

ADVANCED GCE
MATHEMATICS
Core Mathematics 3

4723

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:

None

Thursday 15 January 2009
Morning

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 Find

(i) $\int 8e^{-2x} dx,$

(ii) $\int (4x + 5)^6 dx.$

[5]

2 (i) Use Simpson's rule with four strips to find an approximation to

$$\int_4^{12} \ln x dx,$$

giving your answer correct to 2 decimal places.

[4]

(ii) Deduce an approximation to $\int_4^{12} \ln(x^{10}) dx.$

[1]

3 (i) Express $2 \tan^2 \theta - \frac{1}{\cos \theta}$ in terms of $\sec \theta.$

[3]

(ii) Hence solve, for $0^\circ < \theta < 360^\circ,$ the equation

$$2 \tan^2 \theta - \frac{1}{\cos \theta} = 4.$$

[4]

4 For each of the following curves, find $\frac{dy}{dx}$ and determine the exact x -coordinate of the stationary point:

(i) $y = (4x^2 + 1)^5,$

[3]

(ii) $y = \frac{x^2}{\ln x}.$

[4]

5 The mass, M grams, of a certain substance is increasing exponentially so that, at time t hours, the mass is given by

$$M = 40e^{kt},$$

where k is a constant. The following table shows certain values of t and $M.$

t	0	21	63
M		80	

(i) In either order,

(a) find the values missing from the table,

[3]

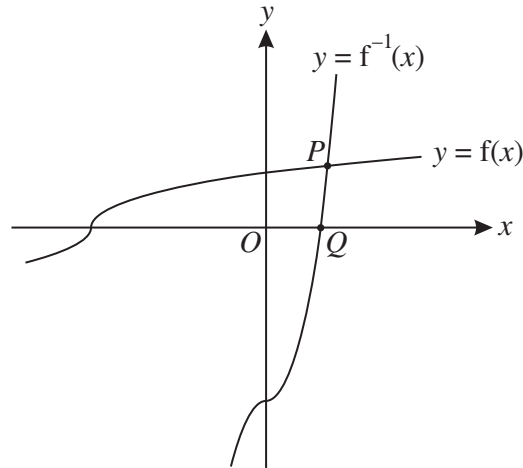
(b) determine the value of $k.$

[2]

(ii) Find the rate at which the mass is increasing when $t = 21.$

[3]

6



The function f is defined for all real values of x by

$$f(x) = \sqrt[3]{\frac{1}{2}x + 2}.$$

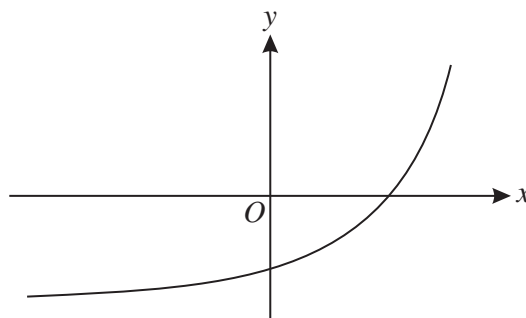
The graphs of $y = f(x)$ and $y = f^{-1}(x)$ meet at the point P , and the graph of $y = f^{-1}(x)$ meets the x -axis at Q (see diagram).

- (i) Find an expression for $f^{-1}(x)$ and determine the x -coordinate of the point Q . [3]
- (ii) State how the graphs of $y = f(x)$ and $y = f^{-1}(x)$ are related geometrically, and hence show that the x -coordinate of the point P is the root of the equation

$$x = \sqrt[3]{\frac{1}{2}x + 2}. \quad [2]$$

- (iii) Use an iterative process, based on the equation $x = \sqrt[3]{\frac{1}{2}x + 2}$, to find the x -coordinate of P , giving your answer correct to 2 decimal places. [4]

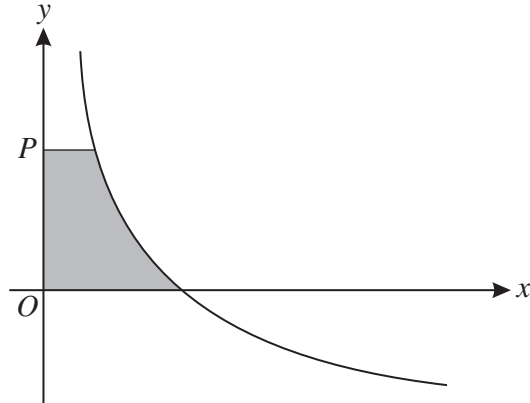
7



The diagram shows the curve $y = e^{kx} - a$, where k and a are constants.

- (i) Give details of the pair of transformations which transforms the curve $y = e^x$ to the curve $y = e^{kx} - a$. [3]
- (ii) Sketch the curve $y = |e^{kx} - a|$. [2]
- (iii) Given that the curve $y = |e^{kx} - a|$ passes through the points $(0, 13)$ and $(\ln 3, 13)$, find the values of k and a . [4]

8



The diagram shows the curve with equation

$$y = \frac{6}{\sqrt{x}} - 3.$$

The point P has coordinates $(0, p)$. The shaded region is bounded by the curve and the lines $x = 0$, $y = 0$ and $y = p$. The shaded region is rotated completely about the y -axis to form a solid of volume V .

(i) Show that $V = 16\pi \left(1 - \frac{27}{(p+3)^3} \right)$. [6]

(ii) It is given that P is moving along the y -axis in such a way that, at time t , the variables p and t are related by

$$\frac{dp}{dt} = \frac{1}{3}p + 1.$$

Find the value of $\frac{dV}{dt}$ at the instant when $p = 9$. [4]

9 (i) By first expanding $\cos(2\theta + \theta)$, prove that

$$\cos 3\theta \equiv 4 \cos^3 \theta - 3 \cos \theta. \quad [4]$$

(ii) Hence prove that

$$\cos 6\theta \equiv 32 \cos^6 \theta - 48 \cos^4 \theta + 18 \cos^2 \theta - 1. \quad [3]$$

(iii) Show that the only solutions of the equation

$$1 + \cos 6\theta = 18 \cos^2 \theta$$

are odd multiples of 90° . [5]