

ADVANCED GCE
MATHEMATICS
Further Pure Mathematics 2

4726

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:

- Scientific or graphical calculator

Thursday 27 May 2010
Morning

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

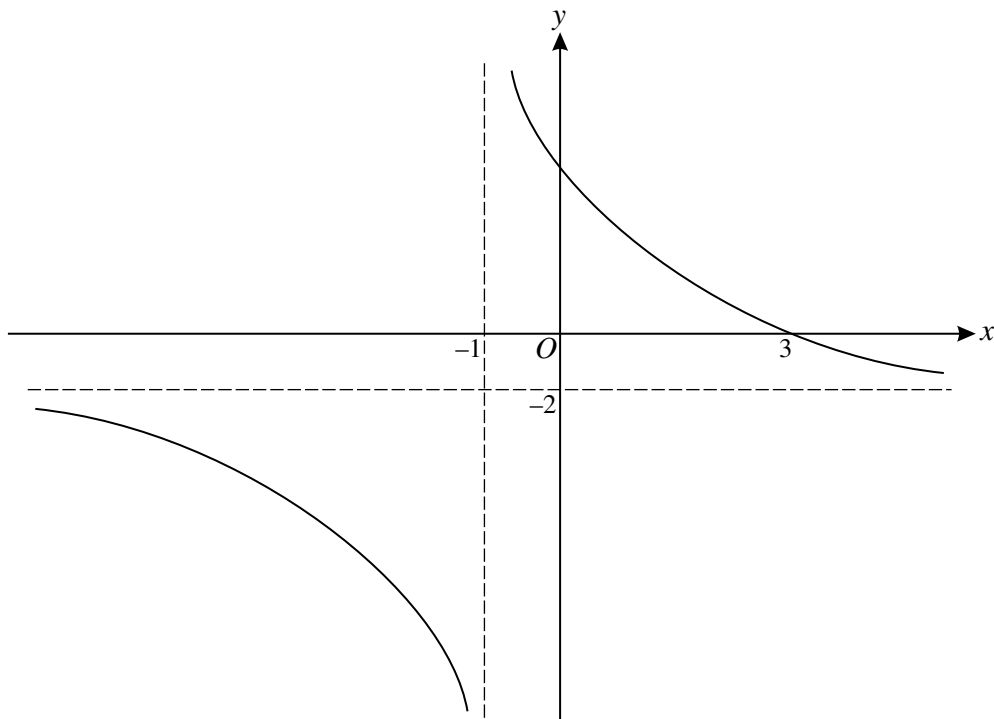
INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

- 1 It is given that $f(x) = \tan^{-1} 2x$ and $g(x) = p \tan^{-1} x$, where p is a constant. Find the value of p for which $f'(\frac{1}{2}) = g'(\frac{1}{2})$. [4]
- 2 Given that the first three terms of the Maclaurin series for $(1 + \sin x)e^{2x}$ are identical to the first three terms of the binomial series for $(1 + ax)^n$, find the values of the constants a and n . (You may use appropriate results given in the List of Formulae (MF1).) [6]
- 3 Use the substitution $t = \tan \frac{1}{2}x$ to show that

$$\int_0^{\frac{1}{3}\pi} \frac{1}{1 - \sin x} dx = 1 + \sqrt{3}. \quad [6]$$

4



The diagram shows the curve with equation

$$y = \frac{ax + b}{x + c},$$

where a , b and c are constants.

- (i) Given that the asymptotes of the curve are $x = -1$ and $y = -2$ and that the curve passes through $(3, 0)$, find the values of a , b and c . [3]
- (ii) Sketch the curve with equation

$$y^2 = \frac{ax + b}{x + c},$$

for the values of a , b and c found in part (i). State the coordinates of any points where the curve crosses the axes, and give the equations of any asymptotes. [4]

5 It is given that, for $n \geq 0$,

$$I_n = \int_0^{\frac{1}{2}} (1 - 2x)^n e^x dx.$$

(i) Prove that, for $n \geq 1$,

$$I_n = 2nI_{n-1} - 1. \quad [4]$$

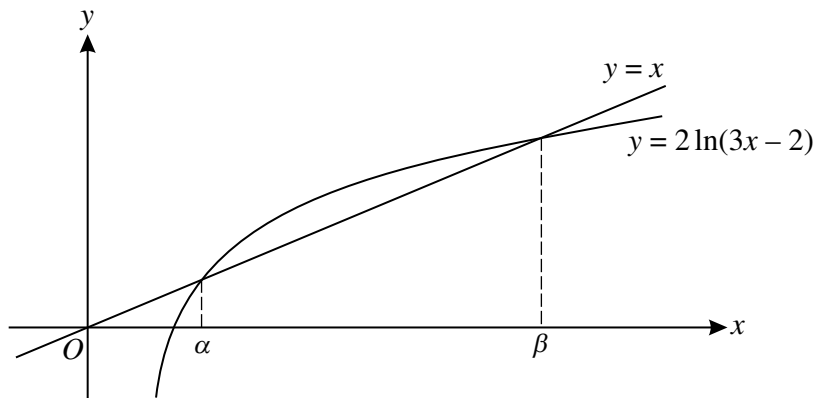
(ii) Find the exact value of I_3 . [4]

6 (i) Show that $\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{x^2 + 1}}$. [2]

(ii) Given that $y = \cosh(a \sinh^{-1} x)$, where a is a constant, show that

$$(x^2 + 1) \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - a^2 y = 0. \quad [5]$$

7



The line $y = x$ and the curve $y = 2 \ln(3x - 2)$ meet where $x = \alpha$ and $x = \beta$, as shown in the diagram.

- (i) Use the iteration $x_{n+1} = 2 \ln(3x_n - 2)$, with initial value $x_1 = 5.25$, to find the value of β correct to 2 decimal places. Show all your working. [2]
- (ii) With the help of a 'staircase' diagram, explain why this iteration will not converge to α , whatever value of x_1 (other than α) is used. [3]
- (iii) Show that the equation $x = 2 \ln(3x - 2)$ can be rewritten as $x = \frac{1}{3}(e^{\frac{1}{2}x} + 2)$. Use the Newton-Raphson method, with $f(x) = \frac{1}{3}(e^{\frac{1}{2}x} + 2) - x$ and $x_1 = 1.2$, to find α correct to 2 decimal places. Show all your working. [4]
- (iv) Given that $x_1 = \ln 36$, explain why the Newton-Raphson method would not converge to a root of $f(x) = 0$. [2]

[Questions 8 and 9 are printed overleaf.]

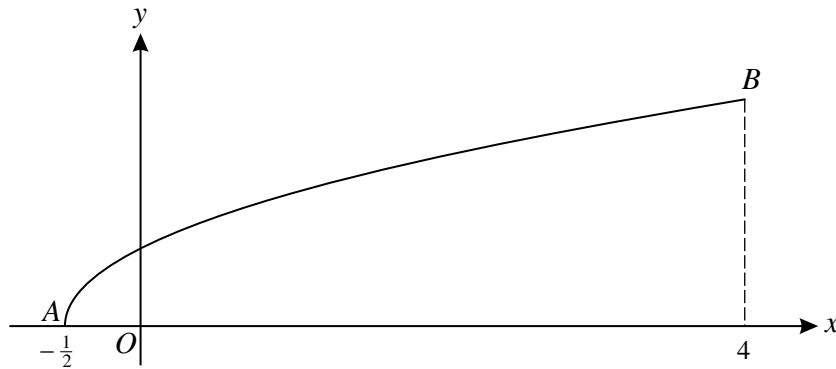
- 8 (i) Using the definition of $\cosh x$ in terms of e^x and e^{-x} , show that

$$4 \cosh^3 x - 3 \cosh x \equiv \cosh 3x. \quad [4]$$

- (ii) Use the substitution $u = \cosh x$ to find, in terms of $5^{\frac{1}{3}}$, the real root of the equation

$$20u^3 - 15u - 13 = 0. \quad [6]$$

9



The diagram shows the curve with equation $y = \sqrt{2x + 1}$ between the points $A(-\frac{1}{2}, 0)$ and $B(4, 3)$.

- (i) Find the area of the region bounded by the curve, the x -axis and the line $x = 4$. Hence find the area of the region bounded by the curve and the lines OA and OB , where O is the origin. [4]
- (ii) Show that the curve between B and A can be expressed in polar coordinates as

$$r = \frac{1}{1 - \cos \theta}, \quad \text{where } \tan^{-1}\left(\frac{3}{4}\right) \leq \theta \leq \pi. \quad [5]$$

- (iii) Deduce from parts (i) and (ii) that $\int_{\tan^{-1}(\frac{3}{4})}^{\pi} \operatorname{cosec}^4\left(\frac{1}{2}\theta\right) d\theta = 24$. [4]

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