Mark Scheme (Results)

January 2009
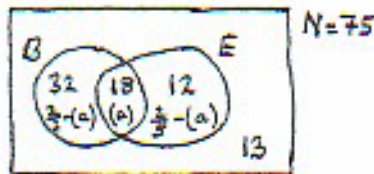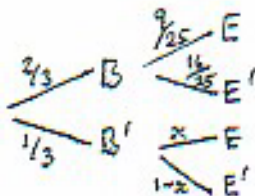
GCE Mathematics (6683/01)
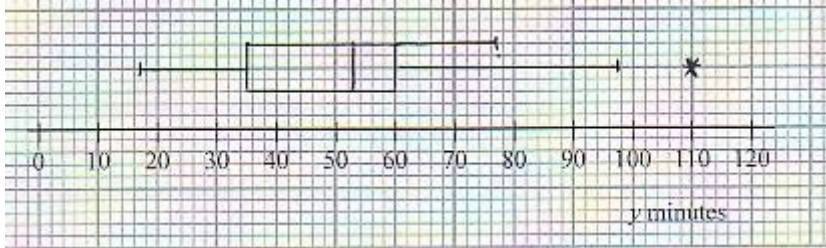
Question Number	Scheme	Marks
1 (a)	$S_{xx} = 57.22 - \dfrac{(21.4)^2}{10} = 11.424$ $S_{xy} = 313.7 - \dfrac{21.4 \times 96}{10} = 108.26$	M1 A1 A1 (3)
(b)	$b = \dfrac{S_{xy}}{S_{xx}} = 9.4765...$ $a = \bar{y} - b\bar{x} = 9.6 - 2.14b = (-10.679...)$ $y = -10.7 + 9.48x$	M1 A1 M1 A1 (4)
(c)	Every (extra) <u>hour</u> spent using the programme produces about <u>9.5 marks improvement</u>	B1ft (1)
(d)	$y = -10.7 + 9.48 \times 3.3, = 20.6$ awrt 21	M1,A1 (2)
(e)	Model may not be valid since [8h is] outside the range [0.5 - 4].	B1 (1) **[11]**
(a)	M1 for a correct expression 1st A1 for AWRT 11.4 for S_{xx} 2nd A1 for AWRT 108 for S_{xy} Correct answers only: One value correct scores M1 and appropriate A1, both correct M1A1A1	
(b)	1st M1 for using their values in correct formula 1st A1 for AWRT 9.5 2nd M1 for correct method for a (minus sign required) 2nd A1 for equation with a and b AWRT 3 sf (e.g. $y = -10.68 + 9.48x$ is fine) Must have a full equation with a and b correct to awrt 3 sf	

(c)	B1ft for comment conveying the idea of <u>b marks per hour</u>. Must mention value of b but can
	         ft their value of b. No need to mention "extra" but must mention "marks" and "hour(s)"
	         e.g. " …9.5 times per hour …" scores B0	

(d)	M1 for sub $x = 3.3$ into their regression equation from the end of part (b) A1 for awrt 21	
(e)	B1 for a statement that says or implies that it may <u>not</u> be valid because <u>outside the range</u>. They do not have to mention the values concerned here namely 8 h or 0.5 - 4	

Question Number	Scheme	Marks
2 (a)	E = take regular exercise B = always eat breakfast $P(E \cap B) = P(E \mid B) \times P(B)$	M1
	$\quad\quad = \dfrac{9}{25} \times \dfrac{2}{3} \ = \ 0.24 \quad\text{or}\quad \dfrac{6}{25} \quad\text{or}\quad \dfrac{18}{75}$	A1 (2)
(b)	$P(E \cup B) = \dfrac{2}{3} + \dfrac{2}{5} - \dfrac{6}{25}$ or $P(E' \mid B')$ or $P(B' \cap E)$ or $P(B \cap E')$	M1
	$\quad = \dfrac{62}{75} \quad\quad\quad\quad\quad = \dfrac{13}{25} \quad\quad = \dfrac{12}{75} \quad\quad = \dfrac{32}{75}$	A1
		M1 A1 (4)
	$P(E' \cap B') = 1 - P(E \cup B) = \dfrac{13}{75}$ or $0.17\dot{3}$	
(c)	$P(E \mid B) = 0.36 \neq 0.40 = P(E)$ or $P(E \cap B) = \dfrac{6}{25} \neq \dfrac{2}{5} \times \dfrac{2}{3} = P(E) \times P(B)$	M1 A1 (2)
	So E and B are <u>not</u> statistically independent	
		[8]

(a)	M1 for $\frac{9}{25} \times \frac{2}{3}$ or $P(E \mid B) \times P(B)$ <u>and</u> at least one correct value seen. A1 for 0.24 or exact equiv. NB $\frac{2}{5} \times \frac{2}{3}$ alone or $\frac{2}{5} \times \frac{9}{25}$ alone scores M0A0. Correct answer scores full marks.
(b)	1st M1 for use of the addition rule. Must have 3 terms and some values, can ft their (a) <u>Or</u> a full method for $P(E' \mid B')$ requires 1 - $P(E \mid B')$ and equation for $P(E \mid B')$: (a) $+ \frac{x}{3} = \frac{2}{5}$ <u>Or</u> a full method for $P(B' \cap E)$ <u>or</u> $P(B \cap E')$ [or other valid method] 2nd M1 for a method leading to answer e.g. $1 - P(E \cup B)$ <u>or</u> $P(B') \times P(E' \mid B')$ <u>or</u> $P(B') - P(B' \cap E)$ <u>or</u> $P(E') - P(B \cap E')$ <u>Venn Diagram</u> 1st M1 for diagram with attempt at $\frac{2}{5} - P(B \cap E)$ or $\frac{2}{3} - P(B \cap E)$. Can ft their (a) 1st A1 for a correct first probability as listed or 32, 18 and 12 on Venn Diagram 2nd M1 for attempting 75 - their (18 + 32 + 12)
(c)	M1 for identifying suitable values to test for independence e.g. $P(E) = 0.40$ and $P(E \mid B) = 0.36$ <u>Or</u> $P(E) \times P(B) = ...$ and $P(E \cap B) = $ their (a) [but their (a) $\neq \frac{2}{5} \times \frac{2}{3}$]. Values seen somewhere A1 for correct values and a correct comment
	 Diagrams You may see these or find these useful for identifying probabilities.

<u>**Common Errors**</u>
(a) $\frac{9}{25}$ is M0A0
(b) $P(E \cup B) = \frac{53}{75}$ scores M1A0
$1 - P(E \cup B) = \frac{22}{75}$ scores M1A0
(b) $P(B') \times P(E') = \frac{1}{3} \times \frac{3}{5}$
scores 0/4

Question Number	Scheme	Marks
3 (a)	$E(X) = 0 \times 0.4 + 1 \times 0.3 + ... + 3 \times 0.1, \quad = \quad 1$	M1, A1 (2)
(b)	$F(1.5) = \big[P(X \leq 1.5) = \big] \, P(X \leq 1), \quad = 0.4 + 0.3 = 0.7$	M1, A1 (2)
(c)	$E(X^2) = 0^2 \times 0.4 + 1^2 \times 0.3 + ... + 3^2 \times 0.1 \quad , = \quad 2$ $Var(X) = 2 - 1^2 \, , \quad = 1 \qquad (*)$	M1, A1 M1, A1cso (4)
(d)	$Var(5 - 3X) = (-3)^2 \, Var(X), \quad = 9$	M1, A1 (2)

(e)

Total	Cases	Probability
4	$(X=3) \cap (X=1)$	$0.1 \times 0.3 = 0.03$
	$(X=1) \cap (X=3)$	$0.3 \times 0.1 = 0.03$
	$(X=2) \cap (X=2)$	$0.2 \times 0.2 = 0.04$
5	$(X=3) \cap (X=2)$	$0.1 \times 0.2 = 0.02$
	$(X=2) \cap (X=3)$	$0.2 \times 0.1 = 0.02$
6	$(X=3) \cap (X=3)$	$0.1 \times 0.1 = 0.01$

B1B1B1

M1

A1

Total probability = 0.03 + 0.03 + 0.04 + 0.02 + 0.02 + 0.01 = 0.15

A1 (6)

[16]

(a) M1 for at least 3 terms seen. Correct answer only scores M1A1. Dividing by $k(\neq 1)$ is M0.

(b) M1 for $F(1.5) = P(X \leq 1)$. [**Beware**: $2 \times 0.2 + 3 \times 0.1 = 0.7$ but scores M0A0]

(c) 1st M1 for at least 2 non-zero terms seen. $E(X^2) = 2$ alone is M0. Condone calling $E(X^2) = Var(X)$.

1st A1 is for an answer of 2 or a fully correct expression.

ALT

2nd M1 for $-\mu^2$, condone 2 − 1, unless clearly 2 - . Allow $2 - \mu^2$, with = 1 even if $E(X) \neq 1$

2nd A1 for a fully correct solution with no incorrect working seen, **both** Ms required.

$\underline{\sum (x - \mu)^2 \times P(X = x)}$

1st M1 for an attempt at a full list of $(x - \mu)^2$ values and probabilities. 1st A1 if all correct

2nd M1 for at least 2 non-zero terms of $(x - \mu)^2 \times P(X = x)$ seen. 2nd A1 for $0.4 + 0.2 + 0.4 = 1$

(d) M1 for use of the correct formula. $-3^2 Var(X)$ is M0 unless the final answer is >0.

(e) Can follow through their Var(X) for M1

ALT

1st B1 for all cases listed for a total of 4 or 5 or 6 . e.g. (2,2) counted twice for a total of 4 is B0

2nd B1 for all cases listed for 2 totals }

3rd B1 for a complete list of all 6 cases }These may be highlighted in a table

<u>Using Cumulative probabilities</u>

1st B1 for one or more cumulative probabilities used e.g.2 then 2 or more or 3 then 1 or more

2nd B1 for both cumulative probabilities used. 3rd B1 for a complete list 1, 3; 2, ≥2; 3, ≥1

M1 for one correct pair of correct probabilities multiplied

1st A1 for all 6 correct probabilities listed (0.03, 0.03, 0.04, 0.02, 0.02, 0.01) needn't be added.

2nd A1 for 0.15 or exact equivalent only as the final answer.

Question Number		Scheme	Marks
4	(a)	$Q_2 = 53, \quad Q_1 = 35, \quad Q_3 = 60$	B1, B1,B1 (3)
	(b)	$Q_3 - Q_1 = 25 \Rightarrow Q_1 - 1.5 \times 25 = -2.5$ (no outlier)	M1
		$Q_3 + 1.5 \times 25 = 97.5$ (so 110 is an outlier)	A1 (2)
	(c)		M1 A1ft A1ft (3)
	(d)	$\sum y = 461, \sum y^2 = 24\ 219 \therefore S_{yy} = 24219 - \dfrac{461^2}{10} , = 2966.9$ (*)	B1, B1, B1cso (3)
	(e)	$r = \dfrac{-18.3}{\sqrt{3463.6 \times 2966.9}}$ or $\dfrac{-18.3}{3205.64...} = -0.0057$ AWRT - 0.006 or -6×10^{-3}	M1 A1 (2)
	(f)	r suggests correlation is close to zero so parent's claim is not justified	B1 (1) **[14]**
	(a)	1st B1 for median 2nd B1 for lower quartile 3rd B1 for upper quartile	
	(b)	M1 for attempt to find one limit A1 for both limits found and correct. No explicit comment about outliers needed.	
	(c)	M1 for a box and two whiskers 1st A1ft for correct position of box, median and quartiles. Follow through their values. 2nd A1ft for 17 and 77 or "their" 97.5 and *. If 110 is not an outlier then score A0 here. Penalise no gap between end of whisker and outlier. Must label outlier, needn't be with *. **Accuracy** should be within the correct square so 97 or 98 will do for 97.5	
	(d)	1st B1 for $\sum y$ N.B. $\left(\sum y\right)^2 = 212521$ and can imply this mark 2nd B1 for $\sum y^2$ or at least three correct terms of $\sum(y - \bar{y})^2$ seen. 3rd B1 for complete correct expression seen leading to 2966.9. So all 10 terms of $\sum(y - \bar{y})^2$	
	(e)	M1 for attempt at correct expression for r. Can ft their S_{yy} for M1.	

	(f)	B1 for comment <u>rejecting</u> parent's claim on basis of <u>weak or zero</u> correlation
		    Typical error is "negative correlation so comment is true" which scores B0
		    Weak negative or weak positive correlation is OK as the basis for their rejection.	

Question Number		Scheme	Marks
5	(a)	8-10 hours: width = 10.5 - 7.5 = 3 represented by 1.5cm 16-25 hours: width = 25.5 - 15.5 = 10 so represented by <u>5 cm</u> 8- 10 hours: height = fd = 18/3 = 6 represented by 3 cm 16-25 hours: height = fd = 15/10 = 1.5 represented by <u>0.75 cm</u>	B1 M1 A1 (3)
	(b)	$$Q_2 = 7.5 + \frac{(52-36)}{18} \times 3 = 10.2$$	M1 A1
		$$Q_1 = 5.5 + \frac{(26-20)}{16} \times 2 \left[= 6.25 \text{ or } 6.3\right] \text{ or } 5.5 + \frac{(26.25-20)}{16} \times 2 \ [=6.3]$$	A1
		$$Q_3 = 10.5 + \frac{(78-54)}{25} \times 5 \left[=15.3\right] \quad \text{or } 10.5 + \frac{(78.75-54)}{25} \times 5 \ [=15.45 \backslash 15.5]$$ IQR = (15.3 - 6.3) = <u>9</u>	A1 A1ft (5)
	(c)	$$\sum fx = 1333.5 \Rightarrow \overline{x} = \frac{1333.5}{104} = \qquad\qquad \text{AWRT } \underline{12.8}$$	M1 A1
		$$\sum fx^2 = 27254 \Rightarrow \sigma_x = \sqrt{\frac{27254}{104} - \overline{x}^2} = \sqrt{262.05 - \overline{x}^2} \qquad \text{AWRT } \underline{9.88}$$	M1 A1 (4)
	(d)	$$Q_3 - Q_2 \left[= 5.1\right] > Q_2 - Q_1 \left[= 3.9\right] \quad \text{or} \quad Q_2 < \overline{x}$$ So data is positively skew	B1ft dB1 (2)
	(e)	Use median and IQR, since data is skewed <u>or</u> not affected by extreme values or outliers	B1 B1 (2) **[16]**
	(a)	M1 For attempting both frequency densities $\frac{18}{3}(=6)$ and $\frac{15}{10}$, <u>and</u> $\frac{15}{10} \times$ SF , where SF $\neq 1$	
	(b)	NB Wrong class widths(2 and 9) gives $\frac{h}{1.66...} = \frac{3}{9} \rightarrow h = \frac{5}{9}$ or 0.55... and scores M1A0	
		M1 for identifying correct interval and a correct fraction e.g. $\frac{\frac{1}{2}(104)-36}{18}$. Condone 52.5 or 53	
		1st A1 for 10.2 for median. Using $(n + 1)$ allow awrt 10.3	
			<u>NB:</u>
		2nd A1 for a correct expression for either Q_1 or Q_3 (allow 26.25 and 78.75)	<u>Must see</u>
		3rd A1 for correct expressions for both Q_1 and Q_3	<u>some</u>
	(c)	4th A1ft for IQR, ft their quartiles. Using $(n + 1)$ gives 6.28 and 15.45	<u>method</u>
		1st M1 for attempting $\sum fx$ and $\overline{x}$	
		2nd M1 for attempting $\sum fx^2$ and $\sigma_x, \sqrt{}$ is needed for M1. Allow s = awrt 9.93	

	(d)	1st B1ft for suitable test, values need not be seen but statement must be compatible with
		                values used. Follow through their values
		2nd dB1 Dependent upon their test showing positive and for stating positive skew
		                 If their test shows negative skew they can score 1st B1 but lose the second	

	(e)	1st B1 for choosing median and IQR. Must mention <u>both</u>. }Award independently 2nd B1 for suitable reason } e.g. "use median because data is skewed" scores B0B1 since IQR is not mentioned	

Question Number	Scheme	Marks
6 (a)	$P(X < 39) = P\left(Z < \dfrac{39 - 30}{5} \right)$ $= P(Z < 1.8) = \underline{0.9641}$ (allow awrt 0.964)	M1 A1 (2)
(b)	$P(X < d) = P\left(Z < \dfrac{d - 30}{5} \right) = 0.1151$ $1 - 0.1151 = 0.8849$ (allow $\pm\,1.2$) $\Rightarrow \quad z = -1.2$ $\therefore \dfrac{d - 30}{5} = -1.2$ $\underline{d = 24}$	M1 B1 M1A1 (4)
(c)	$P(X > e) = 0.1151 \quad$ so $\quad e = \mu + (\mu$ - their $d)$ <u>or</u> $\dfrac{e - 30}{5} = 1.2$ or $-$ their z $\underline{e = 36}$	M1 A1 (2)
(d)	$P(d < X < e) = 1 - 2 \times 0.1151$ $= 0.7698$ AWRT $\underline{0.770}$	M1 A1 (2) **[10]**
	Answer only scores all marks in each section BUT check (b) and (c) are in correct order	
(a)	M1 for standardising with σ , $z = \pm\dfrac{39 - 30}{5}$ is OK A1 for 0.9641 or awrt 0.964 but if they go on to calculate $1 - 0.9641$ they get M1A0	
(b)	1st M1 for attempting 1- 0.1151. Must be seen in (b) in connection with finding d B1 for $z = \pm\,1.2$. They must state $z = \pm\,1.2$ or imply it is a z value by its use. This mark is only available in part (b). 2nd M1 for $\left(\dfrac{d - 30}{5} \right)$ = their negative z value (or equivalent)	

(c)	M1 for a full method to find e. If they used $z = 1.2$ in (b) they can get M1 for $z = \pm\,1.2$ here
	        If they use symmetry about the mean $\mu + (\mu$ - their $d)$ then ft their d for M1
	        Must explicitly <u>see</u> the method used unless the answer is correct.	

(d)	M1 for a complete method or use of a correct expression e.g. "their 0.8849" - 0.1151
	     <u>or</u> **If their d < their e** using their values with $P(X < e)$ - $P(X < d)$
	     If their $d \geq$ their e then they can only score from an argument like $1 - 2 \times 0.1151$
	      A negative probability or probability > 1 for part (d) scores M0A0	