

ADVANCED GCE
MATHEMATICS
Core Mathematics 3

4723

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:

- Scientific or graphical calculator

Wednesday 9 June 2010
Afternoon

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 Find $\frac{dy}{dx}$ in each of the following cases:

(i) $y = x^3 e^{2x}$, [2]

(ii) $y = \ln(3 + 2x^2)$, [2]

(iii) $y = \frac{x}{2x+1}$. [2]

2 The transformations R, S and T are defined as follows.

R : reflection in the x -axis

S : stretch in the x -direction with scale factor 3

T : translation in the positive x -direction by 4 units

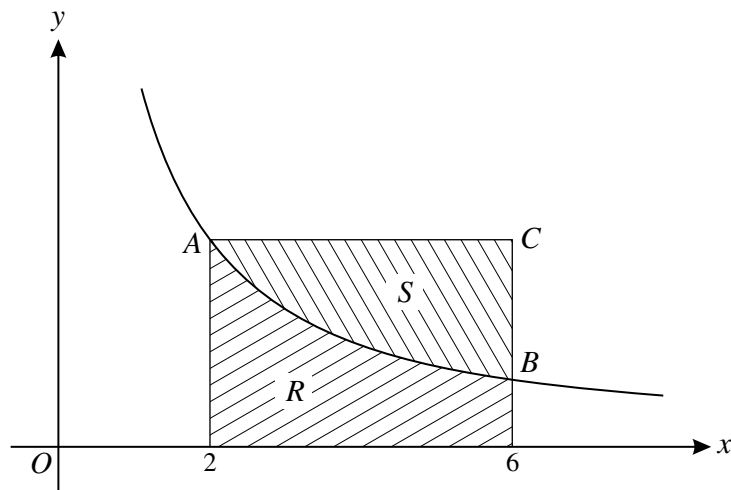
(i) The curve $y = \ln x$ is transformed by R followed by T. Find the equation of the resulting curve. [2]

(ii) Find, in terms of S and T, a sequence of transformations that transforms the curve $y = x^3$ to the curve $y = (\frac{1}{9}x - 4)^3$. You should make clear the order of the transformations. [2]

3 (i) Express the equation $\operatorname{cosec} \theta(3 \cos 2\theta + 7) + 11 = 0$ in the form $a \sin^2 \theta + b \sin \theta + c = 0$, where a , b and c are constants. [3]

(ii) Hence solve, for $-180^\circ < \theta < 180^\circ$, the equation $\operatorname{cosec} \theta(3 \cos 2\theta + 7) + 11 = 0$. [3]

4



The diagram shows part of the curve $y = \frac{k}{x}$, where k is a positive constant. The points A and B on the curve have x -coordinates 2 and 6 respectively. Lines through A and B parallel to the axes as shown meet at the point C . The region R is bounded by the curve and the lines $x = 2$, $x = 6$ and $y = 0$. The region S is bounded by the curve and the lines AC and BC . It is given that the area of the region R is $\ln 81$.

(i) Show that $k = 4$. [3]

(ii) Find the exact volume of the solid produced when the region S is rotated completely about the x -axis. [4]

5 (i) Solve the inequality $|2x + 1| \leq |x - 3|$. [5]

(ii) Given that x satisfies the inequality $|2x + 1| \leq |x - 3|$, find the greatest possible value of $|x + 2|$. [2]

6 (i) Show by calculation that the equation

$$\tan^2 x - x - 2 = 0,$$

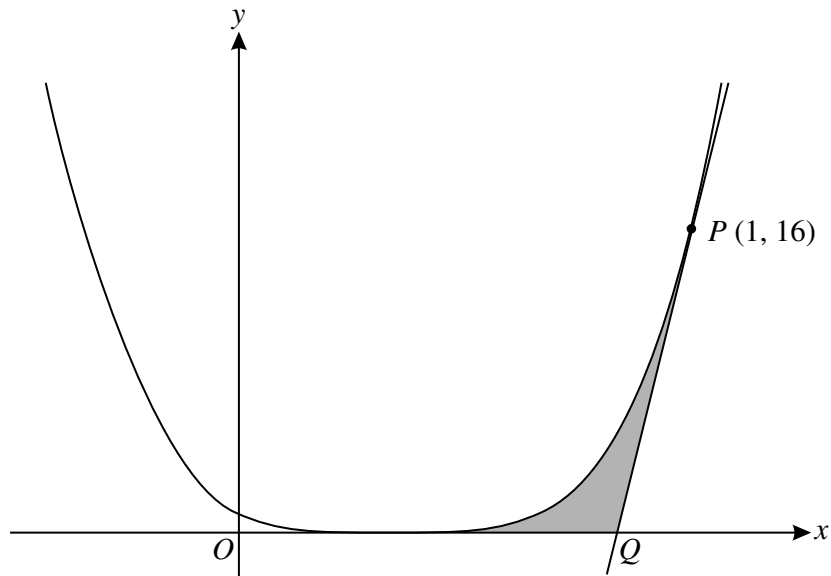
where x is measured in radians, has a root between 1.0 and 1.1. [3]

(ii) Use the iteration formula $x_{n+1} = \tan^{-1} \sqrt{2 + x_n}$ with a suitable starting value to find this root correct to 5 decimal places. You should show the outcome of each step of the process. [4]

(iii) Deduce a root of the equation

$$\sec^2 2x - 2x - 3 = 0. [3]$$

7



The diagram shows the curve with equation $y = (3x - 1)^4$. The point P on the curve has coordinates $(1, 16)$ and the tangent to the curve at P meets the x -axis at the point Q . The shaded region is bounded by PQ , the x -axis and that part of the curve for which $\frac{1}{3} \leq x \leq 1$. Find the exact area of this shaded region. [10]

8 (i) Express $3 \cos x + 3 \sin x$ in the form $R \cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. [3]

(ii) The expression $T(x)$ is defined by $T(x) = \frac{8}{3 \cos x + 3 \sin x}$.

(a) Determine a value of x for which $T(x)$ is not defined. [2]

(b) Find the smallest positive value of x satisfying $T(3x) = \frac{8}{9}\sqrt{6}$, giving your answer in an exact form. [4]

[Question 9 is printed overleaf.]

9 The functions f and g are defined for all real values of x by

$$f(x) = 4x^2 - 12x \quad \text{and} \quad g(x) = ax + b,$$

where a and b are non-zero constants.

- (i) Find the range of f . [3]
- (ii) Explain why the function f has no inverse. [2]
- (iii) Given that $g^{-1}(x) = g(x)$ for all values of x , show that $a = -1$. [4]
- (iv) Given further that $gf(x) < 5$ for all values of x , find the set of possible values of b . [4]



Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations, is given to all schools that receive assessment material and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity. For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.