

Tuesday 17 January 2012 – Morning

A2 GCE MATHEMATICS

4724 Core Mathematics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4724
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 When the polynomial $f(x)$ is divided by $x^2 + 1$, the quotient is $x^2 + 4x + 2$ and the remainder is $x - 1$. Find $f(x)$, simplifying your answer. [3]

2 (i) Find, in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$, an equation of the line l through the points $(4, 2, 7)$ and $(5, -4, -1)$. [3]

(ii) Find the acute angle between the line l and a line in the direction of the vector $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$. [4]

3 The equation of a curve C is $(x + 3)(y + 4) = x^2 + y^2$.

(i) Find $\frac{dy}{dx}$ in terms of x and y . [4]

(ii) The line $2y = x + 3$ meets C at two points. What can be said about the tangents to C at these points? Justify your answer. [2]

(iii) Find the equation of the tangent at the point $(6, 0)$, giving your answer in the form $ax + by = c$, where a , b and c are integers. [2]

4 (i) Expand $(1 - 4x)^{\frac{1}{4}}$ in ascending powers of x , up to and including the term in x^3 . [5]

(ii) The term of lowest degree in the expansion of

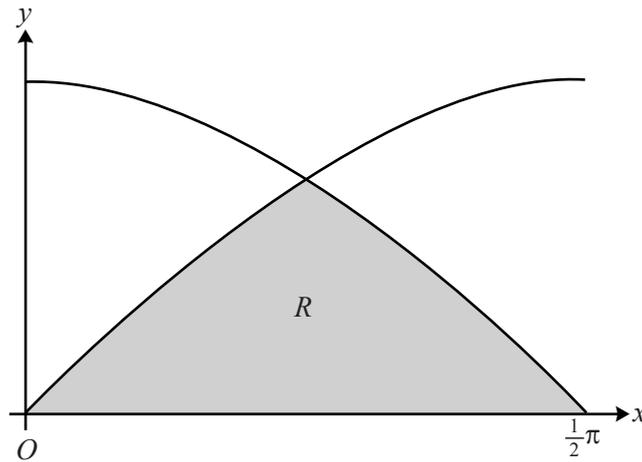
$$(1 + ax)(1 + bx^2)^7 - (1 - 4x)^{\frac{1}{4}}$$

in ascending powers of x is the term in x^3 . Find the values of the constants a and b . [4]

5 Use the substitution $u = \cos x$ to find the exact value of

$$\int_0^{\frac{1}{3}\pi} \sin^3 x \cos^2 x \, dx. \quad [6]$$

6



The diagram shows the curves $y = \cos x$ and $y = \sin x$, for $0 \leq x \leq \frac{1}{2}\pi$. The region R is bounded by the curves and the x -axis. Find the volume of the solid of revolution formed when R is rotated completely about the x -axis, giving your answer in terms of π . [7]

7 The equation of a straight line l is

$$\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}.$$

O is the origin.

(i) Find the position vector of the point P on l such that OP is perpendicular to l . [3]

(ii) A point Q on l is such that the length of OQ is 3 units. Find the two possible position vectors of Q . [3]

8 A curve is defined by the parametric equations

$$x = \sin^2 \theta, \quad y = 4 \sin \theta - \sin^3 \theta,$$

where $-\frac{1}{2}\pi \leq \theta \leq \frac{1}{2}\pi$.

(i) Show that $\frac{dy}{dx} = \frac{4 - 3 \sin^2 \theta}{2 \sin \theta}$. [3]

(ii) Find the coordinates of the point on the curve at which the gradient is 2. [3]

(iii) Show that the curve has no stationary points. [2]

(iv) Find a cartesian equation of the curve, giving your answer in the form $y^2 = f(x)$. [2]

[Questions 9 and 10 are printed overleaf.]

9 Find the exact value of $\int_0^1 (x^2 + 1)e^{2x} dx$. [7]

10 (i) Write down the derivative of $\sqrt{y^2 + 1}$ with respect to y . [1]

(ii) Given that $\frac{dy}{dx} = \frac{(x-1)\sqrt{y^2+1}}{xy}$ and that $y = \sqrt{e^2 - 2e}$ when $x = e$,
find a relationship between x and y . [8]

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