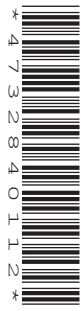
OCR
RECOGNISING ACHIEVEMENT

# Friday 20 January 2012 – Afternoon

## AS GCE MATHEMATICS

**4725** Further Pure Mathematics 1

**QUESTION PAPER**



Candidates answer on the Printed Answer Book.

**OCR supplied materials:**
- Printed Answer Book 4725
- List of Formulae (MF1)

**Other materials required:**
- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes

### INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

### INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

### INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

© OCR 2012 [T/102/2698]

OCR is an exempt Charity

**1** The complex number $a + 5\mathrm{i}$, where $a$ is positive, is denoted by $z$. Given that $|z| = 13$, find the value of $a$ and hence find $\arg z$. **[4]**

**2** The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 3 & 4 \\ 2 & -3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 4 & 6 \\ 3 & -5 \end{pmatrix}$, and **I** is the $2 \times 2$ identity matrix. Given that $p\mathbf{A} + q\mathbf{B} = \mathbf{I}$, find the values of the constants $p$ and $q$. **[5]**

**3** Use an algebraic method to find the square roots of $3 + (6\sqrt{2})\mathrm{i}$. Give your answers in the form $x + \mathrm{i}y$, where $x$ and $y$ are exact real numbers. **[6]**

**4** Find $\sum_{r=1}^{n} r(r^2 - 3)$, expressing your answer in a fully factorised form. **[6]**

**5** **(a)** Find the matrix that represents a reflection in the line $y = -x$. **[2]**

**(b)** The matrix **C** is given by $\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}$.

**(i)** Describe fully the geometrical transformation represented by **C**. **[2]**

**(ii)** State the value of the determinant of **C** and describe briefly how this value relates to the transformation represented by **C**. **[2]**

**6** Sketch, on a single Argand diagram, the loci given by $|z - \sqrt{3} - \mathrm{i}| = 2$ and $\arg z = \frac{1}{6}\pi$. **[6]**

**7** The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} 3 & 0 \\ 2 & 1 \end{pmatrix}$.

**(i)** Show that $\mathbf{M}^4 = \begin{pmatrix} 81 & 0 \\ 80 & 1 \end{pmatrix}$. **[3]**

**(ii)** Hence suggest a suitable form for the matrix $\mathbf{M}^n$, where $n$ is a positive integer. **[2]**

**(iii)** Use induction to prove that your answer to part **(ii)** is correct. **[4]**

**8** **(i)** Show that $\frac{r}{r+1} - \frac{r-1}{r} \equiv \frac{1}{r(r+1)}$. **[2]**

**(ii)** Hence find an expression, in terms of $n$, for

$$\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \ldots + \frac{1}{n(n+1)}.$$ **[4]**

**(iii)** Hence find $\sum_{r=n+1}^{\infty} \frac{1}{r(r+1)}$. **[2]**

© OCR 2012

**9** The matrix $\mathbf{X}$ is given by $\mathbf{X} = \begin{pmatrix} a & 2 & 9 \\ 2 & a & 3 \\ 1 & 0 & -1 \end{pmatrix}$.

**(i)** Find the determinant of $\mathbf{X}$ in terms of $a$. **[3]**

**(ii)** Hence find the values of $a$ for which $\mathbf{X}$ is singular. **[3]**

**(iii)** Given that $\mathbf{X}$ is non-singular, find $\mathbf{X}^{-1}$ in terms of $a$. **[4]**

**10** The cubic equation $3x^3 - 9x^2 + 6x + 2 = 0$ has roots $\alpha$, $\beta$ and $\gamma$.

**(i)** Write down the values of $\alpha + \beta + \gamma$, $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$. **[3]**

The cubic equation $x^3 + ax^2 + bx + c = 0$ has roots $\alpha^2$, $\beta^2$ and $\gamma^2$.

**(ii)** Show that $c = -\frac{4}{9}$ and find the values of $a$ and $b$. **[9]**

© OCR 2012

**THERE ARE NO QUESTIONS WRITTEN ON THIS PAGE**

OCR
RECOGNISING ACHIEVEMENT

**Copyright Information**

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.

© OCR 2012