(4 - x^3)$$ *(1 mark)*

(c) Use the iteration $x_{n+1} = \frac{1}{5}(4 - x_n{}^3)$ with $x_1 = 0.5$ to find x_3, giving your answer to three decimal places. *(2 marks)*

(d) The sketch on **Figure 1** shows parts of the graphs of $y = \frac{1}{5}(4 - x^3)$ and $y = x$, and the position of x_1.

On **Figure 1**, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of x_2 and x_3 on the x-axis. *(2 marks)*

4 (a) Solve the equation $\sec x = \frac{3}{2}$, giving all values of x to the nearest degree in the interval $0° < x < 360°$. *(2 marks)*

(b) By using a suitable trigonometrical identity, solve the equation

$$2\tan^2 x = 10 - 5\sec x$$

giving all values of x to the nearest degree in the interval $0° < x < 360°$. *(6 marks)*

Turn over for the next question

5 The functions f and g are defined with their respective domains by

$$f(x) = 2 - x^4 \qquad \text{for all real values of } x$$

$$g(x) = \frac{1}{x-4} \qquad \text{for real values of } x, \; x \neq 4$$

(a) State the range of f. *(2 marks)*

(b) Explain why the function f does not have an inverse. *(1 mark)*

(c) (i) Write down an expression for $fg(x)$. *(1 mark)*

(ii) Solve the equation $fg(x) = -14$. *(3 marks)*

6 A curve has equation $y = e^{2x}(x^2 - 4x - 2)$.

(a) Find the value of the x-coordinate of each of the stationary points of the curve. *(6 marks)*

(b) (i) Find $\dfrac{d^2y}{dx^2}$. *(2 marks)*

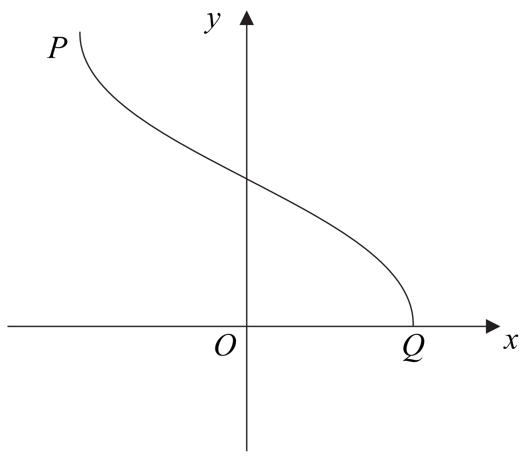
(ii) Determine the nature of each of the stationary points of the curve. *(2 marks)*

7 (a) Given that $3e^x = 4$, find the exact value of x. *(2 marks)*

(b) (i) By substituting $y = e^x$, show that the equation $3e^x + 20e^{-x} = 19$ can be written as $3y^2 - 19y + 20 = 0$. *(1 mark)*

(ii) Hence solve the equation $3e^x + 20e^{-x} = 19$, giving your answers as exact values. *(3 marks)*

8 The sketch shows the graph of $y = \cos^{-1} x$.



(a) Write down the coordinates of P and Q, the end points of the graph. *(2 marks)*

(b) Describe a sequence of two geometrical transformations that maps the graph of $y = \cos^{-1} x$ onto the graph of $y = 2\cos^{-1}(x-1)$. *(4 marks)*

(c) Sketch the graph of $y = 2\cos^{-1}(x-1)$. *(2 marks)*

(d) (i) Write the equation $y = 2\cos^{-1}(x-1)$ in the form $x = \mathrm{f}(y)$. *(2 marks)*

(ii) Hence find the value of $\frac{\mathrm{d}x}{\mathrm{d}y}$ when $y = 2$. *(3 marks)*

9 (a) Given that $y = \frac{4x}{4x-3}$, use the quotient rule to show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{k}{(4x-3)^2}$, where k is an integer. *(2 marks)*

(b) (i) Given that $y = x\ln(4x-3)$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$. *(3 marks)*

(ii) Find an equation of the tangent to the curve $y = x\ln(4x-3)$ at the point where $x = 1$. *(3 marks)*

(c) (i) Use the substitution $u = 4x - 3$ to find $\int \frac{4x}{4x-3}\,\mathrm{d}x$, giving your answer in terms of x. *(4 marks)*

(ii) By using integration by parts, or otherwise, find $\int \ln(4x-3)\,\mathrm{d}x$. *(4 marks)*

END OF QUESTIONS

Figure 1 (for use in Question 3)